Observation of the fundamental length scale of Branched Flow of light

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Abstract: Branched flow is a universal phenomenon in which waves form channels of enhance intensity that keep dividing as they propagate. We experimentally demonstrate the scaling behavior of the fundamental branching length for general anisotropic media.
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The study of waves in random media, such as the atmosphere, the ocean, and biological media, has become increasingly important in areas of directed energy, communication and sensing. A universal wave phenomenon that has been often overlooked is branched flow. It occurs when waves propagate in a smooth disordered potential with correlation length longer than the wavelength. During branched flow, waves form channels of enhanced intensity that keep dividing as they propagate, creating a beautiful pattern that resembles the branches of a tree (Fig. 1). Branched flow was first observed for electrons in semiconductor heterostructures [1], and later in microwave cavities [2], but it can occur for any kind of wave with vastly different wavelengths, for example, it has been proposed as a focusing mechanism of tsunami waves [3]. Recently, branched flow of light was observed for the first time [4] by propagating a laser beam inside a thin liquid film – where the local thickness variation of the membrane lead to a smooth disorder effective index of refraction landscape that generates the branched flow of light.

During branched flow, the smooth disordered potential is characterized by two parameters, its correlation length l_c, and its strength v_0, i.e. the standard deviation of the potential. The fundamental length scale of branched flow is the statistical distance from the input beam to the first branching point and it is given by $d_0 \propto l_c(v_0)^{-2/3}$. This fundamental length scale is independent of the exact spatial structure of the potential and does not depend even on the form of the correlation function, which may be any smooth function; thus, is as universal as the concept of mean free path. This scaling relation has only been previously verified for microwaves transmitted in a random potential for three different isotropic configurations in isotropic potentials which have circular correlation functions [2]. For a general anisotropic disorder medium $l_c \propto (ab)^{4/3}(a^2cos^2\alpha + b^2sin^2\alpha)^{5/6}$, where α is the angle between the coordinate axes and the principle axes of the correlation function of the disordered medium, the parameters a and b are the inverse correlation lengths along the principle axes [6]. This relation has never been experimentally verified before for any kind of waves.

Here, we experimentally verify, for the first time, the scaling of the fundamental branching length for branched flow in general anisotropic media. In our experiments, we couple a plane wave to a thin liquid film and

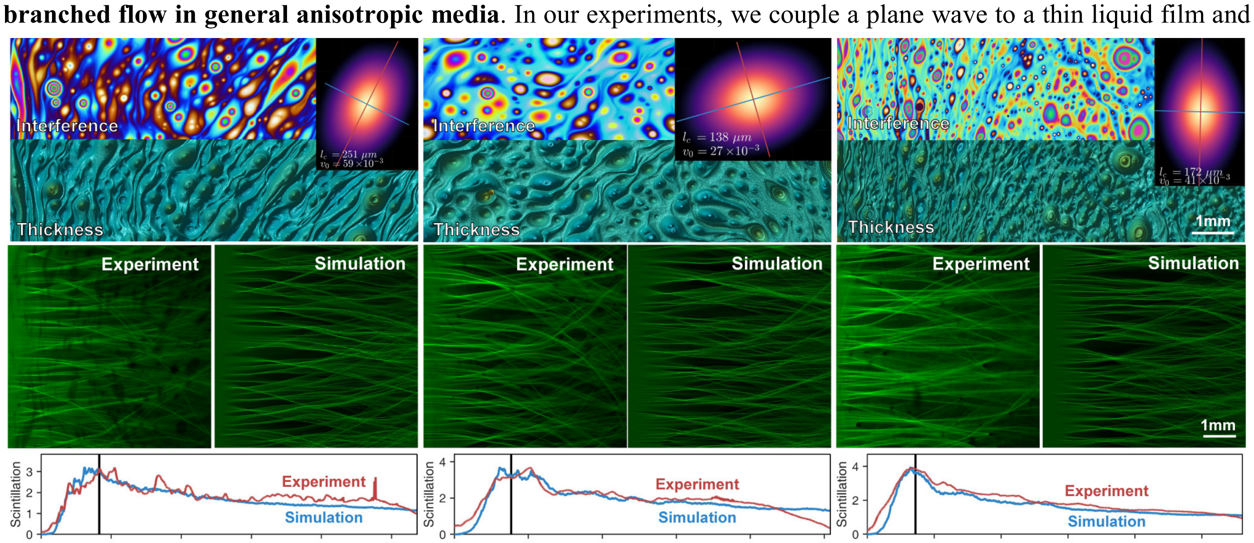

Figure 1: (Left,Center,Right) Three observations of branched flow of light in thin liquid films with anisotropic disorder. For each we show the true-color interference of white light reflected from the membrane follow by the reconstruction of the membrane thickness, which maps to a refractive-index landscape. The inset shows the autocorrelation of the potential that shows the anisotropy of the disorder. **(Middle row)** Microscope images of the observed branched flow of light when a plane wave propagates in the potential landscapes and its corresponding simulation of a plane wave propagating in the reconstructed potential obtained from the interference pattern. **(Bottom row)** (Blue line) respective scintillation index as a function of the propagation distance z, extracted the transverse average of ~25 experimental realizations. (Red line) simulated scintillation index calculated from the numerical propagation. (Black line) Predicted peak position, $d_0 \propto l_c(v_0)^{-2/3}$.

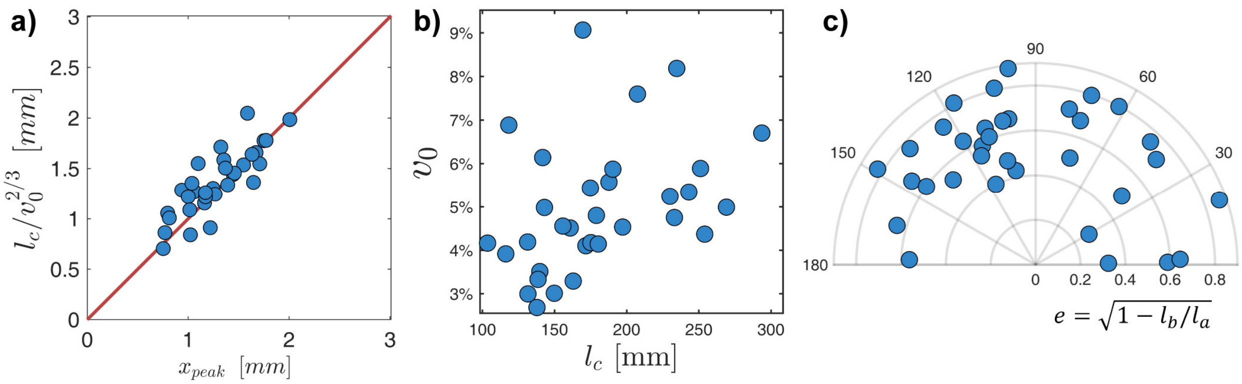

Figure 2: **(a)** Experimental verification of the fundamental branching length: the experimental peak position of the scintillation index is plot versus the theoretical expected value given by the correlation length and strength of the measured potential. **(b)** Parameter space, correlation length and strength of disorder landscapes in the thin liquid films that we generate. The degree of anisotropy of such potentials is shown in **(c),** where we characterize each potential by its "elliptical" correlation function. The graph shows the orientation angle of the ellipse and its anisotropy is measure by its ellipticity, $e = \sqrt{1 - l_b/l_a}$, where l_a and l_b are the correlation lengths of the major and minor principal axis.

observe its propagation inside the membrane, as in [4]. The membrane acts as a two-dimensional slab waveguide that, in our parameter regime, supports a single guided mode. The local thickness variations of the membrane lead to variation of the effective index of refractions. More importantly, by manipulating the thin liquid film we can create disorder landscapes of smooth thickness variations that are mapped to anisotropic disordered correlated potentials that induce branched flow of light.

To characterize the branched flow of light inside the thin liquid film, we couple a plane wave inside the membrane doped with a fluorescent dye and observe only the fluorescents produce by the propagation, as show in Fig. 1. Due to the small fluctuation of the membrane, produce by small air currents, we can record in few seconds many realizations (~25) of branched flow under a potential with the same parameters. From this experimental propagation, we measure the scintillation index: $S(z) = \langle I^2 \rangle / \langle I \rangle^2 - 1$, where I is the local beam intensity, and the average is taken over the transverse coordinate and over multiple realizations of the potential [2,3]. Some examples of the scintillation index measured in our experiments together with the simulated scintillation using the measured effective index landscape are shown in Fig. 1. The position of the peak of the scintillation index gives the fundamental branching length.

To characterize such index of refraction landscapes, we construct an interference microscope in which we illuminate the thin liquid film with RGB illumination, and we observe the true-color interference patterns shown in Fig. 1 top. From this colorful interference pattern, we numerically reconstruct the thickness variation of the membrane (Fig. 1 second row), and finally, we map this thickness to an effective refractive index potential. From this potential landscape, by taking the autocorrelation, we measure l_a and l_b, the correlation lengths of the potential along the principle axes of the medium and α, the angle between the coordinate axes and the principle axes of the medium and by measuring its standard deviation we measure the potential strength, v_0. More importantly, by manipulating the thin liquid film, by mixing or changing the surfactant concentration, we are able to produce a wide range of potential landscapes. We generate 32 different index of refraction landscapes and measure, for each, ~25 realizations of branched flow. More importantly, we sample a very broad parameter space, as shown in Fig 2b, we generate potential with $v_0 = 2\%$ to 8% (where 10% is the theoretical maximum to still get branched flow) and with an effective correlation length that goes from 100λ to 500λ. Respect, to the anisotropy of the potential we are able to generate potential with ellipticity from 0.36 to 0.9 and oriented at practically any angle, Fig 2c. Thanks to this broad parameter range we are able to experimentally verify the scaling relation of the fundamental branching length as shown in Fig. 2a, where we can clearly see that the experimental branching length, the blue dots, follows the expected scaling behavior, red line.

Understanding this fundamental branching length is important to bring branch flow closer to applications. In the same way as diffusion in random media has found a plethora of application, we expect in the near future that branch flow in smooth disorder media will find interesting applications.

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