the effects of an arbitrary dispersion relation. Using this laser, we have, for example, generated solitons with pure dispersion of orders 4, 6, 8 and 10 [2,3].

In another series of experiments [4] we programmed the phase mask so as to have the effect of a dispersion relation with $J = 1,\ldots,5$ equivalent, equally-spaced, maxima. Such systems were previously only investigated numerically [5–7]. As illustrated by the red curves in the first column of Figure 1, this means that, in a moving frame, the dispersion relation exhibits J peaks that are equally spaced in frequency and which have the same wavenumber and the same curvature. At each of these frequencies we might expect the formation of a soliton–in this case we therefore expect J identical solitons that are equally spaced in frequency (blue curves in first column of Figure 1) and that travel at the same group velocity. The interferograms, included as insets, show that these solitons are coincident in time and they thus interact by cross-phase modulation and four-wave mixing [1]. Since these solitons form coherent objects, they beat in the time domain with a period that is inversely proportional to the frequency spacing, as confirmed by the blue curves in the second column of Figure 1. The dashed curves have a hyperbolic secant shape and show that the pulses that would occur when $J = 1$, act as an envelope for the measured pulses. The insets show the same results on a logarithmic scale, and confirm that the pulse envelopes have approximately hyperbolic secant shapes. The resulting pulses are reminiscent of J-slit interference patterns, in indeed the physics of the two interference processes are very similar. The orange curves, show that the solitons are unchirped, as expected.

A theoretical analysis of the nonlinear process shows that the envelopes satisfy the nonlinear Schrödinger equation [1], with the dispersion coefficient corresponding to that of the curvature of each of the maxima, and with a nonlinear coefficient that increases monotonically with J–this means that the effect of the nonlinearity increases with J. This can be understood as follows: by modulating the envelope by a carrier that depends on J, the energy in the pulse exhibits nodes and antinodes. Even though nonlinear effects vanish in the nodes, this is more than overcome by the enhanced nonlinear effects near the anti-nodes. Using $J = 1$ as a reference, the theory predicts that for $J = 2$ the enhancement $\mathscr{E}_2 = 3/2$, whereas for larger J we find $\mathscr{E}_3 = 2.14$, $\mathscr{E}_4 = 2.81$, and $\mathscr{E}_5 = 3.48$. This is in good agreement with the experimental results of $\mathscr{E}_2 = 1.49$, $\mathscr{E}_3 = 2.17$, $\mathscr{E}_4 = 2.71$ and $\mathscr{E}_5 = 3.85$. We have further shown that in the continuum limit, i.e., when J is sufficiently large, the enhancement increases by 0.687 when a single peak is added to the dispersion relation and so $\mathscr{E}_J \to 0.687J$ in this limit [8]. The theory predicts the relative energies of the J frequency components, which arise from their mutual nonlinear interactions. Again, these values are confirmed experimentally. A remarkable result is that these relative energies adjust themselves such that the enhancement factor is a global maximum. In other words, any other ratio of the relative energies leads to a lower value of the enhancement. As J increases, the energies of the frequency components follow a universal curve.