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Analytical model for coherent perfect absorption in one-dimensional photonic structures

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Coherent perfect absorption (CPA) is the phenomenon where a *linear* system with low intrinsic loss strongly absorbs two incident beams but only weakly absorbs either beam when incident separately. We present an analytical model that captures the relevant physics of CPA in one-dimensional photonic structures. This model elucidates an absorption-mediated interference effect that underlies CPA—an effect that is normally forbidden in Hermitian systems but is allowed when conservation of energy is violated due to the inclusion of loss. By studying a planar cavity model, we identify the optimal mirror reflectivity to guarantee CPA in the cavity at resonances extending in principle over any desired bandwidth. As a concrete example, we design a resonator that produces CPA in a 1-μm-thick layer of silicon over a 200-nm bandwidth in the near-infrared.

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Absorption plays a crucial role in many optical applications—sometimes it is desirable to minimize it as in optical fibers, or to enhance it as in solar cells. New schemes exploiting useful consequences of controlling the spatial distribution of optical losses [1–3], or in combination with judiciously placed optical gain [4,5], are now emerging as part of the burgeoning effort on non-Hermitian photonics [6]. To increase absorption in an optical system, a material introducing heavy losses may be inserted. However, there are critical scenarios where this direct approach cannot be implemented. Cost or design considerations may allow for only a thin layer of the lossy material to be included, as in thin-film solar cells. Alternatively, in some arrangements the overall absorption does not increase even when more loss is incorporated, as is the case in certain interferometers.

Coherent perfect absorption (CPA) [7] is a new optical scheme that produces high absorption in systems that have low intrinsic losses [8,9]. By interfering two beams in a lossy material—typically contained in a multipass interferometer such as a Fabry–Perot (FP) resonator—increased absorption is observed with respect to that experienced by each beam separately. The effect appears counterintuitive: while a *single* beam is *weakly* absorbed [Fig. 1(a)], adding a second beam results in *both beams* being *completely* absorbed [Fig. 1(b)]. This *linear* phenomenon appears to contradict the accepted dictum that absorption is anathema to interference.

In this Letter, we present an analytical model for CPA in one-dimensional (1D) photonic structures [10–12]. From this perspective, we find that CPA is an absorption-mediated interferometric effect in systems where conservation of energy is violated by including loss. Traditionally, it is thought that losses reduce interferometric visibility. In contrast, absorption in CPA *produces* an interferometric effect that is normally forbidden in lossless structures. Our analytical model, beside its conceptual clarity, provides the basis for optimizing the structure parameters by establishing the general criteria for maximizing CPA. We apply our analysis to a FP resonator consisting of a thin absorbing layer between two symmetric mirrors. We find that the optimal mirror reflectivity depends on the intrinsic single-pass absorption in the layer, which implies that mirrors with wavelength-dependent reflectivity are required for any real material over an appreciable bandwidth. Bragg mirrors, or other mirrors with a flat spectral reflection band, are thus *not* useful candidates to help achieve CPA over large bandwidths. We apply this model to a 1-μm-thick silicon film sandwiched between appropriately designed aperiodic multilayer dielectric mirrors to achieve CPA in the near-infrared where silicon's single-pass absorption is only a few percent.

We first present a simple example to illustrate how *adding* a beam to a *linear* passive optical system may *increase* the overall absorption. In Fig. 1(c) we depict a balanced Mach–Zehnder interferometer in which a dielectric layer with attenuation factor γ is placed in one arm. For symmetric 50/50 beam splitters, the sum of the two interferometer outputs is $P_t = P_1 + P_2 = \frac{1}{2}\{1+\gamma\}$, in which case $P_t \to \frac{1}{2}$ when $\gamma \to 0$. That is, total absorption cannot be achieved no matter how high the inserted loss is. Nevertheless, interference may be exploited here to overcome this limitation. If instead of directing the beam to a single port at the interferometer entrance, we divide the input beam between the two entrance ports with an appropriate relative phase [Fig. 1(d)], then both beams can be directed together to one arm, leading either to complete extinction [$P_t \to 0$, Fig. 1(d)-ii] or complete transparency [zero

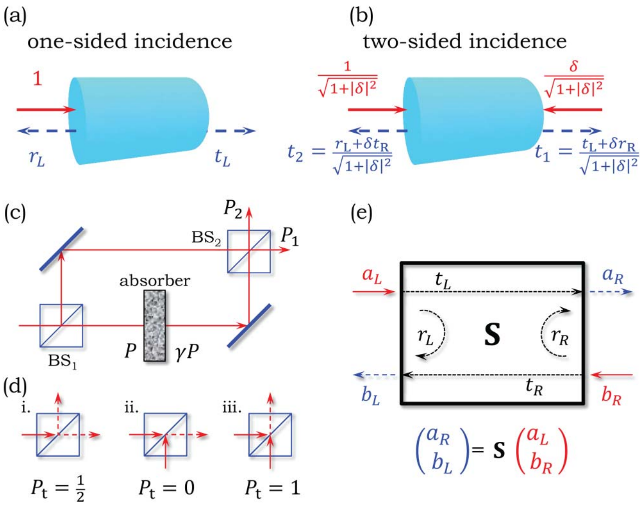

Fig. 1. (a) One-sided and (b) two-sided incidence schemes. (c) A balanced Mach–Zehnder interferometer (formed of two symmetric beam splitters BS_1 and BS_2) containing an absorber having an attenuation factor γ. (d) Three configurations for the beams at the interferometer input beam splitter BS_1 that enable control over the path to which the beam is directed to. (e) A generic two-port optical system described by a scattering matrix $\boldsymbol{S}$. The red solid arrows are the inputs (a_L and b_R) and the dashed blue arrows the outputs (b_L and a_R).

absorption, $P_t = 1$, Fig. 1(d)-iii]. This example highlights that interference *outside* the lossy medium may indeed help increase absorption in a linear structure by directing the incident energy solely to the lossy channel. We show below that CPA is related to this effect when a low-loss material is placed in a multipass configuration.

To determine the general criteria for a lossy system to achieve CPA, we consider a generic 1D optical configuration [Fig. 1(e)] described by a 2×2 scattering matrix $\boldsymbol{S} = \begin{pmatrix} t_L & r_R \\ r_L & t_R \end{pmatrix}$. Here t_L, r_L, t_R, and r_R are the field transmission and reflection coefficients for left (L) or right (R) incidence. We assume the left and right ambient media are the same for simplicity. As a starting point, we consider a lossless or Hermitian system, where the conservation of energy implies that

$$T_L + R_L = 1, T_R + R_R = 1, \Re\{(t_L^* r_R + r_L^* t_R)e^{i\phi}\} = 0, \quad \textbf{(1)}$$

where $T_L = |t_L|^2$, $R_L = |r_L|^2$, $T_R = |t_R|^2$, $R_R = |r_R|^2$, $\Re\{.\}$ denotes taking the real part, and ϕ is an arbitrary relative phase between the left- and right-incident fields. The last condition in Eq. (1) implies that $R_L = R_R$, $T_L = T_R$, and $\theta_L + \theta_R = \pi$, where θ_L and θ_R are the phase differences between the reflection and transmission coefficients for left- and right-incidence, respectively. If fields are incident from the left and right with relative complex amplitude $\delta = |\delta|e^{i\phi}$, a configuration we term hereon *two-sided incidence* [Fig. 1(b)], then the *normalized outgoing fields* to the right and left are $t_1 = \frac{t_L + \delta r_R}{\sqrt{1+|\delta|^2}}$ and $t_2 = \frac{r_L + \delta t_R}{\sqrt{1+|\delta|^2}}$, respectively, with $T_1 = |t_1|^2$ and $T_2 = |t_2|^2$. Finally, if the system is symmetric, i.e., the left and right may be seamlessly interchanged, then $\boldsymbol{S} = e^{i\beta}\begin{pmatrix} t & ir \\ ir & t \end{pmatrix}$ and $r^2 + t^2 = 1$, where r and t are real and β is an overall phase. That is, in a lossless symmetric system the reflection and transmission coefficients are in quadrature, $\Re\{t_L^* r_L\} = 0$.

We now proceed to a consideration of *lossy* systems, where the conditions in Eq. (1) are no longer satisfied. First, since $T_L + R_L \leq 1$ and $T_R + R_R \leq 1$, we define left and right absorptivities $\mathcal{A}_L = 1 - \{T_L + R_L\}$ and $\mathcal{A}_R = 1 - \{T_R + R_R\}$, respectively (in general, $\mathcal{A}_L \neq \mathcal{A}_R$). That is, $\mathcal{A}_L$ and $\mathcal{A}_R$ are the fractions of light absorbed upon left *or* right incidence, respectively, which we term one-sided absorption. Furthermore, we *no longer* have $\Re\{(t_L^* r_R + r_L^* t_R)e^{i\phi}\} = 0$, and in the case of a symmetric system $\Re\{t_L^* r_L\} \neq 0$. This feature deserves special attention since it is essential for CPA. In a lossless symmetric system, the relative phase between the reflection and transmission coefficients is not arbitrary: it is constrained to $\pm\frac{\pi}{2}$. As such, the reflected and transmitted beams *cannot* interfere. This phase constraint is lifted once loss is included, and the reflected and transmitted beams can now interfere.

For *two*-sided incidence on a lossy system, due to interference, the total absorption $\mathcal{A}_t = 1 - \{T_1 + T_2\}$ may *not* be equal to the weighted sum of $\mathcal{A}_L$ and $\mathcal{A}_R$. For a symmetric system ($\mathcal{A}_L = \mathcal{A}_R$), it is straightforward to show that

$$\mathcal{A}_t = \mathcal{A}_L - \frac{4|\delta|}{1+|\delta|^2}\Re\{t_L^* r_L\}\cos\phi. \quad \textbf{(2)}$$

This equation indicates that *two*-sided absorption $\mathcal{A}_t$ may be larger or smaller than that expected from *one*-sided absorption $\mathcal{A}_L$—according to the relative phase ϕ of the two beams and the phase θ of the *interference* term $\Re\{t_L^* r_L\}$. This term is normally equal to zero in lossless systems, but may become nonzero when loss is introduced. This *absorption-mediated interference* effect, the second term in Eq. (2), is what enables CPA. It is important to note that the interference occurs between the fields *outside* the system, just as in the Mach–Zehnder example in Fig. 1(c), with external destructive interference associated with enhanced absorption within it. The normally uncoupled fields in the Hermitian case $\Re\{t_L^* r_L\} = 0$ are now coupled through the mediation of the introduced loss, whereupon $\Re\{t_L^* r_L\} \neq 0$.

We can now determine the general criteria for achieving maximal CPA, $\mathcal{A}_t = 1$. From Eq. (2), such a goal requires simultaneously satisfying the following conditions:

$$(\text{I})|\delta| = 1, \ (\text{II}) \cos\theta\cos\phi = -1, \ (\text{III})|r_L| = |t_L|; \quad \textbf{(3)}$$

$|\delta|$ and ϕ are set by the incident fields, while θ, $|r_L|$, and $|t_L|$ are determined by the system characteristics. These conditions correspond in fact to one of the eigenvalues of $\boldsymbol{S}$ being zero, signifying a "dark" eigenstate that is completely absorbed by the system [7]. Condition (II) requires that $(\theta, \phi) = (0, \pi)$ *or* $(\theta, \phi) = (\pi, 0)$. Condition (III) indicates that a strongly reflecting or transmitting system is not optimal. Instead, we need to arrange for equal reflection and transmission coefficients.

It is crucial to appreciate that the above analysis is independent of any details of the 1D optical system. The conditions in Eq. (3) provide a general recipe for constructing a device demonstrating CPA. The typical scenario envisioned is to start from a material or structure that exhibits low intrinsic loss and to then construct around it a *lossless* system that enables CPA. Equation (3) may then be used to optimize the CPA effect and reach $\mathcal{A}_t = 1$. Furthermore, maintaining $|r_L| = |t_L|$ over a large bandwidth is challenging since the absorption of all materials is wavelength-dependent. We thus anticipate that the characteristics of the lossless systems sandwiching the lossy layer must also be wavelength-dependent.

We now analyze a specific model system. Figure 2(a) depicts a FP resonator consisting of two mirrors M_1 and M_2 that sandwich a lossy dielectric layer of thickness d and *nondispersive* complex refractive index $n + in'$. The mirrors are identical, lossless, but *not* necessarily symmetric; each has the scattering matrix $\boldsymbol{S}_M = \begin{pmatrix} te^{i\beta} & -re^{i(2\beta-\alpha)} \\ re^{i\alpha} & te^{i\beta} \end{pmatrix}$, assuming the mirror is flanked on one side with vacuum and the other by the absorptive medium, $r^2 + t^2 = 1$, α and β are the left transmission and reflection phases, respectively. The resonator transmission and reflection coefficients are

$$t_{\rm L} = (1 - R)e^{i(2\beta-\alpha)} \frac{e^{i(kd+\alpha)}}{e^{k'd} - Re^{-k'd}e^{i(2kd+2\alpha)}},$$
$$r_{\rm L} = -\sqrt{R}e^{i(2\beta-\alpha)} \frac{e^{k'd} - e^{-k'd}e^{i(2kd+2\alpha)}}{e^{k'd} - Re^{-k'd}e^{i(2kd+2\alpha)}}, \tag{4}$$

and the one-sided absorption $\mathcal{A}_{\rm L}$ is

$$\mathcal{A}_{\rm L} = T_{\rm L}\left\{\cosh 2\,k'd + \frac{1+R}{1-R}\sinh 2\,k'd - 1\right\}, \tag{5}$$

where $R = |r|^2$, $k = n\frac{\omega}{c}$, $k' = n'\frac{\omega}{c}$, ω is the angular frequency, c is the speed of light in vacuum, and the resonance order m corresponds to the round-trip phase $2kd + 2\alpha$ being equal to $2\pi m$. We plot $T_{\rm L}$ and $R_{\rm L}$ against normalized frequency for such a cavity in Fig. 2(b), and plot $\mathcal{A}_{\rm L}$ in Fig. 2(d), which shows that absorption occurs on resonance where $T_{\rm L}$ is high, per Eq. (5). For future reference, we introduce the quantity $\mathcal{A} = 1 - e^{-2\,k'd}$, which is the single-pass absorption in the thin layer when *not* contained within the FP resonator—or the intrinsic absorption in the film.

Increasing the intrinsic losses $k'd$ does *not* necessarily increase $\mathcal{A}_{\rm L}$. Surprisingly, *increasing* n' indefinitely *decreases* absorption. The reason is that light transmitted through M_1 in this case fails to reach M_2, thereby disrupting the FP interference. Consequently, the reflected fraction $R_{\rm L} \approx R$ of light from M_1 remains undiminished and $\mathcal{A}_{\rm L} \to 1 - R \approx 0$ for large R. This result suggests the following question: for a layer of a lossy material having a given value of $k'd$ (especially $k'd \ll 1$), what is the *maximum* one-sided absorption $\mathcal{A}_{\rm L}^{(m)}$ that can be achieved by surrounding this layer with symmetric mirrors? It can be shown that $\mathcal{A}_{\rm L}^{(m)} = \frac{1}{2}\cosh^2 k'd$ on resonance when mirrors having dispersive reflectivity $R^{(1)} = \frac{3-e^{2\,k'd}}{3-e^{-2\,k'd}}$ are used—a condition that applies only when $e^{2\,k'd} < 3$ [Fig. 3(a)] and which optimizes the interplay between absorption and FP interference. Therefore, even in the limit $k'd \ll 1$, one may still achieve *at least* 50% absorption on resonance. As $k'd$ *increases*, the mirror reflectivity required to optimize one-sided absorption *decreases*. When $k'd$ increases to reach $e^{2\,k'd} = 3$, the amplitude of the sum of all FP round-trip amplitudes $\frac{1}{1-Re^{-2\,k'd}} = 1$, such that $R_{\rm L} = 0$, $T_{\rm L} = e^{-2\,k'd}$, and $\mathcal{A}_{\rm L}^{(m)} = \mathcal{A}$; the FP resonator no longer offers any enhancement. When $e^{2\,k'd} > 3$, the optimal one-sided absorption remains $\mathcal{A}_{\rm L}^{(m)} = 1 - e^{-2\,k'd}$, which is achieved with $R^{(1)} = 0$; that is, adding the mirrors does not improve the absorption above that of a single pass. In other words, it is impossible to improve the absorption in a thin layer by placing it in a cavity formed of *symmetric* reflectors if the intrinsic absorption in the layer is equal to or exceeds 66.7%.

Consider now *two*-sided incidence [Fig. 2(a)-ii; $|\delta| = 1$] where the total absorption $\mathcal{A}_t$ [Eq. (2)] is the sum of $\mathcal{A}_{\rm L}$ [Eq. (5)] and an absorption-mediated interference term,

$$-2\Re\{t_{\rm L}^* r_{\rm L}\} = \frac{4\sqrt{R}}{1-R}(-1)^m T_{\rm L}\sinh k'd, \tag{6}$$

where $m = 1, 2, \ldots$ is the resonance order, resulting in either even-m or odd-m resonances having enhanced absorption [Fig. 2(e)]. It is straightforward to show that maximal CPA [Eq. (3)] is achieved on resonance when $R^{(2)} = e^{-2\,k'd}$. However, this choice for $R^{(2)}$ does *not* maximize $\mathcal{A}_{\rm L} = 1 - \frac{1}{2}\,\mathrm{sech}^2\,k'd \le \mathcal{A}_{\rm L}^{(m)}$ for a given $k'd$, as shown in Fig. 3(b), but it does ensure that $\mathcal{A}_t = 1$. Indeed, $\mathcal{A}_{\rm L}$ may even be less than the single-pass absorption $\mathcal{A}$. Note that it is *always* possible

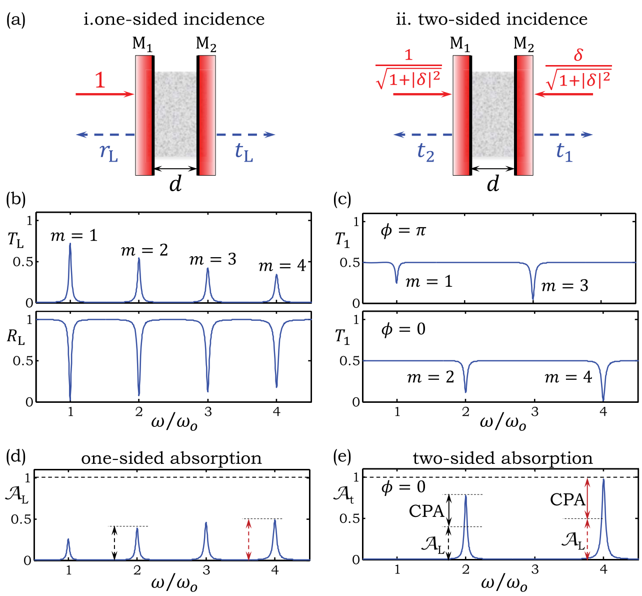

Fig. 2. (a) Schematic of a symmetric FP cavity formed of mirrors M_1 and M_2 sandwiching a lossy dielectric. (b) One-sided transmission $T_{\rm L}$ and reflection $R_{\rm L}$ coefficients against normalized frequency showing the first four resonances; $\omega_{\rm o} = \frac{\pi c}{nd}$, m is the resonance order, $n' = 0.003$, and $R = 0.9$. (c) Two-sided transmission T_1, with $|\delta| = 1$. Top and bottom panels show T_1 for $\phi = \pi$ (achieving CPA for odd-m resonances) and $\phi = 0$ (for even-m resonances), respectively. (d) One-sided absorption. (e) Two-sided absorption.

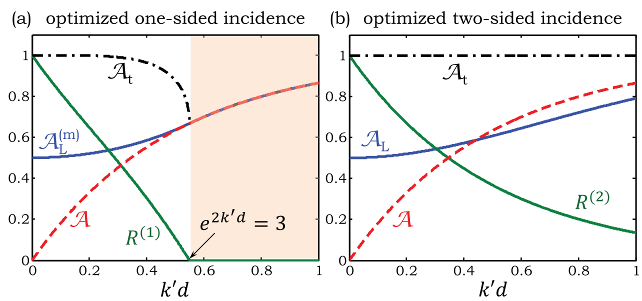

Fig. 3. (a) The maximal one-sided absorption $\mathcal{A}_{\rm L}^{(m)}$ and corresponding optimal mirror reflectivity $R^{(1)}$ for a given value of $k'd$. The dashed red curve $\mathcal{A}$ corresponds to the single-pass absorption $\mathcal{A} = 1 - e^{-2\,k'd}$, and the dashed-dotted curve is the two-sided absorption $\mathcal{A}_t$ achieved if these mirrors having reflectivity $R^{(1)}$ are used. The shaded region corresponds to $e^{2\,k'd} > 3$, a regime in which it is optimal to set $R^{(1)} = 0$, resulting in $\mathcal{A}_{\rm L}^{(m)} = \mathcal{A}_t = \mathcal{A}$. (b) The maximal two-sided absorption $\mathcal{A}_t = 1$ (CPA), the optimal mirror reflectivity $R^{(2)}$ for a given value of $k'd$, and the corresponding $\mathcal{A}_{\rm L}$.

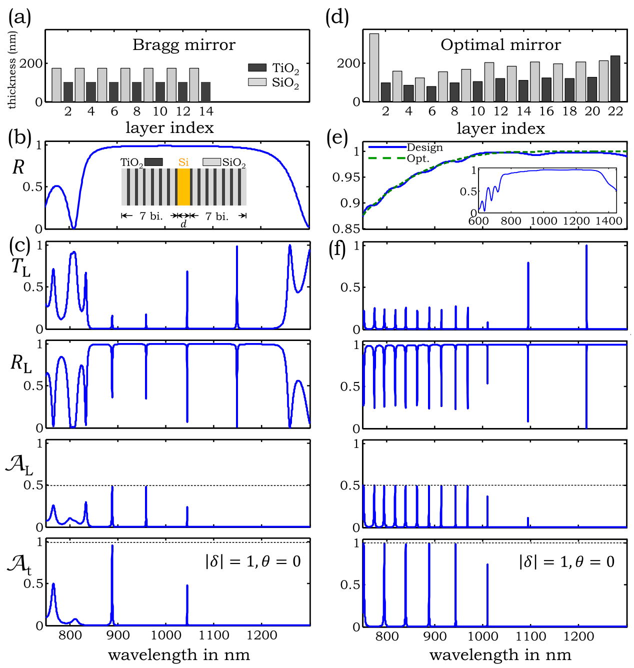

Fig. 4. (a) Layers of a seven-bilayer Bragg mirror with central wavelength 1 μm and (b) its reflection. Inset shows the overall target structure; $d = 1$ μm. (c) The four panels from top to bottom correspond to the one-sided transmission T_L, reflection R_L, absorption A_L, and two-sided absorption $\mathcal{A}_t$. (d) Layers of a mirror designed to optimize CPA in Si in the near-infrared. (e) Reflection of the mirror in (d), compared to the ideal design, assuming incidence from air from the left and a Si substrate on the right. Inset shows R over a wider spectral range. (f) Same as in (c) but using the mirror in (d).

to enhance absorption in the two-sided incidence configuration regardless of $k'd$. Finally, for $k'd \ll 1$, $R^{(1)} \approx R^{(2)} \approx 1\text{–}2\,k'd$, so that the same mirror simultaneously optimizes $\mathcal{A}_L$ and $\mathcal{A}_t$.

We now proceed to a specific realistic example of a dispersive medium and elucidate the impact of wavelength-dependent absorption. We examine a detailed model consisting of two symmetric multilayer mirrors sandwiching a layer of silicon (Si) and take into account the dispersion in both R and n'. Si has high optical absorption in the visible, which drops rapidly in the infrared. We consider here whether we can achieve CPA in a 1-μm-thick layer of Si in the vicinity of $\lambda = 1$ μm where Si is only weakly absorbing and the single-pass absorption is $\mathcal{A} = 0.0064$ (using the parameters for Si in [13]).

We first sandwich the thin Si layer between two Bragg mirrors [Fig. 4(b), inset] each formed of seven bilayers of SiO_2 and TiO_2 with refractive indices of 1.45 and 2.46 and thicknesses 172.4 and 101.6 nm, respectively [Fig. 4(a)], chosen such that the center of the bandgap is at $\lambda = 1$ μm [Fig. 4(b)] and the total device thickness is <5 μm. In Fig. 4(c) we plot the spectral dependence of four relevant quantities for the full structure: the transmission T_L, reflection R_L, and absorption $\mathcal{A}_L$ for incidence from the left, and the two-sided absorption $\mathcal{A}_t$ for two beams with $\delta = 1$. Since R is approximately equal at all resonances but $k'd$ drops rapidly in Si at longer wavelengths, $\mathcal{A}_L < 0.5$ and $\mathcal{A}_t < 1$.

The strategy to remedy this situation is clear: replace the Bragg mirrors with others whose reflection is low at the shorter wavelengths (where absorption in Si is higher) and high at longer wavelengths (where absorption in Si drops). To optimize CPA in Si, a mirror with dispersive reflection $R = e^{-2\,k'd}$ is required to replace the Bragg mirror. A design for such a mirror (obtained using the package FilmStar, FTG Software) is shown in Fig. 4(d) consisting of 22 alternating layers of SiO_2 and TiO_2. This mirror's reflection R, assuming left-incidence from air and a Si substrate on the right, is shown in Fig. 4(e), compared to the ideal target $R = e^{-2\,k'd}$. By sandwiching a 1-μm-thick Si film between two such mirrors symmetrically (total device thickness is <7.9 μm), we now obtain a structure in which complete CPA is achieved within the spectral range where the designed R approaches the ideal target. We plot in Fig. 4(f) T_L, R_L, $\mathcal{A}_L$, and $\mathcal{A}_t$ for this structure, and we confirm that CPA is indeed achieved within the spectral range 750–1000 nm. The number and locations of the resonances are determined by the thickness and refractive index of the Si layer and the mirror spectral phase, which is markedly different from that of a Bragg mirror.

In conclusion, we have presented a general model for 1D photonic structures that establishes the criteria for achieving maximal CPA. On this understanding, CPA is the enhancement in absorption driven by an absorption-mediated interference effect. Using this recipe, we have demonstrated that one may optimize near-infrared CPA in a thin Si film placed in a planar cavity. CPA is achieved in this lossy Fabry–Perot cavity model *only* at its resonance frequencies. *Broadband* CPA, which would have important applications in solar energy for example, cannot be accomplished with these means, and further research is necessary to identify suitable mechanisms and structures for its demonstration. One potential avenue is the use of so-called white-light cavities that are designed to broaden the resonance linewidths [14]. Combining such an approach with our model promises to deliver CPA over a continuous spectrum.

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