Performance Comparison of Grating-Assisted Integrated Photonic Delay Lines

Seyfollah Toroghi, *Member, OSA*, Chris Fisher, *Student Member, OSA*, Saeed Khan,
and Sasan Fathpour, *Senior Member, IEEE, Senior Member, OSA*

***Abstract*—Incorporation of grating waveguides in optical delay lines based on integrated microring resonator structures is proposed. Specifically, transmittive apodized grating waveguides are implemented in two common microring configurations, namely, coupled-resonator optical waveguides (CROWs) and side-coupled integrated spaced sequence of resonators (SCISSORs). The performance of these proposed devices is compared with standard CROWs and SCISSORs (i.e., with no gratings), as well as with grating-assisted waveguides (cascaded complementary apodized grating waveguides). It is shown that, for the same minimum bit rate operation, the proposed grating-assisted SCISSORs exhibit the best performance in terms of delay tunability and delay per footprint. However, the maximum true time delay of grating-assisted waveguides possesses a close to linear dependence on length, while the true delay in mircoresonator-type structures (with or without gratings) do not increase efficiently with the number of rings. Also, it is argued that the grating-assisted waveguides are expected to have the lowest insertion loss per delay time.**

***Index Terms*—Apodized grating, CROW, integrated photonics, microring resonator, optical waveguide, photonic integrated circuits, SCISSOR, silicon photonics, tunable optical delay line.**

I. INTRODUCTION

ON-CHIP optical buffers or delay lines are important components in a variety of applications, including optical communication systems and networks [1]–[4] and optical beam-forming control of phased-array antennas (PAAs) [5]. Such time delay units should not only possess large true delay times with low insertion loss, but they should also be tunable, have large spectral bandwidth for high information trafficking and have small footprint to be suitable for photonic integrated circuits.

Although large optical delay times can be achieved in long spiral optical waveguides, tuning the delay is challenging in them. It has been demonstrated that the delay time can be tuned and enhanced (both in terms of spectral bandwidth and true delay time) in two microring-resonator configurations (commonly known as SCISSORs [6] and CROWs [7]), as well as in photonic crystal (PhC) waveguides [8].

We have proposed and demonstrated that tunable optical delay times can also be achieved in apodized grating waveguides [9]–[12]. The optical delay time, bit rate and tunability of grating waveguides can be controlled using engineered profiles for as flat as possible dispersion of the delay time and hence large bandwidth. Particularly, cascading two inward and outward complementary apodized profiles is studied in detail [10]–[12]. The results of such grating-assisted waveguides (GA-WGs) are promising devices and outperform SCISSORs, CROWs and PhC delay lines in various metrics. For instance, the measured loss per delay in grating-assisted waveguides is $\sim 39\ \mathrm{dB/ns}$ [12], while the figure is $\sim 60\ \mathrm{dB/ns}$ in both SCISSORs [6] and CROWs [7] and is much higher in PhC waveguides ($\sim$160 dB/ns [8]).

However, GA-WG delay lines suffer from relatively large footprint (few millimeters long waveguides for $>$100 ps true time delay [10], [12]), which makes them less ideal for integrated photonic systems. It is, therefore, intriguing to combine the concept of apodized grating and microring-type structures and study the performance of the hybrid devices for delay line application. Such grating-assisted SCISSORs (GA-SCISSORs) and CROWs (GA-CROWs) are proposed in this paper, for the first time.

Several figures of merit are studied, e.g., attainable maximum true time delay, delay tunability, delay times bit-rate product, tunability times bit-rate product, insertion loss per absolute delay ($\mathrm{dB/ns}$) and delay per footprint ($\mathrm{ns/mm^2}$). These figures are compared when grating waveguides are incorporated into three different types of optical delay lines, namely, SCISSORs, CROWs and straight waveguides. Also, the effect of the grating loss on the total delay time and the insertion loss is discussed to show that even with considering additive loss for the gratings, the presented configuration could still be suitable for compact on-chip optical delay lines.

II. Proposed Devices and Their Modeling

The proposed grating-assisted devices are schematically shown in Fig. 1. The transfer matrix method [13] is used to calculate the transmission coefficient, T_G, and group-velocity delay in GA-WGs. The standard transmission and optical delay formulation of SCISSORs and CROWs [14] can also be modified, using the transfer matrix method, to account for the impact of the incorporated gratings. Such a model is developed here for GA-SCISSORs and GA-CROWs and is outlined in the Appendix.

The example platform for all the studied structures is silicon-on-insulator (SOI) channel waveguides with 220 nm thickness

---

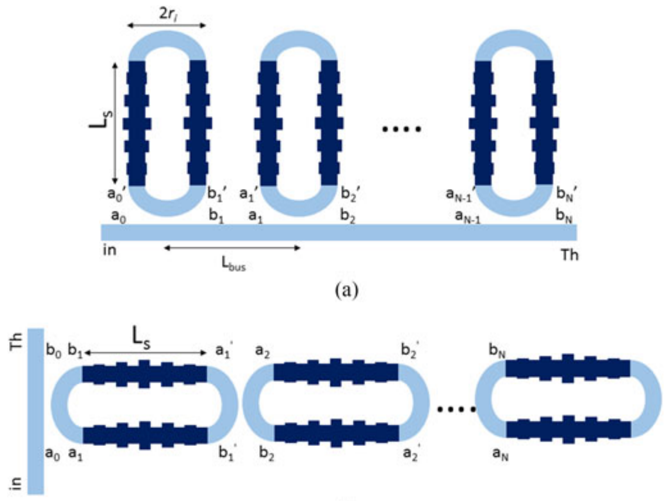
Manuscript received July 28, 2016; accepted October 5, 2016. Date of publication October 18, 2016; date of current version November 7, 2016. The work was supported by the U.S. National Science Foundation under Award ECCS 1128208.

S. Toroghi, C. Fisher, and S. Khan are with CREOL, The College of Optics and Photonics, University of Central Florida, Orlando, FL 32816 USA (e-mail: storoghi@creol.ucf.edu; cdfisher@knights.ucf.edu; saeedk@creol.ucf.edu).

S. Fathpour is with CREOL, The College of Optics and Photonics, University of Central Florida, Orlando, FL 32816 USA, and also with the Department of Electrical Engineering and Computer Science, University of Central Florida, Orlando, FL 32816 USA (e-mail: fathpour@creol.ucf.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JLT.2016.2619182



(a)

(b)

Fig. 1. Schematic of implemented gratings in an all-pass (a) SCISSOR and (b) CROW configurations.

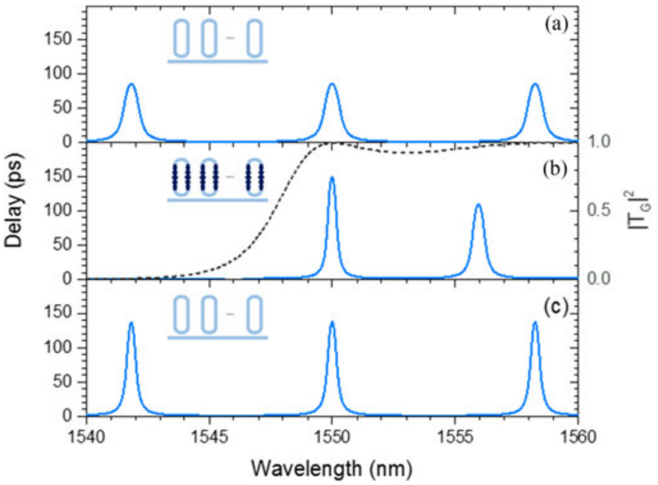


Fig. 2. Effect of grating and coupling coefficient on the delay spectrum of SCISSORs (with 10 rings): (a) No gratings and cross-coupling coefficient, $t = 0.85$; (b) The same t as in (a), but with gratings with lengths $L_{\mathrm{s}} = 50\,\mu$m; and (c) Cross-coupling coefficient, $t = 0.9$ and no gratings. In all cases, the average ring radius, r_{i}, is slightly varied to ensure a resonance wavelength of 1,550.00 nm, but this variation has negligible effect on the attainable delay. The dashed gray line in (b) represents the transmission of the incorporated grating, $|T_G|^2$, ensuring minimum reflection at the operating wavelength.

and 500 nm width, unless stated otherwise. The effective index of the waveguides is calculated using COMSOL. The waveguide propagation loss is assumed to be 1.5 dB/cm in both the waveguide and grating segments (though the impact of potentially higher loss in the gratings is also studied and discussed later).

Overall, the total dispersion of the proposed structures, from which the delay spectrum is calculated, is the effective combination of the dispersion of the microring and grating components. Figure 2(a)-(c) show the optical delay for three different SCISSOR structures with and without gratings and with different self-coupling coefficient, t (see Appendix). For the same coupling coefficient to the bus waveguide and same perimeter,

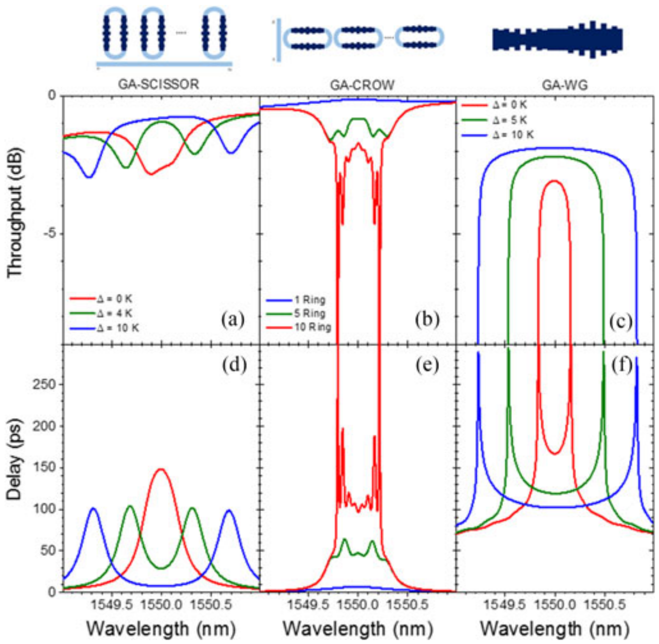


Fig. 3. (a) Transmission and (d) optical delays of a GA-SCISSOR with 10-rings for three different temperature differences Δ of 0, 4 and 10 K between two sets of shorter and longer than average rings; (b) Transmission and (e) optical delays of a GA-CROW configuration including 1, 5 and 10 rings. (c) Transmission and (f) optical delays of a GA-WG with length of 5 mm for three different temperature differences Δ of 0, 5 and 10 K between the complementary inward and outward segments.

incorporating an appropriate apodized grating function in the ring resonators can lead to larger optical delay times, compared to that in standard ring resonators (compare Fig. 2(b) with delay of 149 ps, versus Fig. 2(a) with delay of 86 ps). This enhancement can be explained by the larger group index (lower group velocity) of the grating-assisted rings compared to that of bare rings, which is in turn due to multiple reflection in the grating segments, i.e., a "slow-light" effect.

However, the impact of introducing the gratings in mircorings is not as impressive as the comparison of Figs. 2(a) and 2(b) may imply. This is because the dispersion profile can also be alternatively tailored, to a certain extent, by varying t. An optimized SCISSOR with no grating is shown in Fig 2(c) with delay of 137 ps, which is ~7.5% less than the GA-SCISSOR in Fig. 2(b). The corresponding t values of the three cases in Fig. 2 are given in the caption.

III. RESULTS AND DISCUSSIONS

Figures 3(a) and (d) show the transmission and optical delay time spectra for a 10-ring GA-SCISSOR configuration. Each arm's grating has a length of 50 μm. A super-Gaussian function (order 6 with full width at half maximum (FWHM) of 50 μm) is employed for apodization. These parameters have been acquired after optimizing the delay response for different parameters, such as outward or inward gratings, apodization profile, super-Gaussian order and FWHM. The same apodization profile is employed for GA-CROWs and GA-WGs, to be discussed later.

The bandwidth of SCISSORs can be tuned using coupling coefficient between the ring and the bus waveguide, waveguide width and ring perimeter [2]. The opposite change in the temperature of two sets of rings (smaller and larger radii, r_i, than the average radius) has been used to tune the optical delay of the whole configuration. The temperature variation in rings with smaller radii (shorter wavelength resonances) decreases, while it increases in those with larger radii (longer wavelength resonances). The results show that with increasing the temperature difference, Δ, in the two groups of rings from 0 to 10 K, the delay time drops from 149 to 8 ps, accompanied with a decrease in insertion loss from $\sim$2.7 to $\sim$0.8 dB. Therefore, the GA-SCISSORs configuration offers 141 ps of tunability in delay time with 18 dB/ns loss per delay. The bit rate limited by group-velocity dispersion (GVD) is estimated to be $\sim 15\,\mathrm{Gb/s}$.

The second studied configuration is the CROW, in which tuning of optical delay can be achieved using on- and off-resonances of directly-coupled rings [15]. The corresponding results in Figs. 3(b) and (e) demonstrate that the delay time can be continuously tuned using different number of rings locked into the optical path. With increasing the number of involved rings from 1 to 10, the delay increases from 7 to 97 ps, showing a tunability of 90 ps. The corresponding total insertion loss increases from 0.12 to 1.8 dB, which can be interpreted as 18 dB/ns loss per delay. Similar to the GA-SCISSOR, the corresponding GVD-limited bit rate is $15\,\mathrm{Gb/s}$.

As discussed in the Introduction, we have previously theoretically studied and experimentally demonstrated GA-WG delay lines [9]–[12]. However, to make a consistent comparison with the device proposed here, GA-WGs are also simulated here. The studied GA-WGs comprise a 5-mm-long straight waveguide with two cascaded outward and inward apodized grating sections, each with a length of 2.5 mm. The largest width of the gratings in the middle reaches 600 nm for the outward case, while the smallest width of grating is 430 nm for the inward case.

Figures 3(c) and (f) show the transmission and optical delay of these GA-WGs for three different temperature differences between the inward and outward gratings, respectively. The temperature difference between the outward and inward gratings has been used to control the transmission and delay spectrum of the cascaded structure, which in turn leads to tunable optical delay and bandwidth [10]–[12]. The present results show that increasing the temperature difference between the outward and inward gratings leads to an increased delay from 102 to 167 ps, accompanied with an increased insertion loss from 1.9 to 3.1 dB. For the largest delay time, the bit rate limited by group-velocity-dispersion (GVD) is $15\,\mathrm{Gb/s}$. Consequently, the GA-WG structure shows 65 ps of delay tunability and 18.5 dB/ns of loss per delay.

Figure 4 presents the GVD-limited bit rate versus delay for the five studied delay line configurations, including 5-mm GA-WGs, 10-ring GA-SCISSORs and 10-ring GA-CROWs, as well as standard SCISSORs and CROWs with no gratings. In all cases, it has been ensured that the configurations can operate at a minimum bit rate of $15\,\mathrm{Gb/s}$.

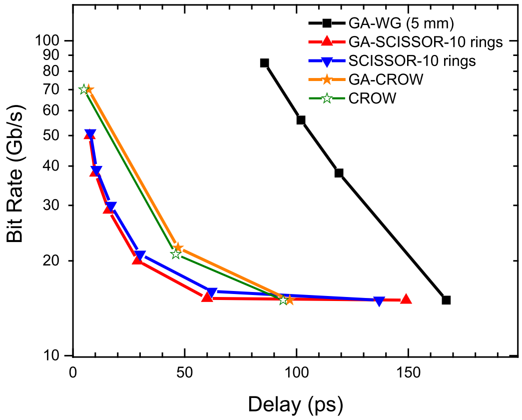


Fig. 4. GVD-limited bit rate versus delay for different combination of the studied 5 delay line configurations, all operating at a minimum bit rate of 15 Gb/s.

The results show that GA-WGs presents the highest true optical delay time of 167 ps. That is a close to linear delay per device length of $\sim 33\,\mathrm{ps/mm}$, which is a factor of 4 higher than straight waveguides with the same average group refractive index. In other words, the cascaded gratings in straight waveguides "slow down" the light by a factor of 4 and of course in addition offer the possibility of thermally or electrically tuning the delay [9]–[12].

The results in Fig. 4 show that the GA-SCISSORs possess the highest delay tunability among all the configurations. Also, the GA-SCISSORs possess 9% higher absolute delay compared to that of SCISSORs for identical 10 rings.

The absolute delay time of GA-CROWs shows slight improvement over CROWs (97 versus 95 ps). As discussed below, delay improvement factors comparable to GA-SCISSORs are likely to be achieved for GA-CROWs with further optimization. Nonetheless, both CROWs and GA-CROWs have the smallest delay values among the studied configurations.

It is evident from the results presented above that introducing gratings in waveguides improves the maximum true time delay by 300% compared to bare waveguides, but the enhancement is more modest in SCISSORs (9%). As mentioned in Section II, this is attributed to the dominance of the microring's dispersion over the gratings'. Mathematically, optimizing t in Eq. (2) of the Appendix can, to a certain extent, have the same effect of $T_{Gi1}.T_{Gi2}$.

The improvement of delay due to introducing gratings is even smaller for the case of CROWs ($\sim 2\%$). This is because optimizing the coupling coefficients between the many involved rings (parameter t_i in the Appendix) is a complicated mathematical problem to be handled manually and may require techniques such as genetic algorithm. Otherwise, issues of fabrication tolerance notwithstanding, it is expected that improvements comparable to GA-SCISSORs be attainable for GA-CROWs.

It is meanwhile noted that both the transmission and delay time spectra of the GA-CROW structures show fine ripples in the stop band of Figs. 3(b) and (e). These are the Fabry-Perot-like oscillations due to the finite number of coupled resonators, called finite-size effect [16]. These ripples can, in principle,

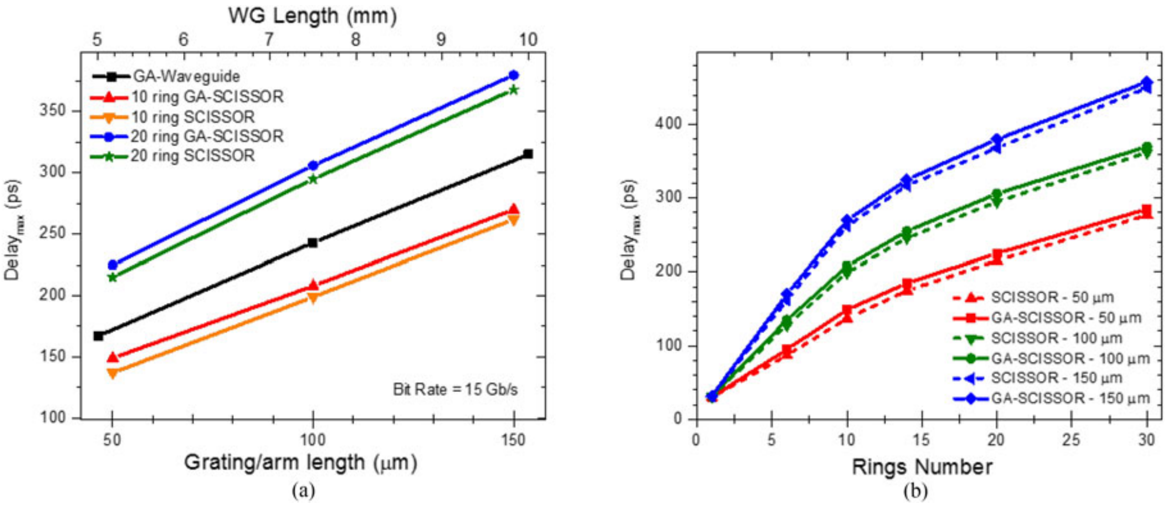


Fig. 5. Maximum delay (a) versus the length of the grating in GA-WGs (top axis in mm) and length of gratings in GA-SCISSOR and straight arms in SCISSORs (bottom axis in μm); (b) versus number of the ring resonators in SCISSORs and GA-SCISSORs with three different grating/arm length.of 50 μm, 100 μm, and 150 μm, all at identical bit rate of 15 Gb/s.

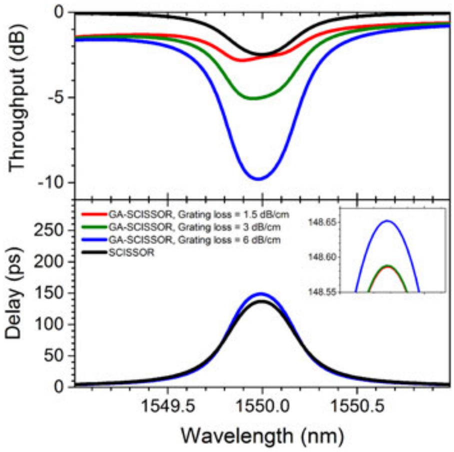


Fig. 6. Throughput and optical delay time for three GA-SCISSOR structures with varied grating propagation loss of 1.5, 3 and 6 dB/cm, while waveguide propagation loss is kept constant as 1.5 dB/cm. The black lines show the results for a SCISSOR structure considering waveguide propagation loss of 1.5 dB/cm.

be reduced with customized adiabatic coupling coefficients between adjacent rings [17], but the issue remains a discouraging shortcoming for CROW-based optical delay lines, since it leads to low signal fidelity.

The effect of grating length in GA-WGs, as well as grating length and number of rings in SCISSORs and GA-SCISSORs is also investigated. Summarized in Fig. 5, the absolute delay time increases almost linearly with grating length in GA-WGs. The same is true for GA-SCISSORs with or without gratings. It has to be clarified that, in the case of SCISSORs with no gratings, the bottom axis in Fig. 5 refers to when the gratings in GA-SCISSORs are replaced with bare waveguides.

Figure 5(a) shows that SCISSORs and GA-SCISSORs with 20 rings have >350 ps true time delay, which are higher than even the longest (10 mm) GA-WGs with a corresponding value of 315 ps. However, to keep the minimum operation bit rate fixed for increasing number of rings (15 Gb/s in our examples), the coupling coefficient and ring perimeter have to be optimized in GA-SCISSORs. The result is that the delay does not increase linearly with the number of rings, as evident in Fig. 5(b). For example, for 150 μm grating length, the GA-SCISSORs' delay only increases from 270 to 380 ps (40% increase), when the number of rings is doubled from 10 to 20. Therefore, the weaker dependence of delay as a function of ring numbers is a disadvantage of ring-resonator-type devices when very long delays of greater than 1 ns are demanded for PAAs [5]. Meanwhile, for 20 rings, the GA-SCISSORs' delay is only 5% higher than the SCISSORs' (compare to the aforementioned 9% figure for 10 rings). This observation suggests that introducing gratings in resonator-type structures becomes less impactful as the number of rings is increased.

As discussed before, all the three structures of Fig. 3 have about the same insertion loss per delay of $\sim 18\,\mathrm{dB/ns}$. This is not surprising, since an identical propagation loss of $1.5\,\mathrm{dB/\,cm}$ is assumed for both the bare and grating waveguide segments of all structures, and the coupling loss in the directional coupler regions of the ring-type devices is ignored. It is, however, known that grating waveguides typically exhibit higher insertion loss than non-corrugated waveguides, due to the mode mismatch and scattering at each grating period interface. To study the influence of this effect, the throughput and optical delay time were calculated for three different propagation losses in the grating segments of the GA-SCISSOR configuration as shown in Fig. 6. The considered grating propagation loss are 1.5, 3 and $6\,\mathrm{dB/cm}$. All other parameters (including the non-corrugated waveguide propagation loss of $1.5\,\mathrm{dB/\,cm}$) are kept identical to those in Section II. Note that the additional grating loss is due to the scattering and mode mismatch and is not related to the material loss or absorption. Therefore, index change does not take place while adding extra grating loss to the system. The results show that higher grating propagation loss only increases the throughput loss from 3 to 10 dB as grating loss increases by a factor of 4, while the attainable optical delay stays almost the same.

TABLE I
SUMMARIZED CALCULATED METRICS FOR DIFFERENT OPTICAL DELAY LINE STRUCTURES STUDIED AT THE SAME BIT RATE OF 15 GB/S

Device Type	Delay (ps)	Tunability (ps)	Delay × Bit Rate	Tunability × Bit Rate	Delay per Footprint (ns/mm^2)
GA-WG (5 mm)	167	80	2.51	1.20	3.34
GA-WG (10 mm)	315	145	4.73	2.18	3.15
CROW (10 rings)	95	88	1.43	1.32	7.90
GA-CROW (10 rings)	97	90	1.46	1.35	8.10
SCISSOR (10 rings)	137	129	2.06	1.96	11.42
GA-SCISSOR (10 rings)	149	141	2.24	2.16	12.42
SCISSOR (20 rings)	215	207	3.23	3.11	8.96
GA-SCISSOR (20 rings)	225	217	3.38	3.26	9.38

It should be also noted that, in the case of ring-resonator-based optical delay lines, there is an extra loss mechanism due to the coupling between the bus waveguide and rings for SCISSORs or between the adjacent rings in CROWs, which in turn leads to larger loss per delay and consequently underperforming in these optical delay lines. This argument is supported by experimental results, i.e., ~50% higher loss per delay time in SCISSORs and CROWs (60 dB/ns [6], [7]) over GA-WGs (~39 dB/ns [12]). Meanwhile, PhC waveguides have exhibited the highest loss (~160 dB/ns [8]) due to the large scattering loss of the etched holes. Considering all the above factors contributing to loss, it might be difficult to draw general conclusions on which configuration offers the lowest insertion loss per delay, but it appears that the GA-WGs offer the best performance in this regard.

Table I summarizes the performances of the five optical delay lines discussed in this work at the same minimum bit rate of 15 Gb/s. Only grating length of 50 μm, similar to those in Figs. 3 and 4, have been summarized for GA-CROWs and GA-SCISSORs. Two lengths of 5 and 10 mm are considered for GA-WGs and two ring numbers of 10 and 20 have been tabulated for GA-SCISSORs. Evidently, the CROWs and GA-CROWs possess the shortest attainable maximum delay. The better device performance belongs to GA-WGs or GA-SCISSORs, but each has its own advantages and disadvantages, as follows.

The 10-mm-long GA-WG shows the longest delay time when compared to resonator types with 10 rings. Higher delays could be achieved in SCISSORs and GA-SCISSORs devices with incorporating more rings (e.g., see Fig. 5). However, as discussed, the dependence of delay on the number of rings is sublinear in ring-based devices and the loss per delay is potentially higher. The GA-SCISSOR offers the highest delay tunability. Also, the delay can be tuned all the way to close to zero values. These are clear advantages over GA-WGs. Finally, Table I also summarizes the delay per footprint of the studied devices ($\mathrm{ns/mm^2}$), which indicates the largest possible delay in the smallest possible real estate on the chip. In this regard, the GA-SCISSOR clearly has the best performance as well.

IV. CONCLUSION

In summary, incorporation of grating structures is proposed to enhance the performance of ring-resonator-type delay lines. A modified transfer matrix method is employed to study the optical delay in GA-CROWs and GA-SCISSORs and their performance is comparted with GA-WGs and CROWs and SCISSORs with no gratings. It is concluded that for the same minimum operation bit rate, GA-WGs offer the strongest dependency of delay and lowest loss per delay as a function of device size. The delay of SCISSORs and CROWs (with or without grating) improves modestly with the number of rings. Nonetheless, GA-SCISSORs possess the highest tunability and smallest footprint.

APPENDIX

It is well-known that the true time delay, τ, of a linear time-invariant (LTI) system with transfer function $T(\omega) = |T(\omega)| \exp\{j\Phi(\omega)\}$ can be calculated from the derivative of the phase response at operating angular frequency ω_0 or wavelength λ_0, i.e.,

$$\tau = -\left.\frac{d\Phi(\omega)}{d\omega}\right|_{\omega=\omega_0} = \frac{\lambda_0^2}{2\pi c}\left.\frac{d\Phi(\lambda)}{d\lambda}\right|_{\lambda=\lambda_0}. \tag{1}$$

Formulations for calculating $T(\omega)$ in GA-SCISSORs and GA-CROWs are developed in the following.

A. GA-SCISSORs

Figure 1(a) shows the proposed GA-SCISSOR configuration with implemented outward apodized grating in each straight part of the race-track-shaped microring resonator. The transmission coefficient of a straight coupled microring [14] is modified to include the impact of the implemented gratings:

$$T_i = \frac{b_i}{a_{i-1}} = \frac{t - T_{Gi1}.T_{Gi2}.\ e^{-j\left(\beta - j\frac{\alpha}{2}\right).2\pi r_i}}{1 - t^*.T_{Gi1}.T_{Gi2}\ e^{-j\left(\beta - j\frac{\alpha}{2}\right).2\pi r_i}}\ , \tag{2}$$

where t is the self-coupling coefficient (identical for all the rings), β is the propagation constant in the waveguide, α is the propagation loss in the waveguide, r_i is the radius of curvature in the i^{th} ring. T_{Gi1} and T_{Gi2} are the complex transmission coefficient of the two gratings in the i^{th} ring, i.e., it includes both the phase and amplitude information. Identical gratings are assumed for all rings in this work. The transmission coefficient of the electromagnetic wave propagating from one ring to the next is not varied also and is given by

$$T_{\mathrm{bus}} = e^{-j\left(\beta - j\frac{\alpha}{2}\right)\,.\ L_{\mathrm{bus}}}, \tag{3}$$

where L_{bus} is the spacing between the two rings. The total transmission of a single-channel grating-assisted SCISSOR including N rings can be calculated using

$$T_{\mathrm{SCISSOR}} = T_1.\,T_{\mathrm{bus}}.T_2.\,T_{\mathrm{bus}}\ldots T_{N-1}.\,T_{\mathrm{bus}}.\,T_N. \tag{4}$$

B. GA-CROWs

Figure 1(b) shows the proposed grating-assisted CROW configuration with outward apodized gratings in each straight part of the ring resonators. The electric field components in this structure with N rings is given by:

$$\begin{bmatrix} a_{N+1} \\ b_{N+1} \end{bmatrix} = P_{N+1} Q_N P_N \ldots Q_2 P_2 Q_1 P_1 \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} \tag{5}$$

or

$$\begin{bmatrix} a_{N+1} \\ b_{N+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}, \tag{6}$$

where Q and P are the propagation and coupling matrices, respectively and are given by:

$$\begin{aligned} P_i &= \frac{1}{j\kappa_i} \begin{bmatrix} -t_i & 1 \\ -1 & t_i^* \end{bmatrix} \\ Q_i &= \begin{bmatrix} 0 & T_{Gi}.e^{-j\left(\beta - j\frac{\alpha}{2}\right).\ \pi r_i} \\ T_{Gi}^{-1}.e^{j\left(\beta - j\frac{\alpha}{2}\right).\ \pi r_i} & 0 \end{bmatrix} \end{aligned} \tag{7}$$

where t_i and κ_i are the self- and cross-coupling coefficients between i^{th} and $(i - 1)^{\mathrm{th}}$ rings, respectively. Considering that the $(N + 1)^{\mathrm{th}}$ ring is virtual, self- coupling coefficient in the N^{th} ring should be 1 (zero cross-coupling). In other words:

$$P_{N+1} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}. \tag{8}$$

Another boundary condition comes from the fact that there is no added signal in the $(N + 1)^{\mathrm{th}}$ ring ($a_{N+1} = 0$). Using these boundary conditions, the throughput of a GA-CROW including N rings can be calculated as:

$$T_{\mathrm{CROW}} = \frac{-A}{B}. \tag{9}$$

REFERENCES

[1] Q. F. Xu, J. Shakya, and M. Lipson, “Direct measurement of tunable optical delays on chip analogue to electromagnetically induced transparency,” *Opt. Express*, vol. 14, pp. 6463–6468, Jul. 10, 2006.

[2] F. N. Xia, L. Sekaric, and Y. Vlasov, “Ultracompact optical buffers on a silicon chip,” *Nature Photon.*, vol. 1, pp. 65–71, Jan. 2007.

[3] D. O’Brien, A. Gomez-Iglesias, M. D. Settle, A. Michaeli, M. Salib, and T. F. Krauss, “Tunable optical delay using photonic crystal heterostructure nanocavities,” *Phys. Rev. B*, vol. 76, Sep. 2007, Art. no. 115110.

[4] F. Morichetti, A. Melloni, A. Breda, A. Canciamilla, C. Ferrari, and M. Martinelli, “A reconfigurable architecture for continuously variable optical slow-wave delay lines,” *Opt. Express*, vol. 15, pp. 17273–17282, Dec. 10, 2007.

[5] S. Fathpour and N. A. Riza, “Silicon-photonics-based wideband radar beamforming: basic design,” *Opt. Eng.*, vol. 49, Jan. 2010, Art. no. 108201.

[6] P. A. Morton, J. Cardenas, J. B. Khurgin, and M. Lipson, “Fast thermal switching of wideband optical delay line with no long-term transient,” *IEEE Photon. Technol. Lett.*, vol. 24, no. 6, pp. 512–514, Mar. 2012.

[7] A. Melloni *et al.*, “Tunable delay lines in silicon photonics: Coupled resonators and photonic crystals, a comparison,” *IEEE Photon. J.*, vol. 2, no. 2, pp. 181–194, Apr. 2010.

[8] Y. A. Vlasov, M. O’Boyle, H. F. Hamann, and S. J. McNab, “Active control of slow light on a chip with photonic crystal waveguides,” *Nature*, vol. 438, no. 7064, pp. 65–69, Nov. 2005.

[9] S. Khan, M. A. Baghban, and S. Fathpour, “Electronically tunable silicon photonic delay lines,” *Opt. Express*, vol. 19, pp. 11780–11785, Jun. 6, 2011.

[10] S. Khan and S. Fathpour, “Complementary apodized grating waveguides for tunable optical delay lines," *Opt. Express*, vol. 20, pp. 19859–19867, Aug. 27, 2012.

[11] S. Khan and S. Fathpour, “Demonstration of tunable optical delay lines based on apodized grating waveguides,” *Opt. Express*, vol. 21, pp. 19538–19543, Aug. 26, 2013.

[12] S. Khan and S. Fathpour, “Demonstration of complementary apodized cascaded grating waveguides for tunable optical delay lines,” *Opt. Lett.*, vol. 38, pp. 3914–3917, Oct. 1, 2013.

[13] P. Yeh, *Optical Waves in Layered Media*. New York, NY, USA: Wiley, 1988, p. 102.

[14] D. G. Rabus, *Integrated Ring Resonators*. New York, NY, USA: Springer, 2007, p. 5.

[15] F. Morichetti, A. Melloni, C. Ferrari, and M. Martinelli, “Error-free continuously-tunable delay at 10 Gbit/s in a reconfigurable on-chip delay-line,” *Opt. Express*, vol. 16, pp. 8395–8405, Jun. 9, 2008.

[16] Y. H. Ye, J. Ding, D. Y. Jeong, I. C. Khoo, and Q. M. Zhang, “Finite-size effect on one-dimensional coupled-resonator optical waveguides,” *Phys. Rev. E*, vol. 69, May 2004, Art. no. 056604.

[17] A. Melloni and M. Martinelli, “Synthesis of direct-coupled-resonators bandpass filters for WDM systems,” *J. Lightw. Technol.*, vol. 20, no. 2, pp. 296–303, Feb. 2002.

Seyfollah Toroghi (M’13) received the Ph.D. degree in optics and photonics from The College of Optics and Photonics (CREOL), University of Central Florida, Orlando, FL, USA, in 2014. He joined as a Research Scientist with Integrated Photonics and Energy Solutions lab at CREOL in 2015. His Ph.D. studies were focused on photo-thermal and Kerr-type nonlinearity in metamaterials and metallic nanostructures. He is currently a Senior Research Scientist at Partow Technologies LLC, Vista, CA, USA. His research interests include design, fabrication, and characterization of integrated photonics components such as optical delay lines, optoelectronic modulators, and nonlinear nanomaterials. He is a member of the OSA and SPIE.

Chris Fisher received the bachelor’s degrees in biomedical and electrical engineering from Michigan Technological University, Houghton, MI, USA, in 2015 and the Master’s degree from CREOL Optics and Photonics Program, University of Central Florida, Orlando, FL, USA. He joined the Integrated Photonics and Energy Solutions lab, CREOL in 2015 and is working toward research interests in photonic delay lines and pulse compression. His Masters work has allowed him to learn techniques in waveguide simulation via COMSOL and MATLAB, and fabrication in a cleanroom environment. He is a student member of the SPIE and OSA.

Saeed Khan received the B.E. degree in electronics engineering from NED University of Engineering and Technology, Karachi, Pakistan, in 2000, the M.S. degree in systems engineering from Pakistan Institute of Engineering and Applied Sciences, Islamabad, Pakistan in 2004, and the M.S. and Ph.D. degrees from University of Central Florida, Orlando, FL, USA, in 2010 and 2013 respectively. From 2004 to 2015, he was with Pakistan Atomic Energy Commission as a Principal Engineer at Islamabad, Pakistan. In 2015, he joined Balochistan University of Information Technology, Engineering and Management Sciences, Quetta, Pakistan, as an Associate Professor. Since 2016, he has been working as a Postdoctoral Associate at CREOL: The College of Optics and Photonics, University of Central Florida. He has been engaged in research on near- and Mid-IR silicon integrated photonics, heterogeneous integrated photonics, nonlinear and quantum integrated optics.

Sasan Fathpour (SM’13) received the Ph.D. degree in electrical engineering from the University of Michigan, Ann Arbor, MI, USA, in 2005. After receiving the Ph.D. degree, he joined the Electrical Engineering Department, University of California, Los Angeles, as a Postdoctoral Fellow and was promoted to a Visiting Assistant Professor in 2007. Prior to joining the Faculty of The College of Optics and Photonics (CREOL), University of Central Florida (UCF), Orlando, FL, USA, in September 2008, he was a Senior Researcher at Ostendo Technologies, Inc. He is currently an Associate Professor of optics and photonics at CREOL. He also holds a joint appointment at the Department of Electrical Engineering and Computer Science, UCF. He received the NSF Career Award in 2012 and the ONR Young Investigator Award in 2013. He received the UCF’s 2015 Reach for the Stars Award and the 2007 UCLA Chancellor’s Award for Postdoctoral Research. He is a co-author of more than 120 publications, including over 50 journal and about 60 conference papers, six book chapters, and three pending patents. He is the co-editor of the book “*Silicon Photonics for Telecommunications and Biomedicine*,” published by CRC Press in 2012. He is a Senior Member of the OSA and SPIE.