# Robust Biphoton Entanglement of Three Topological Modes

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**Abstract:** We numerically show the generation of three-mode biphoton entanglement in silicon photonic topological superlattices. Our results show signatures of robustness to disorder in the resulting entangled state, highlighting a route toward robust complex photonic entanglement. 

## 1. Introduction

Multiphoton entanglement lies at the heart of quantum information science and technology. In the last decade, we have seen a strong push to increase both the number and the dimensionality of entangled photons in order to achieve higher information capacity and richer quantum mechanical effects [1]. However, the quantum correlations in such complex entangled states are fragile, subject to decoherence due to system imperfections.

In parallel, topological photonics [2] – a field that leverages global properties of the bands in the dispersion relation to produce edge modes with extraordinary robustness – has emerged as a fascinating avenue to explore the generation and manipulation of robust quantum states of light [3]. In the last few years, we have witnessed experimental studies of topological protection of biphoton correlations [4,5] as well as of multiphoton two-spatial-mode entanglement [6–8]. More recently, we have extended the number of biphoton spatial modes to three [9], but only one of those three modes was topologically protected, and therefore the resulting entangled state did not show signatures of robustness.

Here, we propose an avenue to increase the dimensionality of the entangled photons to an arbitrarily large number of topological modes by using topological multiband photonic superlattices [10]. Specifically, focusing our analysis in a superlattice with four elements per unit cell, we numerically show that biphoton entanglement of three topological modes in topological superlattices is robust to disorder in the coupling constants. We quantify such robustness by calculating the mean and the variance of the Schmidt number – a measurement of entanglement – as a function of disorder. These results pave the way to realizing complex entangled states with added robustness.

## 2. Results

The platform studied here relies on an array of coupled silicon on insulator waveguides (refractive index $n_{\text{Si}} = 3.48$ at 1550 nm, width $w = 500$ nm, height $h = 220$ nm, length $L = 500\mu$m, and a total of $N = 81$ waveguides). The waveguides are arranged in a superlattice structure with four elements per unit cell, intracell gaps $g_1 = 120$ nm and $g_2 = 238$ nm, and intercell gap $g = 261$ nm, as depicted in Fig.1(a). As shown in Ref. [10], as long as the couplings respect inversion symmetry and the intercell coupling is larger than a particular value at which the largest band gap closes, all the bandgaps in the spectrum are topologically non-trivial. With this in mind, one can engineer an interface between two coupled superlattices of different topological band-gap properties, as represented by the

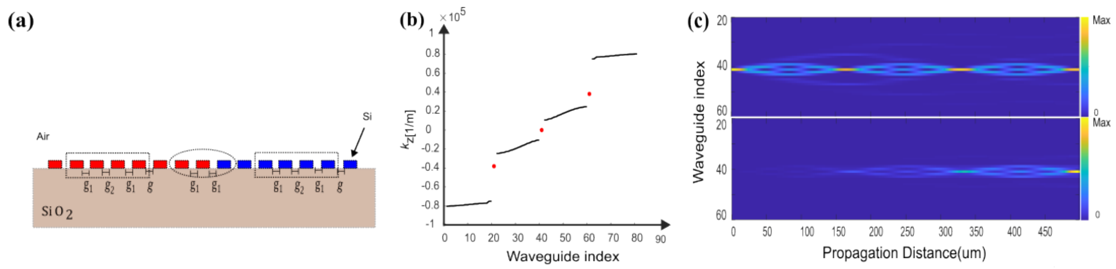


Fig. 1. (a) Silicon photonics supperlatice; (b) Propagation-constant spectrum; (c) Propagation simulation of the pump (top) and photon pairs (bottom).

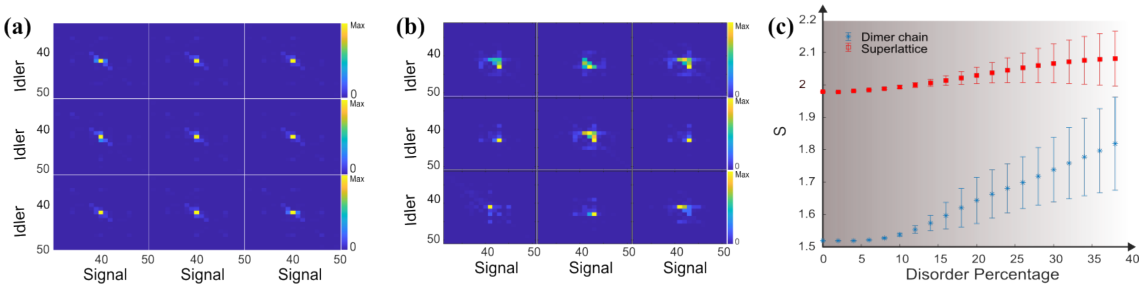


Fig. 2. (a)Biphoton correlation map at the output of the superlattice for nine random iterations of 10% coupling disorde; (b) same for the dimer chain with a short-short defect. (c) Mean and variance of the Schmidt number as a function of disorder for the superlattice (red) and the dimer chain (blue).

transition between red and blue colors in Fig.1(a). The propagation-constant ($k_z$) spectrum of such superlattice is shown in Fig.1(b), exhibiting three band gaps with topologically protected edge modes (red dots). The three modes are co-localized at the interface between the superlattices.

When pumping the lattice at 1550 nm at the center of the interface, a linear combination of the three edge modes is excited, giving rise to the beating pattern shown in the propagation simulations in Fig.1(c) (top panel). In turn, the pump induces the probabilistic generation of photon pairs via the strong nonlinearity of the silicon waveguides (nonlinear parameter $\gamma = 120\ \mathrm{W}^{-1}\mathrm{m}^{-1}$), as shown in Fig.1(c) (bottom panel). The photon pairs are expected to be generated in a superposition of the three topological modes, giving rise to three-mode biphoton entanglement.

Next, we study the robustness of the spatial mode entanglement to disorder in the coupling constants. Fig. 2 (a) illustrates the biphoton correlation at the output of 9 superlattices with different random variations of the nominal coupling strength between each pair of waveguides, drawn independently from a normal Gaussian distribution around the nominal coupling strengths, with a standard deviation of 10%. Notably, the biphoton correlation remains largely unchanged. For comparison, we perform the same robustness test on a platform that also supports three-mode biphoton entanglement, but involving only one topological mode and two trivial modes. This platform, which we recently demonstrated experimentally [9], is based on the dimer chain (or Su-Schrieffer-Heeger model [11]) with a short-short defect. As shown in Fig. 2 (b), in this case the biphoton correlation changes significantly in the presence of the same level of disorder, because the entanglement involves trivial modes. Finally, we quantize this robustness by calculating a lower bound for the Schmidt number ($S$) of the output state at different levels of disorder for the two cases just described. As shown in Fig. 2 (c), the mean of $S$ remains fairly flat up to large values of disorder for the three-mode state in the topological superlattice (red markers), while it increases steeply with disorder in the case involving the trivial modes (blue markers). The variance of $S$ also increases with disorder, as depicted by the error bars, and that increase is significantly stronger in the dimer-chain case.

In summary we have proposed and numerically demonstrated a platform for the generation of robust biphoton three-mode entanglement. This can be easily generalized to an arbitrary number of topological modes.

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