LINE NARROWING IN A SYMMETRY BROKEN LASER*

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The power spectral density of a laser subjected to a symmetry breaking injected signal is calculated via the quantum noise operator approach. The resulting spectrum is sharpened in a manner reminiscent of the line narrowing which occurs in the Lamb–Mössbauer effect. It is further noted that these considerations are potentially of interest in producing high intensity fields in small focal volumes.

The quantum properties of laser radiation are well understood both theoretically and experimentally [1]. Recently, it has been demonstrated that a useful analogy exists between thermodynamic phase transition phenomena and laser threshold behavior [2]. In this context it has been noted that the symmetry breaking mechanism in the laser problem involves a classical signal injected into the laser cavity. It is further noted that the phase fluctuations associated with the laser in such a symmetry broken mode of operation will be markedly changed. This can be seen by looking at the effective free energy G, as a function of the electric field amplitude ρ, and phase ϕ (fig. 1). The injected signal breaks the symmetry of the effective free energy so that the laser will now tend to fluctuate about the injected signal's phase ϕ_0. It will be seen that there is an amusing similarity between the present spectral density for the symmetry broken laser problem and that obtained in the calculation of the spectrum for the Lamb–Mössbauer effect‡ [3].

In the present note we wish to consider the effects of such a symmetry breaking signal on the linewidth of the laser. This problem is a logical extension of the laser–phase-transition analogy and is potentially of interest in considerations involving delivery of a high peak power into a small focal volume. In this latter context we note that if one focuses n "identical" but independent lasers of power P_0 onto a spot of the order of a wavelength, we would in general expect the total power to go as nP_0. This is the case, since due to spontaneous emission, each laser is oscillating with a random phase and the total electric field is proportional to the square root of the number of lasers involved. However, in the case where an external signal is injected into the n lasers in question, the symmetry is broken and the quantum noise fluctuations may be quieted to the extent that the total electric field is now proportional to the number of lasers. In this case we could, in principle, hope to achieve a peak power over a small region which is n^2 times the power of any one laser. This could be potentially interesting in experiments involving small pellets. We would therefore like to know how well we can lock the phase of any given laser by means of a symmetry breaking signal.

We further note that the present notion of n lasers in "parallel" has some obvious advantages over a long chain of amplifiers, e.g., the intensity from any one of our lasers need not be high enough to produce damage to the optical components. However, the purpose of the present paper is not to suggest the design of a practicable scheme, but rather to access the extent to which phase fluctuations may be quieted by injecting into each laser a small fraction of a "master laser". We are therefore interested in the time dependence (spectral density) of the ensemble averaged laser field in the presence of the symmetry breaking signal. We find that the spectrum for the present problem is

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‡ In the present problem the atoms tend to "lock" in phase with the incident field, while in the Lamb–Mössbauer effect, the nuclear coordinates are locked by the lattice, and the subsequent well-known line narrowing results.

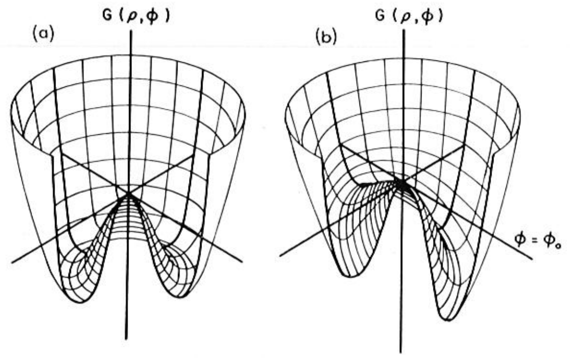

Fig. 1. (a) Effective free energy, $G(\rho, \phi)$ of laser. (b) Effective free energy, $G(\rho, \phi)$ of laser subjected to symmetry breaking signal having phase ϕ_0.

given by a suitably weighted sum of Lorentzians. Furthermore, the spectral half-width at half-maximum is narrowed by the presence of the injected signal.

We begin the calculation by extending the quantum noise operator treatment of a laser to include the presence of an external signal. The equation of motion for the slowly varying part of the radiation field annihilation operator, $A(t)\exp(-\mathrm{i}\nu t)$, is

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = \left.\frac{\partial A(t)}{\partial t}\right|_{\text{laser}} + [A(t), s(A^\dagger(t)\,\mathrm{e}^{-\mathrm{i}\phi_0(t)} - A(t)\,\mathrm{e}^{\mathrm{i}\phi_0(t)})], \qquad (1)$$

where we take the injected signal to be resonant with the laser field, $\phi_0(t)$ is the slowly varying phase of the injected signal, and s is a time independent quantity which designates the influence of the external signal on our laser*. From previous calculations [1,4], we have

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = \tfrac{1}{2}(\mathcal{A} - \mathcal{C})\,A(t) - \tfrac{1}{2}\mathcal{B}\,A(t)A^\dagger(t)A(t) + G(t) + s\,\mathrm{e}^{-\mathrm{i}\phi_0(t)}\,, \qquad (2)$$

where $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are the gain, saturation and loss parameters** [5]. For simplicity, we have tuned the cavity frequency to the center of the atomic spectrum, and $G(t)$ is the quantum Langevin force defined by [1]:

$$\langle G^\dagger(t)\,G(t')\rangle + \langle G(t)\,G^\dagger(t')\rangle = 4\bar{n}D\,\delta(t-t'),$$

where

$$D = \frac{A}{2\bar{n}}\,\frac{1-(n_\mathrm{b}/n_\mathrm{a})}{1+(\bar{n}\mathcal{B}/\mathcal{A})}\left(\frac{1}{2}\,\frac{n_\mathrm{a}+n_\mathrm{b}}{n_\mathrm{a}-n_\mathrm{b}} + \frac{1}{2} + \bar{n}_\mathrm{T}\right).$$

n_a and n_b are the populations of the upper and lower active atomic levels, and $\bar{n}_\mathrm{T}$ is the "thermal" photon number. Making the substitution $A(t) = \rho(t)\,\mathrm{e}^{-\mathrm{i}\phi(t)}$ in eq. (2) gives

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = \tfrac{1}{2}(\mathcal{A} - \mathcal{C})\,\rho(t) - \tfrac{1}{2}\mathcal{B}\rho^3(t) + s\cos[\phi(t) - \phi_0(t)] + \tfrac{1}{2}[G(t)\,\mathrm{e}^{\mathrm{i}\phi(t)} + G^\dagger(t)\,\mathrm{e}^{-\mathrm{i}\phi(t)}], \qquad (3)$$

and

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = -\frac{s}{\rho(t)}\sin[\phi(t) - \phi_0(t)] - \frac{1}{\rho(t)}\,\frac{G(t)\,\mathrm{e}^{\mathrm{i}\phi(t)} - G^\dagger(t)\,\mathrm{e}^{-\mathrm{i}\phi(t)}}{2\mathrm{i}}\,. \qquad (4)$$

Thus, when $|s| \gtrsim$ the RMS value of $G(t)$, which is approximately $\sqrt{\mathcal{A}}$, the phase $\phi(t)$ will tend to fluctuate about the value ϕ_0. For the present discussion we assume the physically reasonable but arbitrary numbers: $2s/\sqrt{\bar{n}} \simeq 1$ Hz, $\mathcal{A} \simeq 1.01\,\mathcal{C}$, $\mathcal{C} \simeq 10^6$ Hz, $\bar{n} \simeq 10^4$. These numbers imply a susceptibility, χ, in the laser medium of 10^{-7} which leads to injected signal fields [4] of $\simeq 10^{-11}$ volt/m. Since the injected signal strength is small, we do not anticipate an appreciable change in the laser field amplitude‡. Thus we can replace $\rho(t)$ by $\sqrt{\bar{n}}$. When $s > \sqrt{\mathcal{A}}$, we can write $\phi(t) = \phi_0(t) + \theta(t)$ where $\theta \ll 1$. The above substitutions in eq. (4) result in

$$\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = -\frac{s}{\sqrt{\bar{n}}}\,\theta(t) - \frac{G(t)\,\mathrm{e}^{\mathrm{i}\phi(t)} - G^\dagger(t)\,\mathrm{e}^{-\mathrm{i}\phi(t)}}{2\mathrm{i}\sqrt{\bar{n}}}, \qquad (5)$$

* The injected electric field $E_0\cos(\nu t + \phi_0(t))$ induces a polarization in the laser medium which couples to the laser field. From eq. (1) we see that s goes as $[\nu\epsilon_0/\hbar V]^{1/2}\chi E_0\cdot\hat{\epsilon}/2$ where χ is the susceptibility of the laser medium, ϵ is the polarization of the radiation field, ν is its frequency, and V is the volume of the cavity.

** $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ may be taken from ref. [5], which work includes temperature effects.

‡ For the usual laser oscillator operating above threshold, amplitude fluctuations are small and thus one replaces $\rho(t)$ by the average amplitude $\sqrt{\bar{n}}$. In the present problem we may account for the influence of the injected signal by replacing ρ by $\sqrt{\bar{n}} + r$. We find that for values of s sufficient to significantly narrow the laser linewidth, r is small compared to $\sqrt{\bar{n}}$.

where terms of order θ^2 are neglected, and we have used the fact that $\dot{\phi}_0(t) \ll \dot{\theta}(t)$. Now the power spectral density (Fourier transform of the second order correlation) is

$$\alpha(\omega) = (\hbar\nu/V\epsilon_0) \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{-\mathrm{i}(\omega-\nu)t}\, \bar{n}$$

$$\times \left\langle \mathrm{e}^{-\mathrm{i}\phi_0(t)} \exp\left[-\frac{1}{2}\int_0^t \mathrm{d}t' \int_0^t \mathrm{d}t'' \left\langle \frac{\mathrm{d}\theta(t')}{\mathrm{d}t'}\frac{\mathrm{d}\theta(t'')}{\mathrm{d}t''}\right\rangle\right]\right\rangle_{\mathrm{s}}, \qquad (6)$$

where $\langle\ \rangle_{\mathrm{s}}$ denotes the appropriate ensemble average over the injected signal coordinates, and $\langle\ \rangle$ involves the usual quantum statistical average associated with the Langevin forces. From eq. (5) and noting that $\langle\langle A(t)\rangle\rangle_{\mathrm{s}}$ varies on a time scale that is long compared to the correlation time of the Langevin forces but short compared to the temporal variation of the injected signal, we obtain

$$\langle\langle A^\dagger(t)\, A(0)\rangle\rangle_{\mathrm{s}} = \bar{n}\langle \mathrm{e}^{-\mathrm{i}\phi_0(t)}\rangle_{\mathrm{s}}$$

$$\times \exp\left[-\frac{D}{2}\,\frac{1-\mathrm{e}^{-2(s/\sqrt{\bar{n}})t}}{2s/\sqrt{\bar{n}}}\right]. \qquad (7)$$

The time $t = 0$ is taken as the point at which the phase of the injected signal coincides with that of the radiation field and $\phi_0(0) = 0$. Eq. (7) may be written as

$$\langle\langle A^\dagger(t)\, A(0)\rangle\rangle_{\mathrm{s}} = \bar{n}\, \langle \mathrm{e}^{-\mathrm{i}\phi_0(t)}\rangle_{\mathrm{s}}\, \mathrm{e}^{-D\sqrt{\bar{n}}/4s}$$

$$\times \sum_{m=0}^{\infty} \frac{\mathrm{e}^{-(2s/\sqrt{\bar{n}})mt}}{m!}\left(\frac{D}{2}\frac{\sqrt{\bar{n}}}{2s}\right)^m .$$

Taking the Fourier transform, we obtain the power spectral density

$$\alpha(\omega) = (\hbar\nu/V\epsilon_0)\, \bar{n}\, \mathrm{e}^{-D\sqrt{\bar{n}}/4s} \qquad (8)$$

$$\times \sum_{m=0}^{\infty}\left(\frac{D}{2}\frac{\sqrt{\bar{n}}}{2s}\right)^m \frac{1}{m!\,[\gamma/2+m(2s/\sqrt{\bar{n}})]}\,\mathcal{L}\left(m\frac{2s}{\sqrt{\bar{n}}}+\frac{\gamma}{2}\right),$$

where

$$\mathcal{L}(\Gamma) = \frac{\Gamma^2}{(\omega-\nu)^2+\Gamma^2},$$

and we have assumed that the external laser has a lorentzian spectrum with a linewidth $1/\gamma$. The sum in eq. (8) converges rapidly and is plotted in fig. 2 for various injected signal values.

In summary we see that the electric field spectrum

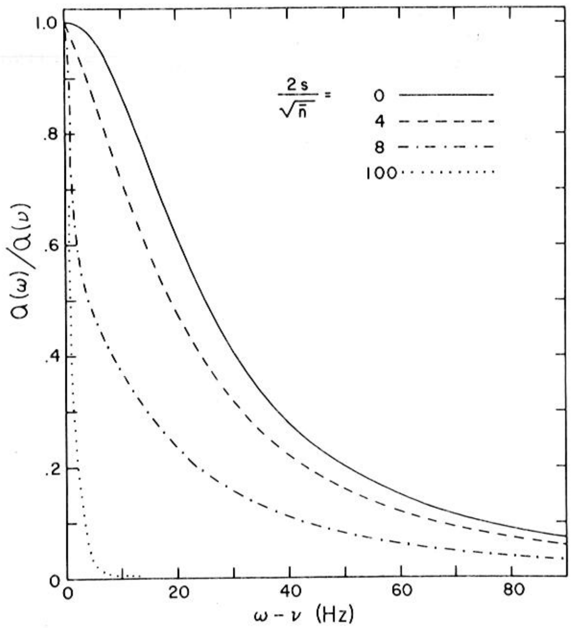

Fig. 2. Normalized spectra of the symmetry broken laser with $\mathcal{A} = 1.01\, \mathcal{C}$, $\mathcal{C} = 10^6$ Hz, $\bar{n} = 10^4$, $D = 50$ Hz, and $2s/\sqrt{\bar{n}} =$ 4 Hz, 8 Hz and 100 Hz compared to the spectrum of the usual laser ($2s/\sqrt{\bar{n}} = 0$) with the same values of $\mathcal{A}$, $\mathcal{C}$, D and $\bar{n}$.

of a laser that is subjected to a weak symmetry breaking signal has gone from a simple lorentzian to a form that is reminiscent of the Lamb–Mössbauer effect with its associated line narrowing. Furthermore, we note that n lasers in "parallel" may be influenced to deliver a total power to a small spot which goes as

$$P_{\mathrm{T}} = n^2 P_0 f(r) \exp\left(-D\sqrt{\bar{n}}/2s\right),$$

where $f(r)$ is a spatial interference factor.

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