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Linking Object Boundaries at Scale: a Common Mechanism for Size and Shape Judgments

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The area over which boundary information contributes to the determination of the center of an extended object was inferred from results of a bisection task. The object to be bisected was a rectangle with two long sinusoidally modulated sides, i.e. a wiggly rectangle. The spatial frequency and amplitude of the edge modulation were varied. Two object widths were tested. The modulation of the perceived center approximately equaled that of the edges at very low edge modulation frequencies and decreased in amplitude with increasing edge modulation frequency. The edge modulation had a greater modulating effect on the perceived center for the narrower object than for the wider object. This scaling with object width didn't follow perfect zoom invariance but was precisely matched by the scaling of the bisection threshold with width, strongly supporting the idea that the same mechanism determines both the location of the perceived center for these stimuli and its variance. We propose that this mechanism is the linking of object boundaries at a scale determined by the object width.

Shape Scaling Size Separation Bisection

INTRODUCTION

One tends to think of shape perception as ordinarily being veridical and robust, with the more interesting phenomena being examples of how the visual system successfully infers shape from relatively impoverished clues, e.g. shape from shading, shadows, motion, or stereopsis. At least as significant, however, are the failures of the visual system to use shape information that is readily available in the image. For example, a golf ball is seen as a sphere textured with dimples, not as a multi-sided form with known relations between its faces. In general, small-scale detail has little effect on the perceived overall shape of an object. Thus, there will often be important differences between the shape percept and the physical description of the object's spatial properties. Investigation of these differences between the percept and the physics of the stimulus can provide clues as to how the visual system represents shape information. Here we focus on the shape information contained in the profile of an object, i.e. silhouette-based shape, comparing the measurable edge relationships with the perceived relationships.

Consider the objects in Fig. 1 [B. B. Kimia & S. W. Zucker (personal communication) have independently used similar stimuli]. The upper objects differ from the lower ones in average width. The objects on the left differ from the ones on the right in the relative phases of the edge modulation. In the objects on the left in the figure, the edge modulations are 180 deg out of phase with respect to one another, resulting in objects that have a sinusoidally modulated width and a straight central axis. In the objects on the right, the edge modulations are in phase, resulting in objects that have a constant horizontal width and a sinusoidally modulated central axis. Although the upper and lower pairs differ from one another only in width, the effects of the edge modulation on their perceived shapes are appreciably different. When the objects are narrow, the relative phase of the edge modulation has a profound effect: one looks straight, the other wiggly. When the objects are wider, however, they look quite similar to one another: the relative phase of the edge modulation is much less apparent. The perceived shapes of these objects clearly depend on their overall widths, not just on their local boundary characteristics. What mechanism can account for this?

Burbeck and Pizer (1995) have proposed that silhouette-based object shape is found and represented by a mechanism that links opposing sides of simple spatial

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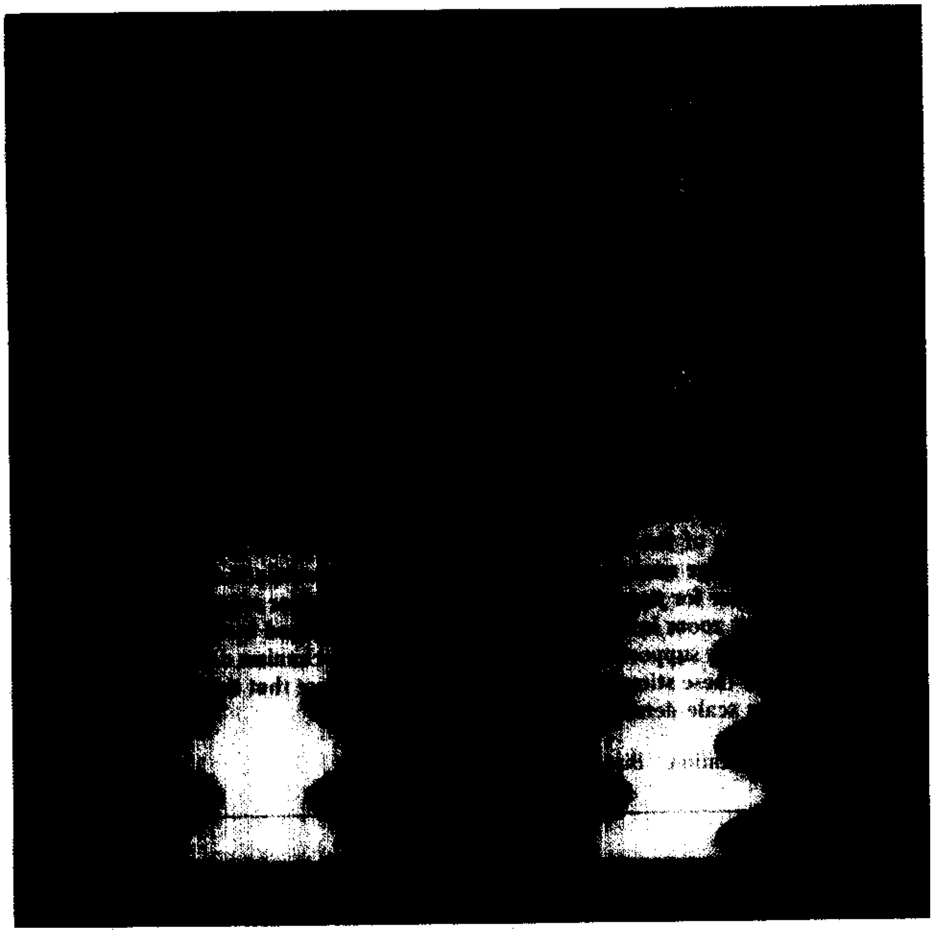

FIGURE 1. Four figures with sinusoidal edge modulations. The upper and lower pairs differ only in their average widths. The left and right members of each pair differ in the relative spatial phase of the sinusoidal edge modulation. In the left member of each pair, the object width is modulated and the central axis is straight. In the right member of each pair, the object width is constant but the central axis is sinusoidally modulated with the edge.

regions through a fuzzy, scaled, medial axis, called a "core". In this model, boundariness detectors of many sizes* respond to the object boundaries, but only those detectors whose sizes are proportional to the local width of the object communicate with one another to form this core. Thus boundary information is acquired over a smaller area for a narrower width than for a wider one. The constant of proportionality (relating the size of the boundariness detectors and the object's width) is assumed to be constant across object widths, to achieve zoom invariance. Such a relationship between the relevant boundary integration area and the object width can account qualitatively for the interaction between object width and the effect of a given edge modulation on perceived shape (as seen in Fig. 1): as the boundary integration area increases, it integrates more cycles, or more of a cycle, of the edge modulation, effectively attenuating the modulation.

The idea that the relevant boundary integration areas increase with the width being spanned was first proposed† to account for some experimental data (Burbeck & Hadden, 1993). In this study, position integration areas were inferred using a two-line separation discrimination task with a background probe mapping out the integration area. Data were obtained for several mean separations between the target lines. The position integration areas thus inferred increased with increasing separation, and the increase paralleled the increase in the separation discrimination threshold, suggesting that the scaling of position integration areas is sufficient to account for the scaling of these thresholds.

In the research reported here, we test the hypothesis that the process of linking boundaries at a scale determined by their separation underlies both the scaling

*The term *boundariness detector* is used to indicate a unit that responds to a sharp gradient of luminance (or texture, color, etc.) in a graded manner, as opposed to a binary boundary or edge detector. The size of a boundariness detector means the size of its receptive field.

†It had been proposed previously that spatial filters with larger receptive fields encoded larger separations, and filters with smaller receptive fields smaller separations [Klein & Levi, 1985; Wilson, 1986], but these models cannot account for – and are not intended to account for – larger-scale distance judgments such as those that are typically involved in object perception. For larger separations, responses of separated units must be linked across space to account for the results [e.g., Toet & Koenderink, 1988; Burbeck, 1987].

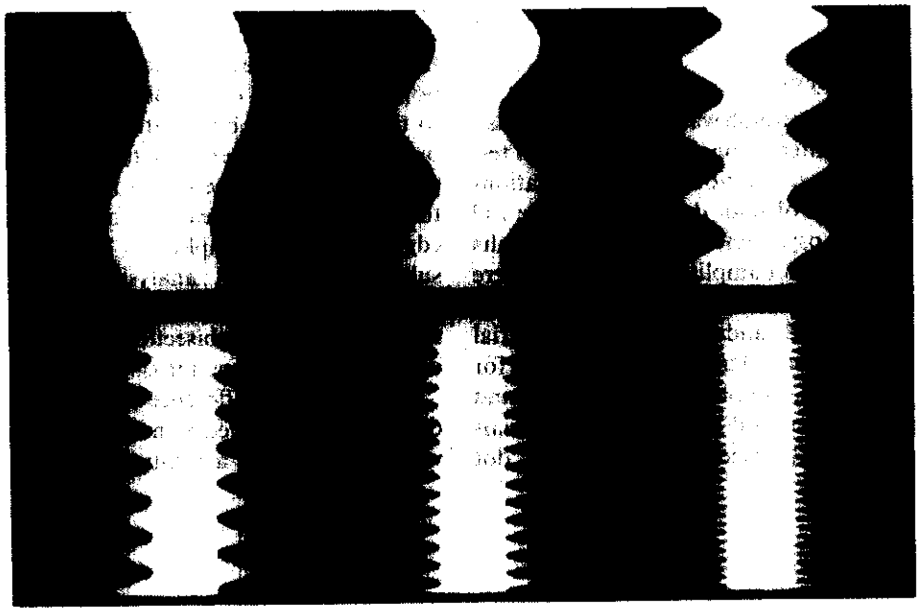

FIGURE 2. A sample of the stimuli used, showing the six edge modulation frequencies for one width (1.5 deg) and one amplitude of modulation (40%). The striping that may be evident in this figure is a reproduction artifact.

of size discrimination thresholds with width, as reported previously, and the attenuation of edge modulation effects with width. We use a bisection task with simple stimuli of the type shown in Fig. 1 to test this common-mechanism hypothesis. Use of this task and stimulus enables us to verify the previous experimental results in a situation where multi-object grouping operations are not relevant [as they may have been in the prior work (Badcock & Hess, 1995)] and permits us to make quantitative connections between the size discrimination threshold and the shape percept.

METHODS

Task

The task was bisection of wiggly-edged stimuli of the type shown in Figs 1 and 2. A black probe dot located near the center of the stimulus served as the bisection target. The observer was asked to indicate, by pressing one of a pair of keys, whether the probe dot appeared to be left or right of the center of the object as measured along a horizontal line through the dot. Observers were instructed to make their observations on the basis of the perceived local center, not on the basis of the perceived center of the overall object. No right/wrong feedback was given. Data were obtained using the method of constant stimuli with the horizontal location of the probe dot being varied between trials.

Stimuli

The wiggly-edged stimuli were 4 deg in height and either 0.75 deg or 1.5 deg in average width. For most of the experiments, the sinusoidal edge modulations on the two sides of the objects were in-phase with one another, creating an object with a sinusoidally modulated central axis and constant width, as shown in the right half of Fig. 1 and in Fig. 2. Six edge modulation frequencies were used, 0.25, 0.5, 1, 2, 4 and 8 c/deg, as shown in Fig. 2. The stimuli were white (142 cd/m^2) on a gray (72 cd/m^2) background which subtended 8 deg in width and 6 deg in height. Viewing was binocular; the viewing distance was 2 m. The room was dark except for the illumination provided by the display.

The peak-to-peak amplitude of the sinusoidal edge modulation took on several values, as given in Table 1.

A probe dot (dia = 0.02 deg) was placed near the left/right center of the wiggly-edged stimulus, in one of two vertical locations on each trial: in line with a leftward jog of the left edge, or in line with a rightward jog of the left edge. The two vertical locations used were at adjacent jogs, nearest the center of the stimulus. The horizontal location of the probe dot was varied from trial to trial. Data were collected at both vertical locations for both widths, both amplitudes of modulation for each width, and for all frequencies of edge modulation.

The stimuli were presented for a duration of 600 msec with abrupt onset and termination. The long viewing duration was used to encourage observers to make local judgments of the relevant edge locations. Previous studies conducted in our lab indicate that separation discrimination judgments are most affected by context during the

TABLE 1. Edge modulation amplitudes

	Amplitude	
Width (deg)	% of width	deg
0.75	20	0.15
	40	0.3
1.5	20	0.3
	40	0.6

first few hundred milliseconds (e.g. Burbeck, 1992; Burbeck & Hadden, 1993).

Procedure and data analysis

The method of constant stimuli was used. On each trial, the probe dot was presented at one of 14 horizontal test locations for each vertical location. The test locations were chosen on the basis of pilot data to cover the full range of the observers' psychometric functions. Results for a single edge modulation amplitude and width were obtained in each experimental session, with the different frequencies being presented randomly from trial to trial during each session. At least 480 trials were conducted for each condition of width, amplitude, frequency, and vertical location. We determined the percentage of trials in which the observer reported that the probe dot appeared to be right of center as a function of the horizontal location of the probe dot. We define zero to be the horizontal center of mass of the object (i.e. the average horizontal center of the object) and represent locations to the left of that by negative numbers and to the right by positive numbers. With this labeling, the local physical center of the object is located at a value equal to plus (rightward peak) or minus (leftward peak) the edge modulation amplitude. The observers' data were subjected to probit analysis, yielding the perceived horizontal center (the 50% point on the best-fitting probit function) and the bisection threshold (the standard deviation of this function).

The horizontal difference between the perceived centers of the object measured at the two vertical locations (in line with a leftward or a rightward peak of the modulated

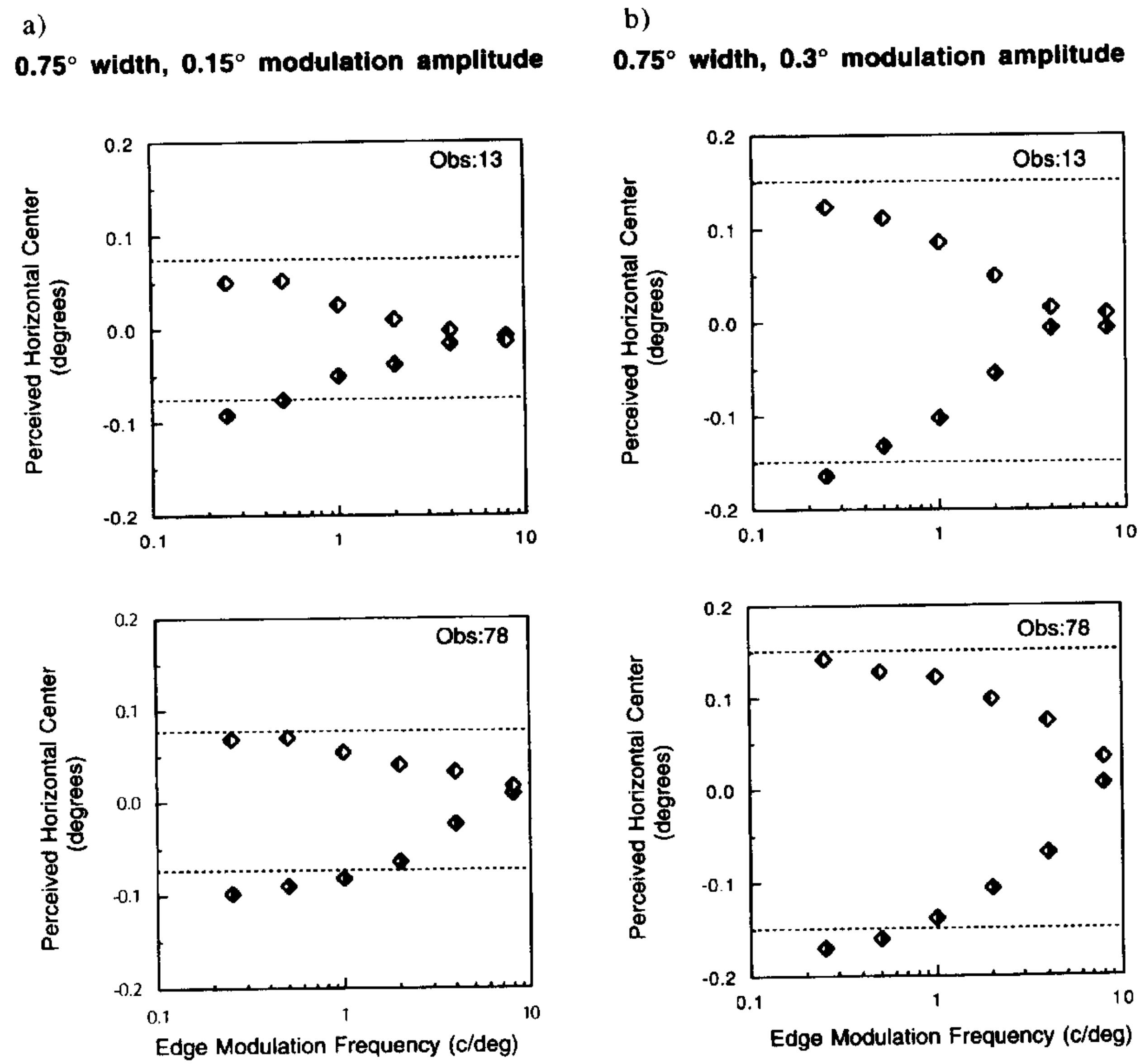

FIGURE 3(a,b). *Caption opposite.*

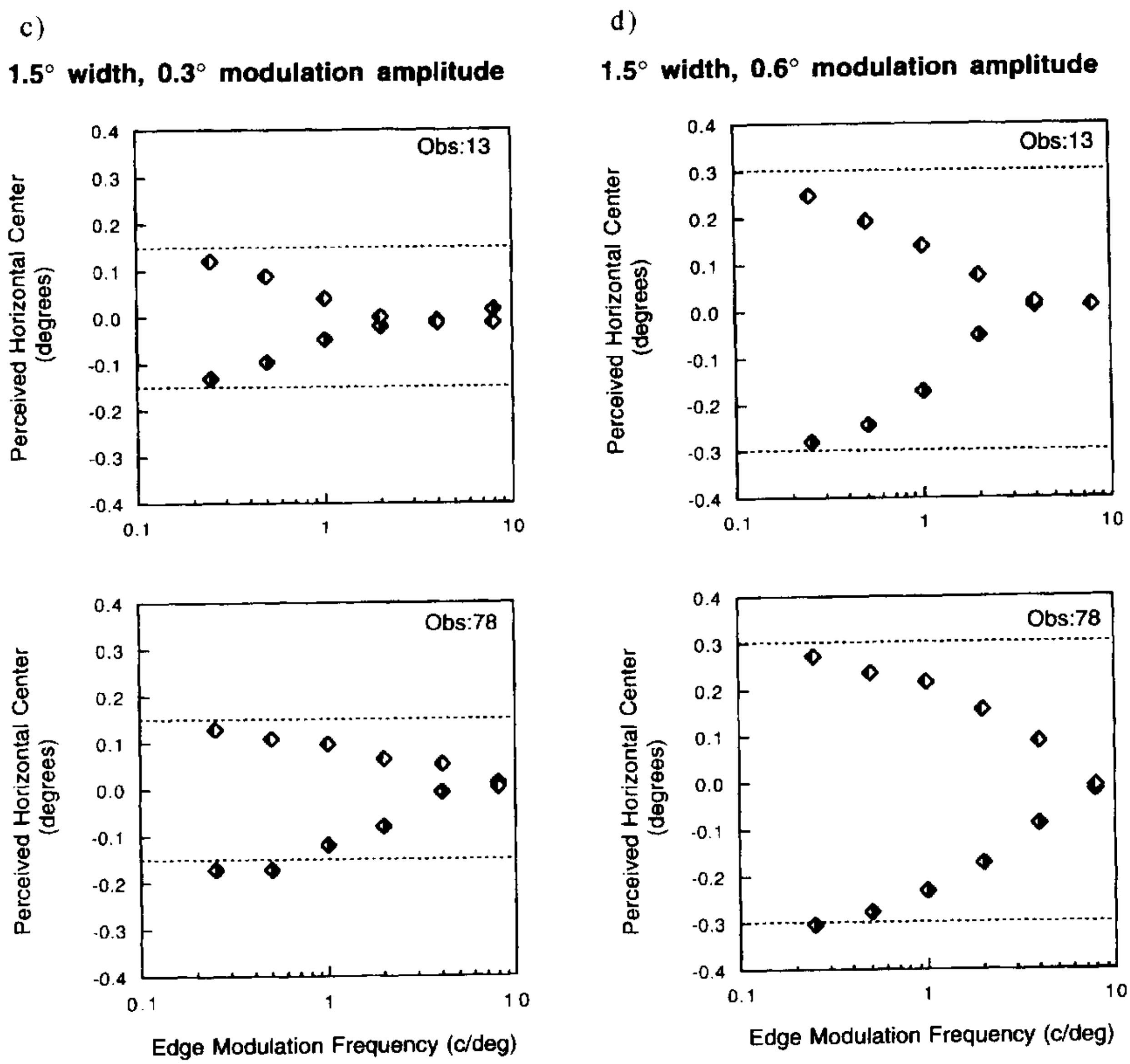

FIGURE 3. Perceived horizontal location of the center when the probe dot was placed at a rightward jog of the edge modulation, shown by the upper curve, and when placed at a leftward jog of the edge, shown by the lower curve in each graph. Error bars were smaller than the symbols. Data are shown for two observers.

edge) was our measure of the effect of the edge modulation on the perceived shape of the stimulus. If the central axis of the object looked straight, then these two measurements should yield the same perceived horizontal center, whether the measurement was made in line with a leftward or a rightward peak. If, on the other hand, the object looked as wiggly as its edge, then the central axis should modulate exactly with the edge. In between these extremes, there would be some horizontal modulation of the perceived central axis.

In other experiments, data were also obtained on objects with straight central axes and modulated widths (e.g. the left half of Fig. 1) as controls. A slightly different range of edge frequencies was used in that study. All other details were identical.

Observers

Two paid students served as observers in this experiment. Both had normal vision. They were highly experienced in psychophysical tasks but were naive as to the purpose of this experiment.

RESULTS

Perceived center at leftward and rightward peaks of the edge modulation

The basic results are shown in Fig. 3. Each graph in this figure shows two plots: the horizontal location of the perceived center measured in line with a leftward peak and that of the perceived center measured in line with a rightward peak. These data are plotted as a function of the frequency of the edge modulation for a given width and edge modulation amplitude. The ordinate is the distance of the perceived center from the average horizontal center of the object. Locations that are left of center are shown as negative values on the ordinate. Data are shown for two observers. The two dotted lines in each graph indicate the measured local horizontal centers of the stimuli at the two vertical locations used. (Recall that zero represents the horizontal center of mass of the object.) The observers' results clearly deviate substantially from the measured local horizontal centers.

At low spatial frequencies of the edge modulation, the perceived centers at the leftward and rightward peaks were near the measured local centers (the dotted lines) and quite far from one another, indicating that the perceived center modulated with the edge. As edge modulation frequency was increased, the difference between the two perceived centers diminished until, at the highest frequencies tested, the two were aligned, indicating that the center appeared straight. If the observers were making a purely local bisection judgment, there would be no effect of edge modulation frequency: the data would lie along the dotted lines. Instead, the data suggest that the observer is acquiring edge information for this task across a substantial spatial extent in spite of the experimental instructions to make a local, one-dimensional judgment of the center between two peaks. We conclude that the observer does not have precise spatial information about the locations of the two peaks available for use in this task.

The convergence of the two perceived centers with increasing frequency, i.e. the perceived straightening of the center of the object, is consistent with the perception of the object as a whole: it too appears to straighten. Thus, the results of the bisection task appear to capture the effect of the edge modulation on the perceived shape. This result is consistent with the idea that some basic shape information inheres in a medial representation.

Control experiments conducted with stimuli in which the two edge modulations were out of phase show the expected straight middles for these stimuli. These results are shown in Fig. 4.

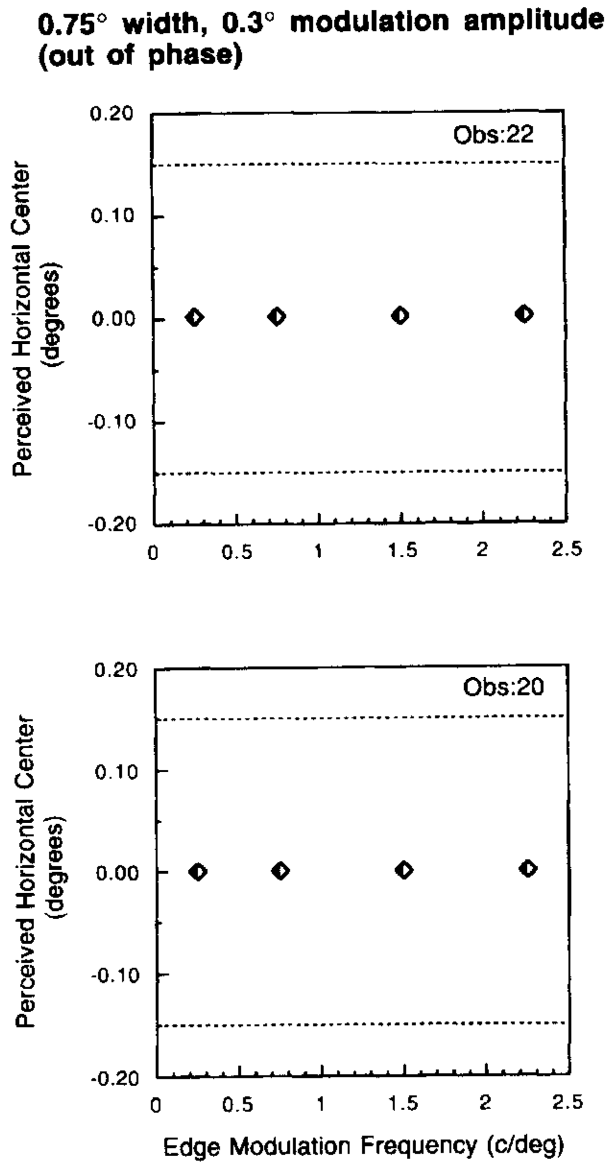

FIGURE 4. Perceived horizontal location of the center when the probe dot was placed at a rightward jog of the left edge and when placed at a leftward jog of the left edge. The data for the leftward and rightward jogs superimpose, so only one data set is visible. Data are shown for two observers. The horizontal axis in this figure is not logarithmic, as it is in the other figures, because a different set of frequencies was used in this study.

Effect of object width on perceived central modulation

Our hypothesis is that the boundary integration areas underlying shape perception are the same as those underlying distance judgments and that these boundary integration areas increase in size with increasing object width. If this is true, then for a given edge modulation, increasing an object's width should decrease its perceived wiggliness because the larger associated boundary integration areas will damp the effect of the edge modulation more than will the smaller ones associated with a narrower object.

Figure 5 shows the interaction between object width and the effect of the edge modulation. In this figure, we represent the effect of edge modulation as the horizontal difference between the perceived center locations measured at the two vertical locations on each object. This measure captures the perceived modulation of the center of the stimulus (see Methods). The larger its value, the larger the effect of the edge modulation on the perceived centers. The data shown in Fig. 5 were obtained with an edge modulation amplitude of 0.3 deg for the two object widths, 0.75 and 1.5 deg. Each graph in this figure shows two curves, one for the wider and one for the narrower object. Results are shown for two observers.

As was apparent in Fig. 3, the effect of edge modulation on the perceived center decreases as the edge frequency increases. At the lowest edge frequency, the center modulated with the edge; at the highest edge frequency, the centers were aligned with one another, i.e. the objects appeared nearly straight. This dependence on the edge modulation frequency suggests that information about

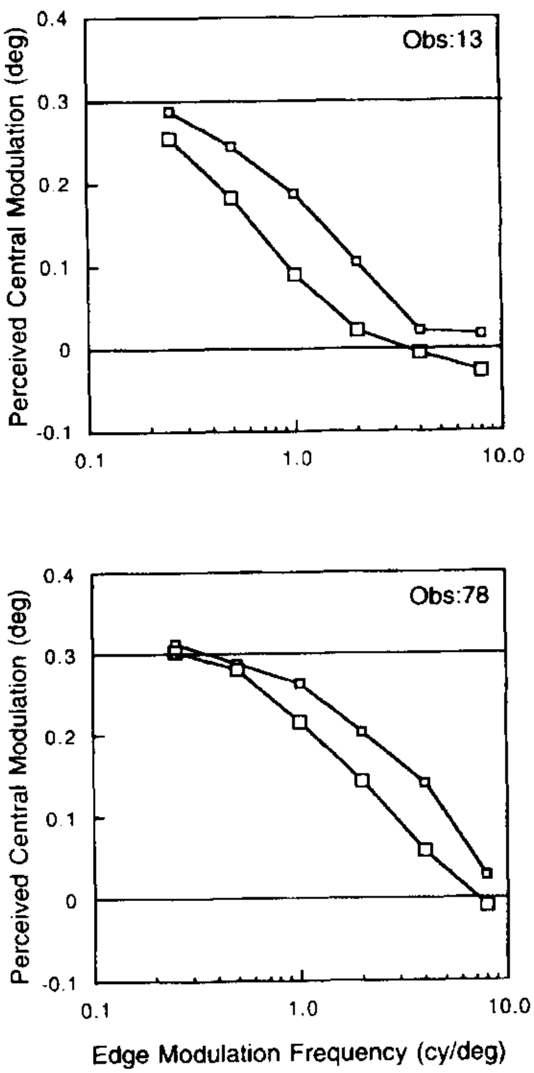

FIGURE 5. Perceived central modulation for two observers as a function of the frequency of the sinusoidal edge modulation. In each graph, data are shown for objects of two widths: 0.75 deg width is shown by small symbols; 1.5 deg width is shown by larger symbols. The edge modulation amplitude was 0.3 deg. The error bars were smaller than the larger symbols and are not shown.

edge location is gathered over an extended spatial area for this task.

The data also show clearly that the effect of edge modulation depended on the object's width. For a given edge frequency, the narrower object had a larger perceived central modulation than did the wider object. This corresponds to the percept. For a given edge frequency, the narrower object appeared more wiggly than the wider object. As previously noted, this result is consistent with the idea that the area over which edge information is integrated increases with the width of the object.

Perfect zoom invariance?

Given the—at least approximate—invariance of shape perception to zoom, it is to be expected that the perceived shape would change when the object width was changed but the parameters of the edge modulation were kept constant. Zoom invariance requires that the edge modulation be scaled with the object's size. In our results edge modulation frequency had a different effect for the wider than for the narrower object. If perfect zoom invariance can account for this difference, then we should be able to eliminate it by halving the edge frequency (i.e. doubling the period) and doubling the edge modulation amplitude for the wider object. We assumed that, for the relatively local central judgments our observers made, our stimuli were long enough that length was not a factor, and we held length constant.

Data were obtained using edge modulation amplitudes of 0.15 for the 0.75 deg object and 0.3 deg for the wider object, giving a constant 20% modulation across object widths. Data were also obtained with a 40% amplitude at both widths. We scaled the edge modulation frequency by shifting the curve for the wider object by a factor of 2 to the right on the horizontal axis, so that the data for the wide object at frequency f were compared to those for the narrow object at frequency $2f$.

Figure 6 shows data for the two edge modulation percentages, 20 and 40% of the object's width, with the curve for the wider (1.5 deg object) translated along the horizontal axis by a factor of 2. The perceived central modulation is plotted as a percentage of the edge modulation amplitude because the question of interest is whether the perceived central modulation bears the same relationship to the edge modulation for the two widths, after the frequency-scaling has been done. Data are shown for two observers for each modulation amplitude.

Simple scaling of the edge modulation by a factor of 2, i.e. by the relationship between the measured widths of these objects, accounts for the effect of width only approximately. For one observer in one condition, Fig. 6(a), the fit is excellent, but for the other three data sets, it is not. There are clear differences between the data for the narrow object and for the wider one when amplitude and modulation are scaled by a factor of 2.

A common mechanism for size discrimination and shape perception

The failure of perfect zoom invariance shown above provides us with a rigorous test of our hypothesis that a common underlying mechanism, namely an increase in the relevant boundary integration area with object width, underlies both size discrimination and (silhouette-based) shape perception. If there is a common underlying mechanism responsible for the increase in bisection thresholds with width and for the decrease in edge modulation effect with width, then these two phenomena ought to scale with width in the same way. In light of the above finding that zoom invariance did not hold perfectly for our perceived central modulation data, this means that it should also not hold for our bisection results. More strongly, it means that another scaling factor should hold for both.

The bisection thresholds are shown in Table 2. Thresholds were averaged over edge modulation frequency, which had no systematic effect. Perfect zoom

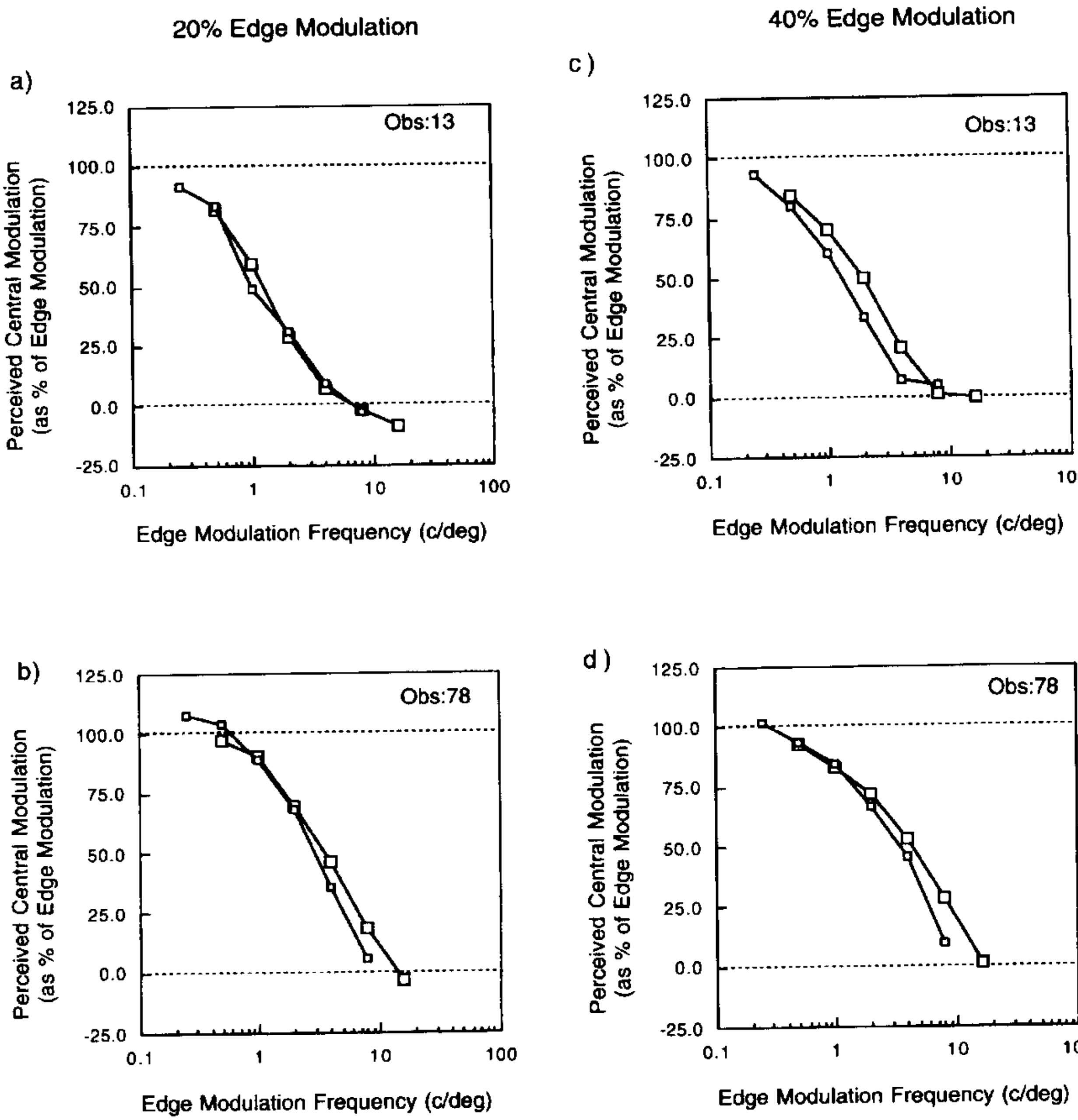

FIGURE 6. Perceived central modulation (as a percentage of the edge modulation amplitude) is shown as a function of edge modulation frequency for two modulation percentages: 20% in (a) and (b); 40% in (c) and (d). The data for the 1.5 deg object have been translated along the horizontal axis by a factor of 2 (see text). Each graph shows data for one observer. Small symbols indicate the results for an object of width 0.75 deg; larger symbols are for an object of width 1.5 deg. Error bars were smaller than the larger symbols and are not shown.

invariance did not hold, just as it didn't hold for the perceived central modulation. Doubling the width consistently produced less than a doubling in the bisection threshold, regardless of edge modulation amplitude. The scaling factor that does hold for the thresholds can be found simply by taking the ratio of the bisection thresholds for the wider and for the narrower objects, for the individual observers and conditions. The crucial test is whether these individual scaling factors can account for the effect of object width on the perceived central modulation.

To determine whether the bisection threshold ratio is the appropriate scaling factor for the perceived central modulation data, we need to determine the effect of scaling the edge modulation of the wider object by this factor. Scaling the frequency of the edge modulation by the bisection threshold ratio is readily done by translating the curve for the wider object along the horizontal axis by that ratio (the same technique that we used to test perfect zoom invariance). Scaling the amplitude of the edge modulation cannot be done by such a simple process because data were obtained at only two amplitudes. These

TABLE 2. Bisection thresholds

Observer	Amplitude	Width	Threshold (deg)
13	0.15	0.75	0.0378 ± 0.00381*
	0.3	0.75	0.0288 ± 0.00184
	0.3	1.5	0.0511 + 0.00601
	0.6	1.5	0.0437 ± 0.00285
78	0.15	0.75	0.0198 ± 0.00188
	0.3	0.75	0.0185 ± 0.00183
	0.3	1.5	0.0309 ± 0.00273
	0.6	1.5	0.0289 ± 0.00200

*Twelve threshold values were measured for each amplitude and width for each observer. 6 edge frequencies × 2 vertical locations. The reported error is $\sigma \sqrt{12}$, where σ is the standard deviation of the distribution of 12 threshold values, assuming they are samples from a normal distribution.

two amplitudes did provide a range within which the scaled data should fall, however (the bisection threshold ratios were near 1.5, and we had data representing ratios of 1 and 2). As it turned out, the perceived central modulation scaled almost perfectly with amplitude, so these flanking values tightly defined the required outcome.

We begin with the data for the 0.75 deg object with 0.3 deg edge modulation, as originally plotted in Fig. 5, for two observers. The question, then, is whether the data for the 1.5 deg object will superimpose if the edge modulation is scaled by the appropriate bisection threshold ratio. All analyses were done individually for each observer. We compare the data for the 0.75 deg object with 0.3 deg edge modulation to the data for the 1.5 deg object with both 0.3 deg and 0.6 deg edge modulation to bracket the effect of scaling the amplitude of the edge modulation. The bisection threshold ratios relating the thresholds for the wider object with 0.3 and 0.6 amplitude to the threshold for the narrower object with 0.3 amplitude were as follows: Observer 13, 1.77 and 1.52; Observer 78, 1.67 and 1.56. If the bisection threshold ratio is the appropriate scaling factor, then the data for the 0.75 deg object should lie between these two curves for the 1.5 deg object (after they have been translated along the horizontal axis by their threshold ratios).

Figure 7 shows data for the 0.75 deg object with 0.3 deg edge modulation plotted together with the two curves obtained with the 1.5 deg object shifted by the appropriate bisection threshold ratios. The two shifted (dashed) curves for the 1.5 deg object differ little (if any) from one another, indicating that perceived central modulation scales almost perfectly with the edge modulation and providing a strong test of the prediction that the curve for the narrower object will lie between them.

The results, shown in Fig. 7, are remarkable. If the bisection threshold ratio accounts for the interaction between object width and the effect of the edge modulation, then the solid curve for the narrower object should lie between or superimpose on the two dashed curves for the wider object. (It should lie between the two curves if the perceived central modulation is not strictly proportional to the edge modulation amplitude, and superimpose on the two curves if it is.) The agreement is excellent. The ratio of the bisection thresholds is precisely the scaling factor required to account for the interaction between object width and the effect of edge modulation. This is strong evidence for the idea that the same mechanism is responsible for the increase in size discrimination thresholds with size (Weber's law for size) and the approximate zoom invariance of shape perception, as measured by the perceived central modulation. This conclusion is heightened by the fact that the assumption of perfect zoom invariance did not account for the data as well.

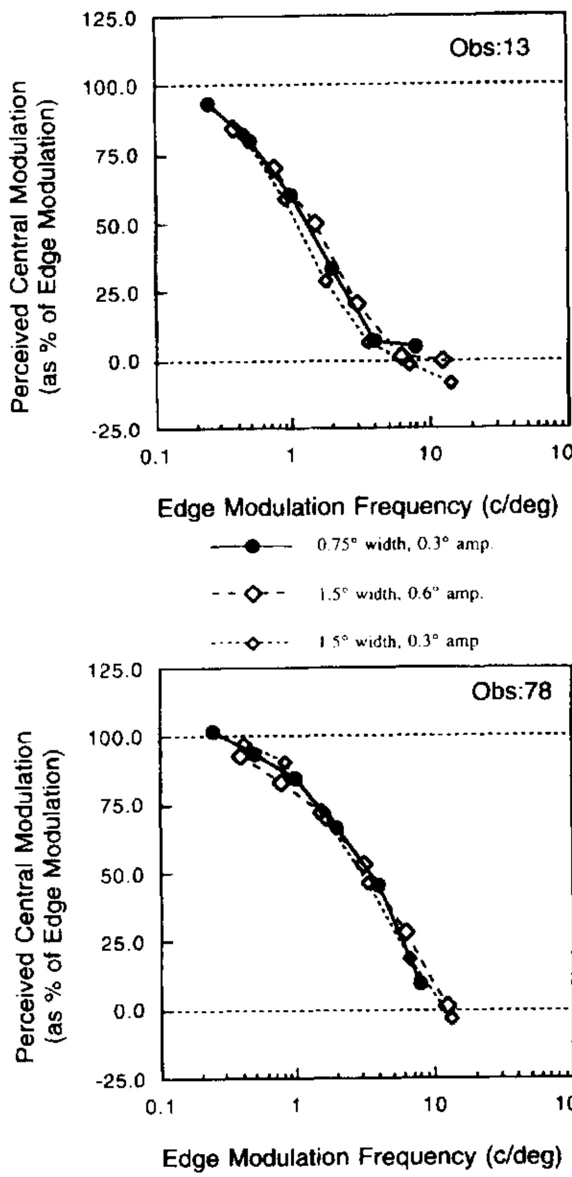

FIGURE 7. Perceived central modulation (as a percentage of the edge modulation amplitude) for the narrow object with 0.3 deg edge modulation amplitude plotted against the data for the wider object translated along the horizontal axis by the bisection threshold ratio (see the text). Data are shown for two observers.

MODELING THE RESULTS

Many psychophysical and physiological results point to there being multiple spatial scales available to detect a given boundary [see Graham (1989) for a review of related results]. We propose that spatial scaling also occurs in the communication between scaled units. Specifically, we propose:

(1) That the creation of object-specific representations involves the linking of opposing boundaries of coherent spatial regions.
(2) That this linking is done by the same process that underlies spatial position judgments, such as bisection and separation discrimination.
(3) That the process that is responsible for both of these abilities links disjoint regions in a scale-dependent way with larger boundary integration areas being linked across larger distances and smaller boundary integration areas being linked across smaller distances.

We have proposed a mechanism for establishing this linking-at-scale (Pizer, Burbeck, Coggins, Fritsch & Morse, 1994; Burbeck & Pizer, 1995) which we call core-based-analysis. Core-based-analysis yields a medial-axis description of simple spatial objects in which the medial axis is created at a scale proportional to the object's width, with a resolution proportional to that width. [See Kovacs and Julesz (1994), for recent psychophysical evidence, and Lee, Mumford and Schiller (1995), for recent physiological evidence favoring the existence of a medial representation.] This scaling of the aperture size with the distance being spanned yields perfect zoom invariance and size discrimination thresholds that scale perfectly with size.

We applied core-based-analysis to some of the stimuli used in this experiment to test its behavior against the observers' results. Because the task was horizontal bisection, we limited the set of boundariness detectors to those with vertical orientations that were horizontally aligned with the probe dot. The boundariness detectors were first derivatives of Gaussians. (For filled images with uniform luminance, one stage of processing suffices. Two stages are generally required to separate the scale at which contrast detection of the boundary information occurs from the scale at which position information is integrated, the latter being determined by the scale of the object.) The scale of the boundariness detectors that were linked to create the core is determined by the linking process itself: one need not know it in advance. A free parameter in the model is the value of the ratio, r/σ, where σ is the scale of the boundariness detector and r is the distance at which it communicates with other boundariness detectors of similar scale. The value of the ratio together with the width of the object determine the values of r and σ that will contribute most to the determination of the core. We checked several (integer) values of this ratio and show here the best fit (from those tested).

We applied the core model to the narrow and wide objects with 40% edge modulation, determining two core locations for each object, one for each of the two vertical locations used in the experiment. The difference in the x-values of these predicted central locations is the predicted perceived central modulation of the object. Results from the core model with a scaling ratio, $r/\sigma = 4$, for the two stimulus widths are shown in Fig. 8(a). Comparable observers' data are shown in Fig. 8(c). The main effects of edge modulation frequency and object width are similar to those seen in the observers' data for this modulation amplitude, but the model predicts a larger effect of object width, as expected because of the zoom invariance assumption. Using the ratio of the observer's bisection thresholds as the scale factor eliminates this discrepancy. Figure 8(b) shows the expected results of the model for the case in which the standard deviation of the Gaussian boundary integration area for the wider object is 1.6 times that for the narrower object (1.6 being the average ratio obtained with the observers). The model captures well the difference between the results for the wide and narrow objects when this smaller scaling factor is used and is an adequate quantitative approximation to the average observers' data.

The shapes of the predicted functions [Fig. 8(a, b)] differ somewhat from those for the observers [Fig. 8(c)]. The observers' data are more nearly linear (on the log-linear coordinates) over much of the measured range, whereas the model predicts a more Gaussian form, with an apparent inflection point. The shape of the boundary integration areas we chose seems the most likely candidate to account for this difference. Even as it stands, however, the model is consistent with the basic characteristics of the observers' performance when the appropriate scaling factor is used.

SUMMARY OF EXPERIMENTAL AND THEORETICAL FINDINGS

The primary result was that observers could not make point-to-point bisection judgments of our edge modulated stimuli. Instead, the data indicated that more extended regions of boundary were the basis for their judgments. Furthermore, the boundary areas that contributed to judgments of the wider object were larger than those that contributed to judgments of the narrower object.

We also found that the well-known scaling of size discrimination thresholds with size accurately matched the scaling of silhouette-based shape perception. Specifically, the ratio of the bisection thresholds accounted for the effect of object width on perceived shape, as measured by the perceived central modulation. We concluded that a common mechanism was responsible for both phenomena.

We compared predictions of the core model of shape representation to observers' responses to the wiggly-edge stimuli. In the core model, boundaries are linked in a size-dependent way through a medial representation. Use of a medial representation was supported by the fact that the perceived central modulation varied in a way that was

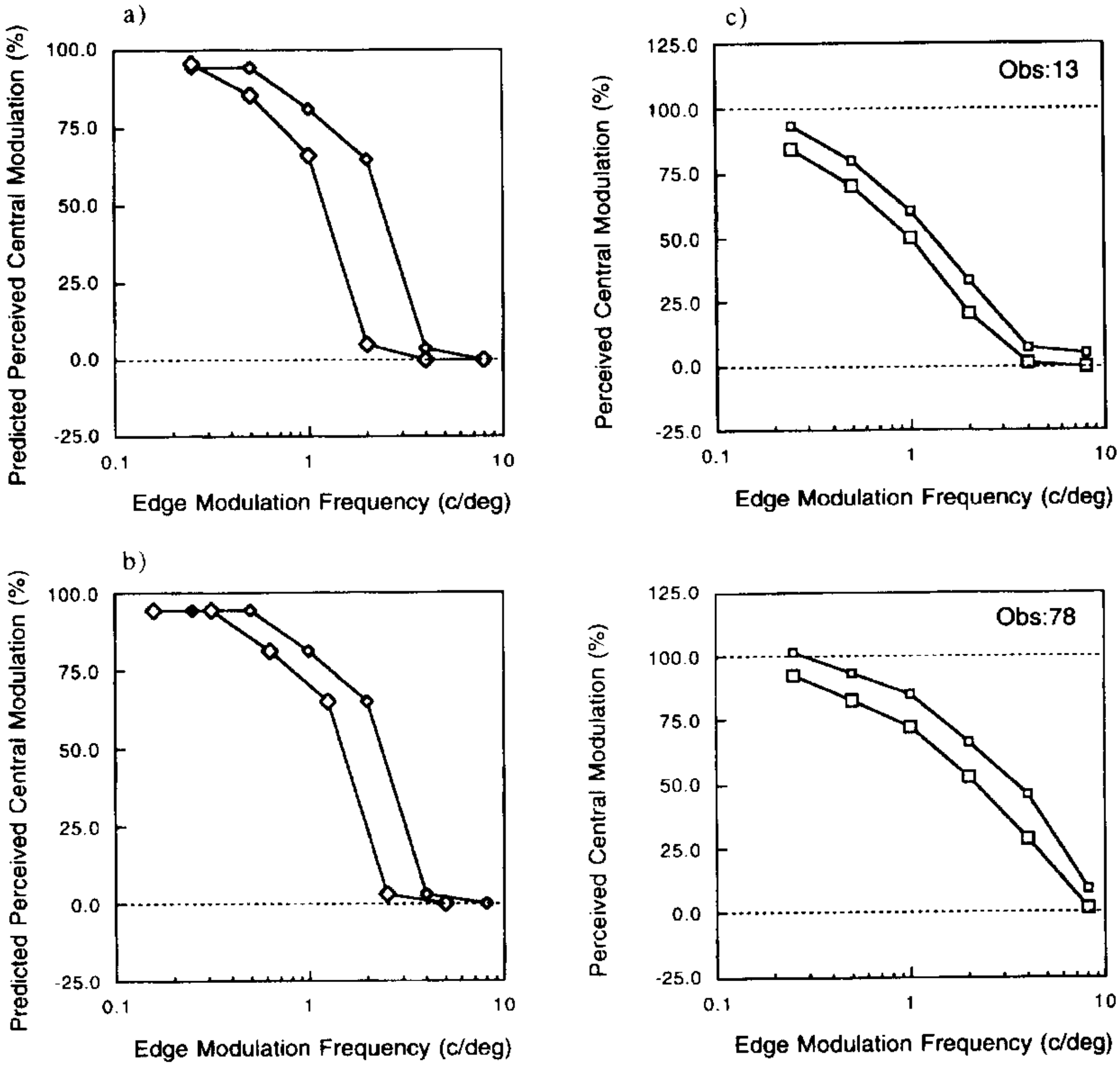

FIGURE 8. (a) Perceived central modulation predicted by core-based-analysis of the 40% edge modulation stimuli used in the experiment. Predictions for the narrower (0.75 deg) object are shown by the small symbols and those for the wider (1.5 deg) object are shown by the larger symbols. (b) Perceived central modulation predicted by core-based-analysis but using a scale factor of 1.6, which is the average bisection threshold ratio obtained from the observers' data. (c) Data from two observers for the same 40% edge modulation stimuli.

consistent with the subjective percept. The core model captured the effect of object size when the scaling factor inferred from the observers' data was used, but predicted too large a difference when zoom invariance was assumed. The model predicted the decrease in perceived central modulation with increasing edge frequency, although the form of that function differed somewhat from the observers' data.

DISCUSSION

Scaling of the relevant boundary integration areas is consistent with a restriction on visual processing that has been suggested by Nakayama (1990) and by Van Essen and Anderson (1990). They have both proposed that there is a limited amount of information that can be held in a given representation, so that if a large area is to be represented, the sampling aperture size must be larger, in absolute terms, than if the area to be represented is smaller. To get our results from such a hypothesis, one only needs to add the requirement that two boundaries can be related only in representations that contain them both. The core model provides a means of determining the appropriate scales for each object and results in an explicit representation of shape. Thus, core-based-analysis is completely consistent with limited resolution ideas.

Our experimental results are also qualitatively consistent with Kimia, Tannenbaum and Zucker's (1992, 1995) curve-evolution model of shape. In their model, boundaries propagate waves of activity which interact when they meet. The distance that the wave travels determines its effective resolution. Thus, their model predicts that the center of the wider object would be less modulated than the center of the narrower one, as we found. The curve-evolution model requires that the object boundary be found first, however, which is itself a difficult problem.

Our experimental results also have interesting implications for the considerable body of work that has been done on 2D distance judgments, dating back to the time of Fechner [for a historical survey of the early results, see Wolfe (1923)]. Our data revealed a strong quantitative connection between the variance of size judgments, i.e. the size discrimination threshold, and the perceived shape of our 2D regions, as measured by the perceived central modulation. The existence of this connection suggests that other experimental results on large-scale size discriminations,* e.g. those on the effects of retinal eccentricity, exposure duration, contrast, spatial frequency characteristics of the targets, effect of distractors etc., may be useful guides for the development of our understanding of silhouette-based shape representation.

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*Small-scale size discriminations are probably limited by distal spatial processes [Klein & Levi, 1985; Wilson, 1986].

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