Evolution dynamics of vectorial Bessel beams

Parinaz Aleahmad,[1*] Hector Moya Cessa,[1,3] Mohammad-Ali Miri,[1] Armando Perez-Leija,[1] Ido Kaminer,[2] Mordechai Segev,[2] and Demetrios N. Christodoulides[1]

[1]*CREOL,/College of Optics, University of Central Florida, Orlando Florida 32816, USA*
[2]*Physics Department and Solid State Institute, Technion-Israel Institute of Technology, Haifa 32000, Israel*
[3]*INAOE, Coordinacion de Optica, Luis Enrique Erro No.1, 72840 Toantzintla, Pue., Mexico*
Parinaz.aleahmad@creol.ucf.edu

Abstract: We investigate the acceleration dynamics of non-paraxial Bessel beams. We show that this acceleration behavior can persist even in the presence of evanescent components. Our study can be useful in plasmonic and other sub-wavelength settings.

OCIS codes: (050.1960) Diffraction theory; (070.7345) Wave propagation.

After the first suggestion and observation of Airy beams [1, 2], the propagation dynamics of other families of such accelerating diffraction-free wave packets have received a great deal of attention. In general, non-diffracting paraxial Airy beams tend to accelerate in a homogeneous environment even in the absence of any index gradient. In recent years, this exotic property has been utilized in a number of applications such as in microparticle manipulation, accelerating light bullets and supercontinuum generation, plasmonics, and in inducing curved plasma channels through filamentation. Lately a new class of non-paraxial Bessel accelerating beams has been proposed [3]. These latter wavefronts are solutions to the Helmholtz equation and they tend to follow circular trajectories with radii of only few wavelengths [3, 4]. Soon after, other 3D vectorial accelerating beams were obtained and observed [5, 6]. These come in the form of Mathieu functions (with elliptical trajectories), spherical Bessel, Weber, and prolate/oblate spheroidal functions. Given that the aforementioned beam configurations are by nature not paraxial, it is therefore important to investigate the role of evanescent fields during their evolution. Evidently, this is an intricate process and can only be resolved through an exact analysis. This is because any numerical attempt will be always limited by apodization effects needed for computation. In this study, we analytically investigate the propagation dynamics of accelerating Helmholtz Bessel wave packets. We show that for higher-order Bessel beams, the acceleration behavior still persists in spite of the presence of exponentially decaying evanescent field components. The intensity profile of these wavepackets along the propagation axis is described in closed form. These results are then generalized in three-dimensional vectorial configurations. Our study could be useful in plasmonic and other sub-wavelength settings.

We begin our analysis by considering the Helmholtz equation in two dimensions $(\nabla^2 + k^2)\{\vec{E}, \vec{H}\} = 0$ where k represents the wave number. Without any loss of generality, we here investigate the transverse-electric mode, i.e., $\vec{E} = E_y(x,z)\hat{y}$. Using Fourier transforms, we then decompose the Helmholtz solution in terms of plane waves. This leads to the diffraction integral, $E_y(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, F(\omega) e^{i\omega x} e^{iz\sqrt{k^2-\omega^2}}$, where z stands for the propagation coordinate and $F(\omega)$ represents the Fourier transform of the input electric field distribution at $z = 0$.

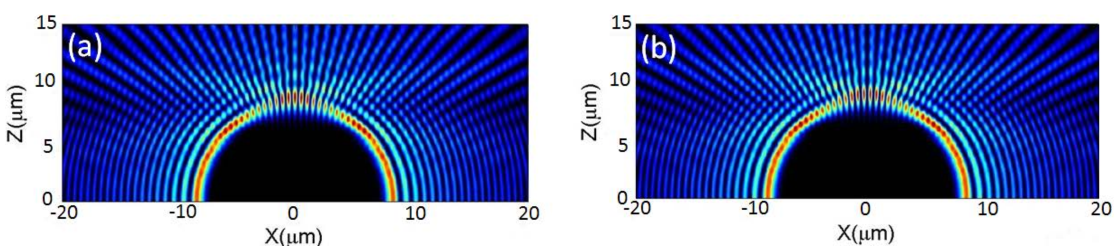

Fig. 1. (color online). Intensity profile of an (a) even-order Bessel accelerating beam when 2m=50, and (b) odd-order Bessel propagation with 2m+1=51.

To study the diffraction dynamics, we first assume that initially everywhere on the *x*-axis $E_y(x,0) = J_\nu(kx)$, where ν represents the order of the Bessel accelerating beam. In this case, we show that the propagation of these wavefronts can have a closed form solution. For example, in the case of an even Bessel function ($\nu = 2m$) this result reads:

$$E_y(x,z) = cos2m\phi\, J_{2m}(kr) + \frac{2i}{\pi}(-1)^m \sum_{n=0}^{\infty}(-1)^n J_{2n+1}(kr)\sin[(2n+1)\phi]\,\frac{4n+2}{(2n+1)^2-4m^2} \quad . \qquad (1)$$

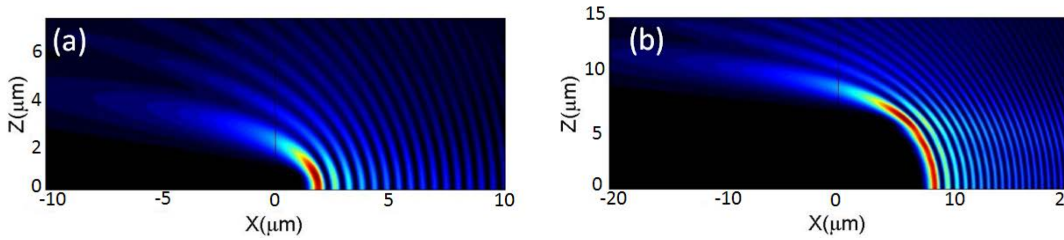

Fig. 2. (color online). Intensity profile resulting from the self-bending dynamics of an even-order Bessel beam (a) with 2m=10, and (b) 2m=50.

A similar result can also be obtained for odd order Bessel beams, i.e.,$\nu = 2m + 1$. Figures 1 (a, b) show two-dimensional plots of the resulting intensity based on these analytical solutions for of an even/odd Bessel distribution with $m = 25$. It is important to stress that this latter arrangements are free of evanescent contributions. As Fig.1 clearly shows, the left and right branches of this Bessel beam interfere and focusing occurs within a distance of few wavelengths. Meanwhile, the optical intensity within the corresponding semicircle remains zero. Note that for even order Bessel beams, the intensity on the *z*-axis is maximum whereas is zero for the odd case (because of destructive interference).

This problem is entirely different when considering only one of the branches of the accelerating Bessel beams-an arrangement very typical in experiments where beam self-bending is sought. In this case, the initial field profile is given by $E_y(x,0) = H(x)J_\nu(kx)$ where $H(x)$ represents a Heaviside step function. Interestingly, this configuration involves an evanescent part which becomes more prominent for lower values of order m. The propagation dynamics of the evanescent component can also be analytically described. For example, if $\nu = 2m$ this part behaves according to:

$$\frac{(-1)^{\mathrm{m}}}{\pi} Im\left[e^{ikx}\sqrt{\frac{i\pi}{2kx}}\,e^{\frac{i}{2kx}(zk+2m)^2}\left(1-\mathrm{erf}\left(\frac{zk+2m}{\sqrt{-2ikx}}\right)\right)\right] \quad . \quad (2)$$

Figures 2 (a, b) show the accelerating two-dimensional intensity patterns resulting from a half-Bessel Helmholtz beam when $m = 5$ and $m = 25$ respectively. These figures also show that the main lobe tends to deteriorate faster at lower orders, as it is evident from Eq. (2). In addition, the intensity features along the *z*-axis can also be described in closed form-allowing a better understanding of the behavior of such vectorial beams.

Finally, the evanescent contributions on the dynamics of other families of 3D accelerating vectorial Helmholtz beams will be also discussed in our talk. To do so, we introduce magnetic and electric vector potentials, $\boldsymbol{A}$ and $\boldsymbol{F}$, from which one can derive the electric and magnetic field components from the relations $\boldsymbol{E} = -\nabla\times\boldsymbol{F} - \frac{1}{i\omega\epsilon}\nabla\times\nabla\times\boldsymbol{A}$, and $\boldsymbol{H} = \nabla\times\boldsymbol{A} - \frac{1}{i\omega\mu}\nabla\times\nabla\times\boldsymbol{F}$. One such example could include self-bending wavefronts in spherical coordinates [6], described by spherical Bessel functions and associated Legendre polynomials. Figures 3(a, b) depict the intensity cross-section and the evolution of such a spherical mode in three dimensions.

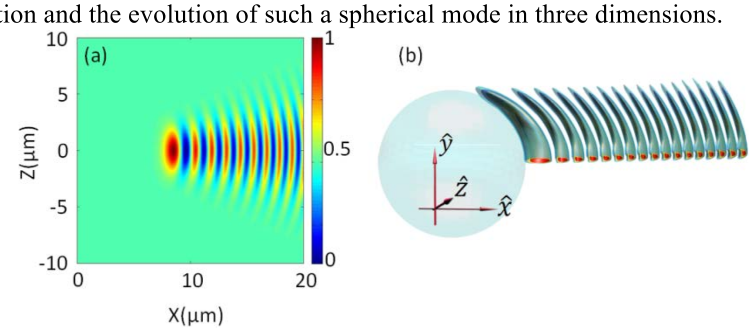

Fig. 3. (color online). (a) Intensity cross section of an accelerating spherical wavefront and (b) its propagation.

In conclusion, we have investigated the acceleration dynamics of non-paraxial Bessel beams and we have shown that their acceleration behavior can persist even in the presence of evanescent components.

This work was supported by an AFOSR (MURI Grant No. FA9550-10-1-0561), BSF, and the Israeli Minister of Defense.

References

[1] G. A. Siviloglou and D. N. Christodoulides, Opt. Lett. 32, 979 (2007).
[2] G.A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, Phys. Rev. Lett. 99, 213901 (2007).
[3] I. Kaminer, R. Bekenstein, J. Nemirovsky, and M. Segev, Phys. Rev. Lett. 108, 163901 (2012).
[4] P. Zhang, Y. Hu, D. Cannan, A. Salandrino, T. Li, R. Morandotti, X. Zhang, and Z. Chen, Opt. Lett. 37, 2820 (2012).
[5] P. Zhang, Y. Hu, T. Li, D. Cannan, X. Yin, R. Morandotti, Z. Chen, and X. Zhang, Phys. Rev. Lett. 109, 193901 (2012)
[6] P. Aleahmad, M. A. Miri, M. S. Mills, I. Kaminer, M. Segev and D. N. Christodoulides, Phys. Rev. Lett. 109, 203902 (2012)