nature photonics

Perspective

https://doi.org/10.1038/s41566-023-01307-9

The bright prospects of optical solitons after 50 years

Received: 31 May 2023

Accepted: 12 September 2023

Andrea Blanco-Redondo[1] ✉, **C. Martijn de Sterke**[2], **Chris Xu**[3], **Stefan Wabnitz**[4] & **Sergei K. Turitsyn**[5]

The idea of temporal solitons in optical fibres as a means to compensate for the chromatic dispersion-induced temporal broadening of pulses via the intensity-dependent refractive index of silica (the Kerr effect) was introduced by Hasegawa and Tappert in a paper that was submitted on 12 April 1973. In this Perspective we present a brief historical overview of how this prediction developed in light of other technological developments made through the decades, followed by an extensive forward-looking discussion on the most exciting opportunities in soliton research.

The idea of temporal solitons in optical fibres, as a means to compensate for the chromatic dispersion-induced temporal broadening of pulses via the intensity-dependent refractive index of silica (the Kerr effect), was introduced by Hasegawa and Tappert in a paper that was submitted on 12 April 1973[1]. At that time, the first generation of fibre-optic transmission systems operating near 800 nm was yet to arrive, and fibre losses were hovering around 20 dB km^{-1}, that is, 100 times larger than in fibre links of today. However, the authors made a remarkable visionary prediction: "if the absorption is small and the nonlinear term can be made comparable to the dispersion term"—which meant 1 W of peak power for a 1 ps pulse—"one can expect to obtain a significant modification of the pulse shape", since the pulses could propagate unchanged for indefinite distances in the form of an optical soliton. Earlier, Zakharov and Shabat had discovered that one of the key mathematical models for fibre optics and nonlinear science, that is, the nonlinear Schrödinger equation (NLSE), is integrable using the powerful inverse-scattering transform method, and they found analytically single- and multi-soliton solutions of this equation[2]. With a background in plasma physics, Hasegawa knew about the existence of solitons in the analogous context of self-focusing that was considered in ref. 2—see Hasegawa's personal perspective on the discovery of optical solitons[3]. This led him to make the imaginative and inspiring prediction that, one day in the future, solitons could transport information across the globe at terabit rates. We now provide a brief historical overview of how this prediction evolved, and aim not at a comprehensive review of the field but instead provide a brief illustration of how optical temporal soliton research has transformed over the decades in conjunction with other technological breakthroughs. For recent historical overviews of temporal optical solitons during the past 50 years, we refer the reader to a feature article[4] and to the latest edition of Agrawal's *Nonlinear Fiber Optics*[5].

Solitons in optical fibres were first observed in 1980 by Mollenauer, Stolen and Gordon at 1.55 μm (ref. 6). In addition to unperturbed propagation in the fundamental soliton regime, their paper reported the observation of higher-order soliton dynamics at higher powers, introducing a new application of solitons, that is, nonlinear optical pulse compression (with a factor of ~3.5-fold). Three years later, Mollenauer and colleagues demonstrated a compression factor of ~27 with solitons of order $N = 13$, transforming 7 ps pulses into 260 fs pulses[7]. By the end of the 1980s, pulses with only a few cycles had been achieved via higher-order soliton compression—see the personal perspective from Taylor on this achievement[8].

Ten years after the first demonstration of temporal solitons, it was realized that optical solitons can survive periodic loss–gain variations of power and be transmitted over ultralong distances if they are periodically amplified, making path-averaged propagation approximately equivalent to the lossless propagation[9]. Fast forward 20 years from the paper of Hasegawa and Tappert: fibre loss in the third window around 1.55 μm is now reduced to 0.2 dB km^{-1}. This, in combination with the invention of the erbium-doped fibre amplifier (EDFA), virtually eliminated the loss limitation to the transmission distance. The focus then became alleviating the optical noise introduced by EDFAs. Noise randomly perturbs solitons: specifically, the frequency, and hence the arrival time of fundamental solitons, is affected by the Gordon–Haus

---

[1] CREOL, The College of Optics and Photonics, University of Central Florida, Orlando, FL, USA. [2] Institute for Photonics and Optical Sciences (IPOS), School of Physics, University of Sydney, Sydney, New South Wales, Australia. [3] School of Applied and Engineering Physics, Cornell University, Ithaca, NY, USA. [4] Dipartimento di Ingegneria dell'Informazione, Elettronica e Telecomunicazioni, Sapienza University of Rome, Rome, Italy. [5] Aston Institute of Photonic Technologies, Aston University, Birmingham, UK. ✉e-mail: Andrea.BlancoRedondo@ucf.edu

timing jitter[10]. Research on the suppression of timing jitter using dispersion-compensating fibres led to the experimental discovery and theoretical description of the dispersion-managed soliton[11–13]. The soliton pioneers of Bell Laboratories—Linn Mollenauer and Pavel Mamyshev—assessed the potential to transmit massive amounts of data over transoceanic distances by combining wavelength-division multiplexing (WDM), EDFAs and dispersion-managed solitons[14].

At the turn of the twenty-first century, the concept of the optical soliton became a commercial reality, when the team of Marconi Solstis, led by Nick Doran, installed a 5,745-km-long terrestrial fibre-optic system based on dispersion-managed soliton technology in Australia[15]. However, in the early 2000s, the introduction of phase-shift keying[16] and coherent communications for ultralong-haul WDM transmission, which enabled electronic compensation of transmission impairments via digital data processing, as well as machine-learning techniques, dramatically reduced commercial interest in on–off-keying modulation solutions, including soliton-based communication systems[17].

Nevertheless, the original optical soliton concept was augmented and reinforced by expansion to new applications, such as, for example, supercontinuum generation[18], frequency combs[19] and mode-locked fibre lasers[20], where solitons and dispersion-managed solitons spontaneously form from optical noise, an impressive example of self-organization in nature. Indeed, it is in the context of ultrashort pulse lasers that optical solitons have found their undisputed realm over the past 20 years: a leap forward in this direction was provided by extending the soliton concept to dissipative systems, where not only does nonlinearity balance dispersion, but also gain balances linear and nonlinear loss mechanisms[21,22], thus expanding soliton theory well beyond the path-average gain–loss dynamics studied in ref. 9. Indeed, dissipative solitons represent the true spatiotemporal eigenmodes of nonlinear optical cavities; hence, they are expected to provide the building blocks for a number emerging photonic technologies and applications, as we will discuss in the next sections. Figure 1 shows examples of the already emerged and emerging soliton technologies and applications, as well as some of the currently investigated novel soliton concepts.

The future of solitons in ultrafast laser technology

Soliton lasers are, arguably, the simplest and most efficient kind of mode-locked laser, since the fibre itself acts as a pulse-shaping mechanism by balancing (anomalous) dispersion and nonlinearity, negating the need for a recompressing stage to achieve transform-limited short pulses[23]. However, the soliton area theorem imposes a fixed relation between the width of the soliton pulse and the energy it carries. For NLSE solitons, the energy E scales as the inverse of the pulse duration T: $E \propto 1/T$. Although arbitrarily short pulses can lead to very high energies, in practice this regime is hard to achieve. This is so because when the characteristic nonlinear length (which is inversely proportional to the pulse peak power) becomes comparable to the length of the laser cavity, the periodic variation of power during round-trip makes the solitons unstable. This leads, first, to the generation of spectrally dispersive resonant sidebands and then to the destruction of the pulses. Research on high-energy solitons in lasers requires a modification of the initial concepts, and we anticipate that new soliton techniques, including, for example, new types of dissipative solitons, adiabatic in-cavity pulse dynamics and more complex nonlinear Poincaré mapping-based laser designs will advance soliton laser technology further. In particular, dissipative laser solitons, which are not limited by the conventional soliton area constraint, are excellent candidates for achieving high pulse energies[24,25]. The fundamental NLSE soliton results from a balance between dispersion and nonlinearity; therefore, by changing either the nonlinear or dispersive properties of the medium, one can scale the soliton characteristics. An exciting avenue for overcoming fundamental soliton energy limitations is through solitons with a fundamentally different energy-width scaling. Recently, there has been growing interest in solitons (or solitary waves) that arise from a balance of the Kerr effect and high even orders of dispersion ($k = 4,6,8...$), for which the energy scales as $E \propto 1/T^{k-1}$ (refs. 26–28), entailing an important energy advantage in the ultrashort regime with respect to NLSE soliton lasers. Advances in the engineering of dispersive properties of waveguides and materials may make these novel solitons a promising approach for tackling the fundamental energy limitations of soliton lasers[20,23,24] while maintaining the principles of simplicity, efficiency and robustness that made soliton lasers interesting to begin with.

An alternative research path to address the energy limitation of a soliton laser involves a reduction of the nonlinearity, using either a large mode area or hollow-core fibres. The soliton energy is inversely proportional to the nonlinear parameter $\gamma = 2\pi n_2/\lambda A_{\mathrm{eff}}$ of the waveguide, where n_2 is the material nonlinear coefficient, λ is the wavelength, and A_{eff} is the effective mode-field area. By either increasing the effective mode-field area A_{eff} or reducing the material nonlinearity n_2, soliton energies can be scaled up by several orders of magnitude in higher-order mode fibres[29], photonic crystal rods[30] and hollow-core photonic bandgap fibres[31]. Soliton generation in anti-resonant hollow-core fibres[32] and large-core hollow capillary fibres[33] can reach beyond microjoule and millijoule levels, respectively. Further to this, the use of these fibres has enabled higher-order soliton compression to subcycle pulses[33].

The approach of scaling up the energy of pulses from fibre lasers by increasing the fibre diameter leads naturally to the multimode fibre soliton laser. The beauty of solitons is that they may not only survive in multimode fibres but even lead to the compensation of both chromatic and modal dispersion via the Kerr nonlinearity, as again predicted by Hasegawa in his visionary 1980 paper[34]. Stimulated by the revival of interest in multimode fibres for space-division-multiplexing communications, it took as long as 33 years until multimode fibre solitons were experimentally observed by Renninger and Wise[35]. The nonlinearity-induced locking of different transverse modes of a fibre laser leads to a practical platform for total or spatiotemporal mode-locking[36], which may enable orders-of-magnitude increases in the pulse energy from mode-locked lasers, with respect to their single-mode counterparts. This is a somewhat remarkable property of multimode fibres, because in a bulk medium the multi-dimensional NLSE features self-focusing, wave-collapse and unstable solitons.

We anticipate that soliton-based technologies such as dissipative solitons, supercontinuum generation and the soliton self-frequency shift (SSFS) will play important roles in further advances in pulsed laser systems, both in single- and multimode fibres, expanding light emission to new spectral bands (for example, to the mid-infrared region[37]) and enhancing the control over lasing through machine-learning methods[38].

The future of solitons in microresonator frequency combs

Kerr microcombs are arguably one of the most important modern practical applications of the soliton concept[39]. Compact, chip-scale microresonators, operating at low power in the dissipative soliton regime with microwave-to-terahertz repetition rates, are already used widely in a range of applications, including metrology, optical communications, microwave devices, biomedical applications, high-resolution spectroscopy and others[19,39]. Frequency-comb technology is underpinned by soliton theory, and in more general terms by nonlinear science that governs the formation of coherent structures to ensure the phase locking of spectral modes in optical combs.

Although the progress in soliton Kerr microcombs has been outstanding over the past decade and various new concepts, such as soliton crystals[40], have arisen in the past few years, some important roadblocks remain for the practical exploitation of microcombs, for instance, the need for self-starting operation (see, for example, the discussion in ref. 41) and power efficiency for conversion of the continuous-wave

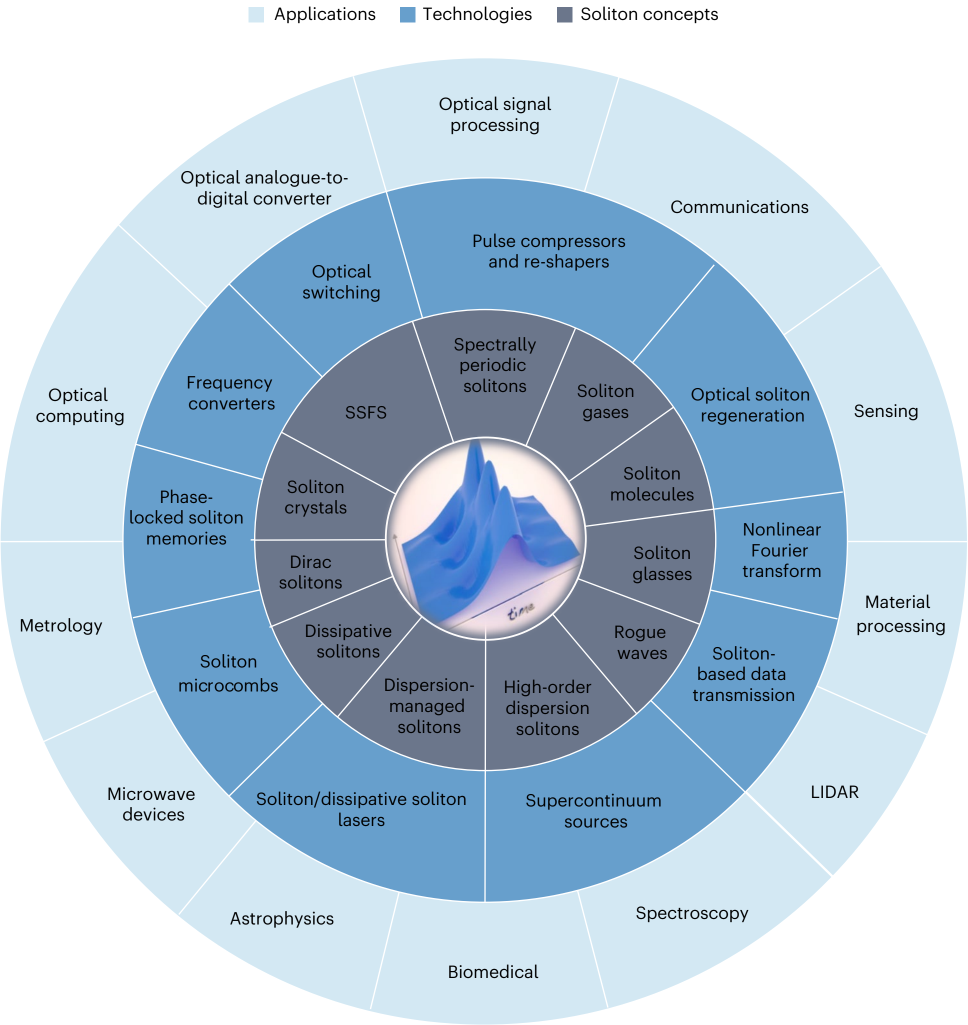

Fig. 1 | Soliton concepts, technologies and applications. Examples of soliton theoretical concepts, technologies, as well as the current and potential future domains of application.

pump laser to the soliton comb. New materials, for example, quadratic nonlinear materials, have the potential for self-starting and ultralow-pump-power soliton frequency combs[42]. Note that an effective quadratic nonlinearity could also be introduced in a cubic nonlinear material, via thermal/optical poling, or by applying a static field.

In relation to the trend of leveraging high-order dispersion for improving performance discussed in the previous section[26–28], it has been suggested that building soliton Kerr microcombs based on quartic dispersion rings can lead to a flattened spectrum, and hence to lower variations between frequency lines, a broader bandwidth and even an increased pump-to-comb conversion efficiency[43]. Although a soliton Kerr resonator based solely on high even-order dispersion has not yet been experimentally realized, there are already promising designs available[44,45] suggesting that this field may be destined to produce some disruption in an already well-established soliton microcomb community.

The future of solitons in biomedical applications

The flexibility and convenience provided by waveguide solitons and the SSFS have already found applications in biomedical imaging, such as multiphoton microscopy and coherent Raman scattering imaging[46], that rely on short-pulsed lasers. Although the advantages of a femtosecond source based on solitons and the SSFS are well recognized (for example, compact and low cost, among others), these sources are mostly used in research laboratories, and many of them have been used for proof-of-concept demonstrations rather than for routine applications. Despite their high costs, until today, the workhorses of multiphoton imaging are still bulk mode-locked lasers and the recently developed high-repetition-rate (several megahertz) non-collinear optical parametric amplifiers (NOPAs). For systems based on solitons and the SSFS to have widespread biomedical applications, the performance of future soliton-based systems must match or exceed that delivered by mode-locked lasers and NOPAs. For example, commercial NOPAs can routinely produce wavelength-tuneable femtosecond pulses at the microjoule level within long-wavelength spectral windows for multiphoton imaging of ~1,300 nm and ~1,700 nm. Such a performance is yet to be matched by systems based on solitons and the SSFS. At wavelengths around 1,300 nm, the generation of energetic, widely wavelength-tuneable solitons is even more challenging because of the requirement of anomalous dispersion. Recently developed anti-resonant hollow-core fibres have a large transmission band and easily tailored dispersion properties. When pumped by a fibre oscillator or a fibre chirped-pulse amplification system around 1,060 nm, soliton generation and the SSFS in such a fibre have the potential to achieve a better performance than that of a NOPA at ~1,300 nm, for example, a pulse energy of 1–10 μJ, broad wavelength tunability around 1,300 nm and a multi-megahertz pulse-repetition rate[29,47]. They may therefore become a low-cost alternative to a NOPA system. Waveguide solitons and the SSFS provide a convenient platform for femtosecond sources that are compact, low cost and robust. Such a platform will be particularly attractive in wavelength regions where no broadband gain medium exists for wavelength-tuneable mode-locked lasers. With improved understanding of nonlinear wave propagation and new innovations in waveguide design and fabrication, soliton systems are a promising technology for a wide range of biomedical applications.

The future of solitons in telecommunications and signal processing

The initial idea of information transmission with solitons was based on using the return-to-zero, on–off-keying modulation format with direct detection, which was fully sufficient for the optical network transmission capacities of the 1990s. The development, at the beginning of this century, of practical coherent detection communication systems enabled a transition to multilevel modulation formats using coding over the optical phase, which leads to a much higher spectral efficiency. In coherent systems, the ability of the fundamental optical soliton to balance dispersion by nonlinearity is less important than the limited soliton spectral efficiency, when compared with Nyquist-type carrier pulses. By contrast, the detection of both amplitude and phase at the receiver opened the possibility of eigenvalue communications, as proposed by Hasegawa and Nyu[48], which again is based on the integrability of the NLSE[2]. The inverse-scattering transform method (also known as nonlinear Fourier transform)[48,49] makes it possible to compensate exactly for both dispersive and nonlinear transmission impairments via electronic post-processing, and introduces fundamentally new modulation formats for nonlinear fibre channels, including multi-soliton signal coding and modulation.

In terms of signal processing, optical solitons could provide a means of implementing the fundamental—but so far elusive—functionality of all-optical information storage[50]. All-optical buffers are a key building block of future transparent optical internet networks: the payload data need to be stored for a short period of time during the processing of the header at an optical switching node. Optical solitons may recirculate for a long time in both active and passive fibre loops with coherent continuous-wave pumping by a holding beam. This was, in fact, the first proposed application of nonlinear Schrödinger solitons in a configuration that leads to soliton-based optical frequency combs[51]. Another exciting potential application of optical solitons may be in optical computing. The use of solitons in the emerging area of neuromorphic computing has been discussed in ref. 52.

Emerging optical soliton concepts and new science being unveiled with solitons

The general concept of the optical soliton as a stable, localized, particle-like object that is formed by nonlinear interactions of distributed light waves is one of the fundamental ideas in nonlinear photonics. These 'particles' can interact through their tails, thus attracting or repulsing each other. This may lead to soliton molecules,[53,54] in which a bound state between two solitons is obtained when each soliton resides in a minimum of the potential formed by the other soliton, so that the effective mutual interaction forces vanish. Soliton molecules can form a bound state that is robust to weak perturbations, to which they respond through damped oscillations. More complex examples include localized nonlinear waves with internal degrees of freedom, that is, optical breathers[55,56]. The interaction between multiple solitons can lead to the formation of soliton crystals (see 'The future of solitons in microresonator frequency combs') or spectrally periodic solitons[56]. These can be thought of as a bound state between solitons, with different spectra that coincide in time. The overlap of solitons at different frequencies leads to strong temporal oscillations, which in turn enhances the nonlinear effects[56–58]. Solitons play an important role in the occurrence of optical rogue waves, that is, waves of extremely high amplitude, which are localized in both space and time. However, owing to space limitations, the discussion of this emerging topic is beyond the scope of the present article. An increase in the number of solitons leads naturally to the concepts of an optical soliton gas and soliton turbulence[59,60]. Interestingly, soliton gas theory may be relevant to the practical application of discrete eigenvalues (multi-solitons) to signal transmission and processing in fibre-optic communications[48,49]. Moreover, many practically important physical systems, for example, fibre lasers, may feature the coexistence of multiple solitons and dispersive waves, thus setting new challenges for the application of soliton theory to optical wave turbulence[61].

Using new materials with different natural or engineered nonlinearity and/or dispersion, fresh perspectives emerge for solitons. Higher-order nonlinearity, more general types of dispersion, which include pure-quartic dispersion and its higher-order variants (see 'The future of solitons in ultrafast laser technology'), Bragg-type structures[62] and other engineered media can lead to new types of soliton and different applications. We have barely scratched the surface—there is no doubt that there are many other interesting nonlinearity types and dispersion relations that could lead to solitons with novel properties.

A fascinating opportunity for optical solitons lies in multi-dimensional systems. In particular, the study of highly multimode optical systems and solitons is a new and exciting field of emerging research:[63,64] multimode solitons (see also 'The future of solitons in ultrafast laser technology') represent a discrete (in the spatial domain) version of the concept of a spatiotemporal light bullet[65], although this has been elusive so far because of its intrinsic tendency to collapse. Because of the guiding parabolic refractive-index profile in graded-index optical fibres, multimode solitons may propagate over tens of thousands of dispersion lengths[66]. Their propagation is accompanied by spatiotemporal periodic oscillations, which lead to the emission of high-intensity dispersive spectral sidebands well into the mid-infrared domain[67]. From a fundamental point of view, the theoretical description of multimode solitons will require an extension of the weak turbulence[68] or thermodynamic[69] approaches to the strong nonlinearity regime, including the temporal dimension, which remains an open problem.

Optical solitons offer fascinating opportunities for analogue physics, enabling in optical laboratories the imitation of otherwise inaccessible or hardly accessible physical phenomena, such as event horizons in black holes, Hawking radiation, the Hawking temperature of two-dimensional black holes[70,71] and others. These unusual applications of optical solitons are based on the observation that some physical phenomena, for example, physics at the event horizon, resemble the dynamics of waves in moving media. Optical solitons are easily accessible and enable high-accuracy fibre-optic experiments, thus providing an excellent platform for a laboratory testbed of many complex physical phenomena.

In conclusion, the science and applications of optical solitons have a rich and distinguished history as well as a bright future. The interplay between dispersive and nonlinear effects, and between gain and loss, potentially in multi-dimensional systems, is likely to raise intriguing questions for physicists and mathematicians, and the robustness of optical solitons to perturbations is expected to remain highly attractive for many engineering applications. Soliton theory paves the way for mastering nonlinear dynamics, providing a platform for new technology and devices, setting targets for the engineering of new materials and structures, and, thus, bringing together the three main drivers of photonics: materials science, optical engineering and new scientific concepts.

References

1. Hasegawa, A. & Tappert, F. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion. *Appl. Phys. Lett.* **23**, 142–144 (1973).

2. Zakharov, V. E. & Shabat, A. B. Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. *J. Exp. Theor. Phys.* **34**, 62–69 (1972).
3. Hasegawa, A. Optical soliton: a memoir of its discovery and future prospects. *Opt. Commun.* **532**, 129222 (2023).
4. Dudley, J. M., Finot, C., Genty, G. & Taylor, R. Fifty years of fiber solitons. *Opt. Photon. News* **34**, 28–33 (2023).
5. Agrawal, G. P. *Nonlinear Fiber Optics* 6th edn (Academic, 2019).
6. Mollenauer, L. F., Stolen, R. H. & Gordon, J. P. Experimental observation of picosecond pulse narrowing and solitons in optical fibers. *Phys. Rev. Lett.* **45**, 1095–1098 (1980).
7. Mollenauer, L. F., Stolen, R. H., Gordon, J. P. & Tomlinson, W. J. Extreme picosecond pulse narrowing by means of soliton effect in single-mode optical fibers. *Opt. Lett.* **8**, 289–291 (1983).
8. Taylor, J. R. Early optical soliton research at Imperial College London. *Opt. Commun.* **536**, 129382 (2023).
9. Kodama, Y. & Hasegawa, A. Guiding-center soliton in optical fibers. *Opt. Lett.* **15**, 1443–1445 (1990).
10. Gordon, J. P. & Haus, H. A. Random walk of coherently amplified solitons in optical fiber transmission. *Opt. Lett.* **11**, 665–667 (1986).
11. Suzuki, M. et al. Reduction of Gordon–Haus timing jitter by periodic dispersion compensation in soliton transmission. *Electron. Lett.* **31**, 2027–2029 (1995).
12. Smith, N. J., Knox, F. M., Doran, N. J., Blow, K. J. & Bennion, I. Enhanced power solitons in optical fibres with periodic dispersion management. *Electron. Lett.* **32**, 54–55 (1996).
13. Gabitov, I. & Turitsyn, S. K. Averaged pulse dynamics in a cascaded transmission system with passive dispersion compensation. *Opt. Lett.* **21**, 327–329 (1996).
14. Mollenauer, L. F. & Mamyshev, P. V. Massive wavelength-division multiplexing with solitons. *IEEE J. Quantum Electron.* **34**, 2089–2102 (1998).
15. Pratt, A. R. et al. 5,745 km DWDM transcontinental field trial using 10 Gbit/s dispersion managed solitons and dynamic gain equalization. In *Optical Fiber Communications Conference* PD26-P1 (Optica Publishing Group, 2003).
16. Gnauck, A. H. et al. 2.5 Tb/s (64×42.7 Gb/s) transmission over 40×100 km NZDSF using RZ-DPSK format and all-Raman-amplified spans. In *Optical Fiber Communication Conference and Exhibit* FC2-FC2 (IEEE, 2002).
17. Hasegawa, A. Soliton-based optical communications: an overview. *IEEE J. Sel. Top. Quantum Electron.* **6**, 1161–1172 (2000).
18. Dudley, J. M., Genty, G. & Coen, S. Supercontinuum generation in photonic crystal fiber. *Rev. Mod. Phys.* **78**, 1135–1184 (2006).
19. Herr, T. et al. Temporal solitons in optical microresonators. *Nat. Photon.* **8**, 145–152 (2014).
20. Tamura, K., Ippen, E. P., Haus, H. A. & Nelson, L. E. 77-fs pulse generation from a stretched-pulse mode-locked all-fiber ring laser. *Opt. Lett.* **18**, 1080–1082 (1993).
21. Rozanov, N. N. Dissipative optical solitons. *Phys. Usp.* **43**, 421–424 (2000).
22. Akhmediev, N. & Ankiewicz, A. in *Dissipative Solitons* Vol. 661 (eds Akhmediev, N. & Ankiewicz, A.) 1–17 (Springer, 2005).
23. Mollenauer, L. F. & Stolen, R. H. The soliton laser. *Opt. Lett.* **9**, 13–15 (1984).
24. Grelu, P. & Akhmediev, N. Dissipative solitons for mode-locked lasers. *Nat. Photon.* **6**, 84–92 (2012).
25. Wise, F. W., Chong, A. & Renninger, W. H. High-energy femtosecond fiber lasers based on pulse propagation at normal dispersion. *Laser Photon. Rev.* **2**, 58–73 (2008).
26. Blanco-Redondo, A. et al. Pure-quartic solitons. *Nat. Commun.* **7**, 10427 (2016).
27. Runge, A. F. J. et al. Infinite hierarchy of solitons: interaction of Kerr nonlinearity with even orders of dispersion. *Phys. Rev. Res.* **3**, 013166 (2021).
28. Runge, A. F. J. et al. The pure-quartic soliton laser. *Nat. Photon.* **14**, 492–497 (2020).
29. Van Howe, J. et al. Soliton self-frequency shift below 1300 nm in higher-order-mode, solid silica-based fiber. *Opt. Lett.* **32**, 340–342 (2007).
30. Horton, N. G. et al. In vivo three-photon microscopy of subcortical structures of an intact mouse brain. *Nat. Photon.* **7**, 205–209 (2013).
31. Ouzounov, D. G. et al. Generation of megawatt optical solitons in hollow-core photonic band-gap fibers. *Science* **301**, 1702–1704 (2003).
32. Chen, Y.-H., Sidorenko, P., Antonio-Lopez, E., Amezcua-Correa, R. & Wise, F. W. Efficient soliton self-frequency shift in hydrogen-filled hollow-core fiber. *Opt. Lett.* **47**, 285–288 (2022).
33. Travers, J. C., Grigorova, T. F., Brahms, C. & Belli, F. High-energy pulse self-compression and ultraviolet generation through soliton dynamics in hollow capillary fibres. *Nat. Photon.* **13**, 547–554 (2019).
34. Hasegawa, A. Self-confinement of multimode optical pulse in a glass fiber. *Opt. Lett.* **5**, 416–417 (1980).
35. Renninger, W. H. & Wise, F. W. Optical solitons in graded-index multimode fibres. *Nat. Commun.* **4**, 1719 (2013).
36. Wright, L. G., Christodoulides, D. N. & Wise, F. W. Spatiotemporal mode-locking in multimode fiber lasers. *Science* **358**, 94–97 (2017).
37. Petersen, C. R. et al. Mid-infrared supercontinuum covering the 1.4–13.3 μm molecular fingerprint region using ultra-high NA chalcogenide step-index fibre. *Nat. Photon.* **8**, 830–834 (2014).
38. Genty, G. et al. Machine learning and applications in ultrafast photonics. *Nat. Photon.* **15**, 91–101 (2021).
39. Kippenberg, T. J., Gaeta, A. L., Lipson, M. & Gorodetsky, M. L. Dissipative Kerr solitons in Kerr microresonators. *Science* **361**, eaan8083 (2018).
40. Cole, D. C., Lamb, E., Del'Haye, P., Diddams, S. A. & Papp, S. B. Soliton crystals in Kerr resonators. *Nat. Photon.* **11**, 671–677 (2017).
41. Rowley, M. et al. Self-emergence of robust solitons in a microcavity. *Nature* **608**, 303–309 (2022).
42. Smirnov, S., Sturman, B., Podivilov, E. & Breunig, I. Walk-off controlled self-starting frequency combs in $\chi^{(2)}$ optical microresonators. *Opt. Express* **28**, 18006–18017 (2020).
43. Taheri, H. & Matsko, A. B. Quartic dissipative solitons in optical Kerr cavities. *Opt. Lett.* **44**, 3086–3089 (2019).
44. Blanco-Redondo, A., Dorche, A. E. & Stern, B. Dual-ring resonators for optical frequency comb generation. US patent 11,402,724 (2022).
45. Yao, S., Liu, K. & Yang, C. Pure quartic solitons in dispersion-engineered aluminum nitride micro-cavities. *Opt. Express* **29**, 8312–8322 (2021).
46. Lee, J. H., van Howe, J., Xu, C. & Liu, X. Soliton self-frequency shift: experimental demonstrations and applications. *IEEE J. Sel. Top. Quantum Electron.* **14**, 713–723 (2008).
47. Xu, C. & Wise, F. W. Recent advances in fibre lasers for nonlinear microscopy. *Nat. Photon.* **7**, 875–882 (2013).
48. Hasegawa, A. & Nyu, T. Eigenvalue communication. *J. Lightwave Technol.* **11**, 395–399 (1993).
49. Turitsyn, S. K. et al. Nonlinear Fourier transform for optical data processing and transmission: advances and perspectives. *Optica* **4**, 307–322 (2017).
50. Pang, M. et al. All-optical bit storage in a fibre laser by optomechanically bound states of solitons. *Nat. Photon.* **10**, 454–458 (2016).
51. Wabnitz, S. Suppression of interactions in a phase-locked soliton optical memory. *Opt. Lett.* **18**, 601–603 (1993).

52. Marcucci, G., Pierangeli, D. & Conti, C. Theory of neuromorphic computing by waves: machine learning by rogue waves, dispersive shocks, and solitons. *Phys. Rev. Lett.* **125**, 093901 (2019).
53. Malomed, B. A. Bound solitons in the nonlinear Schrödinger–Ginzburg–Landau equation. *Phys. Rev. A* **44**, 6954–6956 (1991).
54. Stratmann, M., Pagel, T. & Mitschke, F. Experimental observation of temporal soliton molecules. *Phys. Rev. Lett.* **95**, 143902 (2005).
55. Kuznetsov, E. A. Solitons in a parametrically unstable plasma. *Sov. Phys. Dokl.* **22**, 507–508 (1977).
56. Akhmediev, N. & Korneev, V. I. Modulation instability and periodic solutions of the nonlinear Schrödinger equation. *Theor. Math. Phys.* **69**, 1089–1093 (1986).
57. Zakharov, V. E. & Ostrovsky, L. A. Modulation instability: the beginning. *Physica D* **238**, 540–548 (2009).
58. Lourdesamy, J. P. et al. Spectrally periodic pulses for enhancement of optical nonlinear effects. *Nat. Phys.* **18**, 59–66 (2022).
59. Zakharov, V. E. Turbulence in integrable systems. *Stud. Appl. Math.* **122**, 219–234 (2009).
60. Suret, P. et al. Soliton gas: theory, numerics and experiments. Preprint at https://arxiv.org/abs/2304.06541 (2023).
61. Turitsyna, E. et al. The laminar–turbulent transition in a fibre laser. *Nat. Photon.* **7**, 783–786 (2013).
62. Eggleton, B. J., Slusher, R. E., de Sterke, C. M., Krug, P. A. & Sipe, J. E. Bragg grating solitons. *Phys. Rev. Lett.* **76**, 1627–1630 (1996).
63. Krupa, K. et al. Multimode nonlinear fiber optics, a spatiotemporal avenue. *APL Photon.* **4**, 110901 (2019).
64. Wright, L. G. et al. Physics of highly multimode nonlinear optical systems. *Nat. Phys.* **18**, 1018–1030 (2022).
65. Silberberg, Y. Collapse of optical pulses. *Opt. Lett.* **15**, 1282–1284 (1990).
66. Zitelli, M., Ferraro, M., Mangini, F. & Wabnitz, S. Single-mode spatiotemporal soliton attractor in multimode GRIN fibers. *Photon. Res.* **9**, 741–748 (2021).
67. Wright, L. G., Wabnitz, S., Christodoulides, D. N. & Wise, F. W. Ultrabroadband dispersive radiation by spatiotemporal oscillation of multimode waves. *Phys. Rev. Lett.* **115**, 223902 (2015).
68. Aschieri, P., Garnier, J., Michel, C., Doya, V. & Picozzi, A. Condensation and thermalization of classical optical waves in a waveguide. *Phys. Rev. A* **83**, 033838 (2011).
69. Wu, F. O., Hassan, A. U. & Christodoulides, D. N. Thermodynamic theory of highly multimoded nonlinear optical systems. *Nat. Photon.* **13**, 776–782 (2019).
70. Philbin, T. G. et al. Fiber-optical analog of the event horizon. *Science* **319**, 1367–1370 (2008).
71. Robson, C. W., Di Mauro Villari, L. & Biancalana, F. Topological nature of the Hawking temperature of black holes. *Phys. Rev. D* **99**, 044042 (2019).

Acknowledgements

We are grateful to our many colleagues, with whom we have shared the soliton journey. This paper is dedicated to the memory of Linn Mollenauer, a leading figure in optical fibre soliton experimental science, who passed away on 28 July 2021. When finishing the manuscript, we learned that Vladimir Zakharov, another soliton science pioneer, passed away on 20 August 2023. We acknowledge his profound contributions, which have provided our field with inspiration and with solid mathematical foundations. We thank F. Mangini for help in preparing the figure. C.M.d.S. is supported by the Australian Research Council (ARC) Discovery Project (DP230102200) and the Asian Office of Aerospace R&D (AOARD) (grant no. FA2386-19-1-4067). S.W. is supported by the European Research Council (grant 740355) and the European Union under the Italian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, partnership on 'Telecommunications of the Future' (PE00000001—program 'RESTART'). S.K.T. acknowledges support of the Engineering and Physical Sciences Research Council (project EP/W002868/1). C.X. is supported by NIH/NIBIB R01EB033179 and NIH/NINDS U01NS128660.

Competing interests

The authors declare no competing interests.

Additional information

Correspondence should be addressed to Andrea Blanco-Redondo.

Peer review information *Nature Photonics* thanks John Travers and the other, anonymous, reviewer for their contribution to the peer review of this work.

Reprints and permissions information is available at www.nature.com/reprints.