Evolution dynamics of Helmholtz Bessel beams

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Abstract: We study the dynamics of accelerating Bessel beams and their corresponding apodized version. Presenting a closed analytical form for the intensity profile, we investigate the contribution of evanescent fields during the evolution for both cases.
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Airy beams, as the first class of paraxial non-diffracting accelerating wave packets, were suggested and experimentally observed in 2007 [1, 2]. This class of beams tends to transversely accelerate in free space in the absence of any external force while their intensity distribution remains unchanged. These features have been utilized in various applications such as microparticle manipulation, light bullet creation, and induction of curved plasma channels. However, the dependence of Airy beams to Schrödinger-type equations restricts them to the paraxial regime. In 2012, a new class of non-paraxial accelerating wave-packets has been presented by studying the solutions to the Helmholtz equation [3] and experimentally observed [4]; this type of wave-packets follows circular trajectories with radius of few wavelengths. Recently, other class of non-paraxial accelerating wave-packets, following elliptical (Mathieu beams) and parabolic (Weber beams) trajectories have also been suggested and observed [5, 6]. Moreover, 3D fully vectorial wave-packets in the form of spherical Bessel and oblate/prolate spheroidal functions have been investigated theoretically [6]. It is important to emphasize that in every diffraction-free arrangement, particularly in the non-paraxial regime, evanescent field components can play a crucial role during the evolution. In principle, however, these configurations possess an infinite norm, and as such have to be truncated to be observed experimentally – this may influence the contribution of evanescent filed. As a result, the dynamical evolutions of such accelerating distributions can only be fully understood through an exact analysis. In this study, we present closed form analytical solutions describing the evolution dynamics of both apodized and non-apodized Bessel wavepackets and show this acceleration behavior can persist even in the presence of evanescent components.

We begin our analysis by considering the Helmholtz equation in two dimension $(\nabla^2 + k^2)\{\vec{E},\vec{H}\} = 0$, where k represents the wave number. Without any loss of generality, we can choose the transverse-electric mode, i.e., $\vec{E} = E_y(x,z)\hat{y}$. Using the Fourier transform to decompose the Helmholtz equation in plane waves, we get the familiar diffraction integral as the form $E_y(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, F(\omega) e^{i\omega x} e^{iz\sqrt{k^2-\omega^2}}$, where z is the propagation direction and $F(\omega)$ represents the Fourier transform of the electric field at $z = 0$.

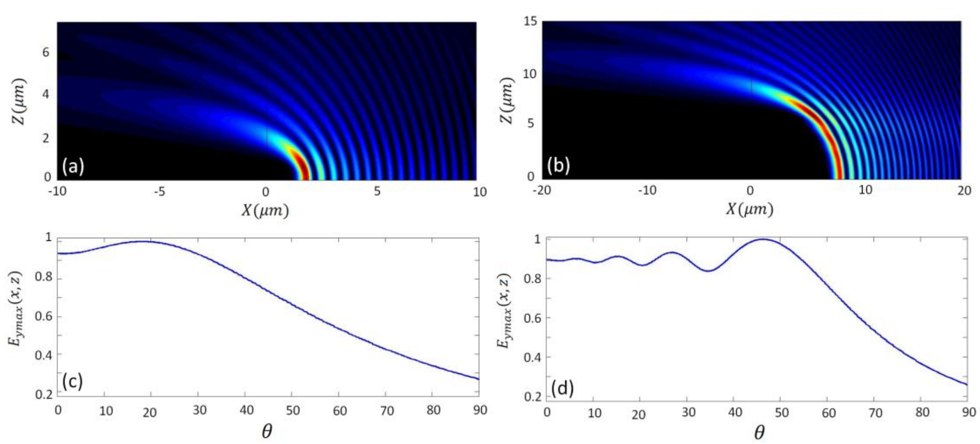

Fig. 1. (color online). Intensity profile resulting from the self-bending dynamics of a Bessel beam (a) with m=5 and (b) m=25. (c, d) Normalized intensity variation of the first lobe with respect to the transverse propagation angle corresponding to the patterns in (a, b).

We first study the dynamical evolution of the "half" Bessel wave-packet – an arrangement very typical in the experiments – where the initial symmetries are broken using a Heaviside step function, $H(x)$; i.e. $E_y(x,0) = H(x)J_\nu(kx)$. In this case, we show that the propagation of these wavefronts have a closed form solution. For example, for $= 2m$, the evolution can be described in the form:

$$E_y(x,z) = \frac{1}{2}J_{2m}(kr)e^{2im\phi} + \frac{i}{\pi}(-1)^m \sum_{n=0}^{\infty}(-1)^n J_{2n+1}(kr)\left[\sin(2n+1)\phi \frac{4n+2}{(2n+1)^2-4m^2} - i\cos(2n+1)\phi \frac{4m}{(2n+1)^2-4m^2}\right] + \frac{(-1)^{\mathrm{m}}}{\pi} Im\left[e^{ikx}\sqrt{\frac{i\pi}{2kx}}\, e^{\frac{i}{2kx}(zk+2m)^2}\left(1-\mathrm{erf}\left(\frac{zk+2m}{\sqrt{-2ikx}}\right)\right)\right]. \quad (1)$$

A similar analytical result can be achieved for propagation of an odd order Bessel function; i.e. $E_y(x,0) = H(x)J_{2m+1}(kx)$. Figures 1 (a, b) show the two-dimensional intensity pattern for the accelerating half even-Bessel wave packets when $m = 5$ and $m = 25$, respectively. One can see from Fig. 1 that for higher orders the beam-envelope travels a longer path and the evanescent part becomes negligible after a few wavelengths of propagation. This effect is authenticated in Eq. 1 where the argument of the error function, the term responsible for evanescent part, increases with the order of the Bessel. Figs. 1 (c, d) plot the intensity variation of the main lobe with respect to the angle of propagation for $m = 5$ and $m = 25$, respectively. As it is clear from these figures, the higher order Bessel profiles experience more oscillations during propagation as the evanescent field contribution fades out faster.

This problem becomes entirely different when an apodized version of this "half" Bessel wave-packet is considered. Here, we choose to truncate the initial electric field by a factor of x^{-1}; i.e. $E_y(x,0) = H(x)J_\nu(kx)/x$. An analytical description similar to the previous case has been reached. Figs. 2 (a, b) show the propagation dynamics of an apodized Bessel wavepackets for $\nu = 10$ and $\nu = 50$, respectively. The corresponding normalized intensity variation during the propagation of the first lobe is illustrated in Figs. 2(c, d). In contrast to the unapodized version, we see that the intensity of the main lobe fades out faster. Furthermore, the oscillations seen in Fig. 1(c, d) vanish because of the apodization.

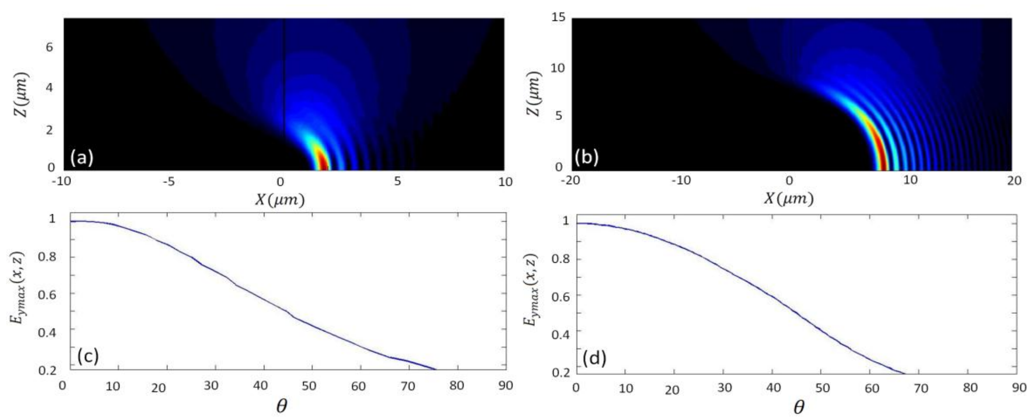

Fig. 2. (color online). Intensity profile resulting from the self-bending dynamics of an apodized Bessel beam (a) with $\nu = 10$ and (b) $\nu = 50$. (c, d) Normalized intensity variation of the first lobe with respect to the transverse propagation angle corresponding to the patterns in (a, b).

In conclusion, we have investigated the acceleration dynamics of both unapodized and apodized non-paraxial Bessel beams, and we have demonstrated that their acceleration behavior can persist even in the presence of evanescent components.

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