The misalignment induced aberrations of TMA telescopes

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Abstract: The next major space-borne observatory, the James Webb Space Telescope, will be a 6.6M field-biased, obscured, three-mirror anastigmat (TMA). Over the used field of view, the performance of TMA telescopes is dominated by 3rd order misalignment aberrations. Here it is shown that two dominant 3rd order misalignment aberrations arise for any TMA telescope. One aberration, field constant 3rd order coma is a well known misalignment aberration commonly seen in two-mirror Ritchey Chretien telescopes. The second aberration, field-asymmetric, field-linear, 3rd order astigmatism is a new and unique image orientation dependence with field derived here for the first time using nodal aberration theory.

OCIS Codes: (220.1010) Aberrations (global); (220.1140) Alignment.

References and links

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1. Introduction

Astronomers have been aligning large (> 30"), two-mirror telescopes for over 100 years. Until recently, most large astronomical telescopes for professional use have been two-mirror forms with either a parabolic (Cassegrain) or a hyperbolic (Ritchey-Chretien) primary mirror (other classes of professional large telescopes typically involve large corrector plates as in Schmidt and Schmidt-Cassegrain or lens groups near the focal plane). The field of view of the Cassegrain form is limited by uncorrected 3rd order coma, which is an aberration that increases linearly with field of view. The field of view of the Ritchey-Chretien form is limited by uncorrected 3rd order astigmatism, which is an aberration that increases quadratically with field of view. The limiting aberration of each of these systems is illustrated in Figure 1 using a Full Field Display (FFD). This type of display plots the magnitude and orientation of, in this case, specific components of a Zernike coefficient decomposition of the wavefront created by tracing a grid of real rays over a grid of field points.

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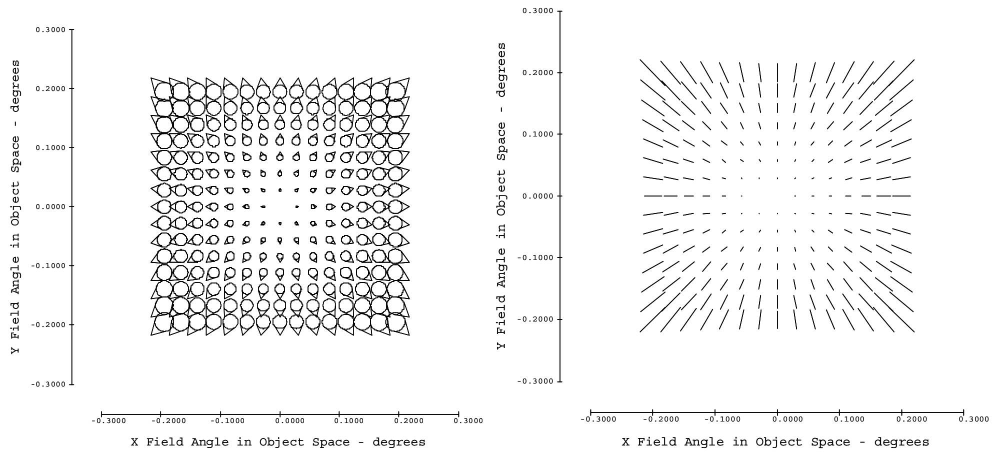

Fig. 1. The dominant residual aberrations of, (a) Cassegrain, 3rd order field linear coma (b) Ritchey-Chretien, 3rd order astigmatism that is quadratic with field.

It is well known from experience and conventional aberration theory (and as explained by nodal aberration theory) that if either of these forms is in a misaligned state, it displays on-axis coma. Nodal aberration theory demonstrates that the Cassegrain telescope continues to be limited by 3rd order coma that increases linearly with field, but in a misaligned state the coma field is offset from the center of the field, as illustrated in Fig. 2(b). The Ritchey-Chretien form also displays on-axis coma in a misaligned state, however in this case because this form is corrected for 3rd order coma in an aligned state, it displays a 3rd order coma field that is constant over the field of view, as illustrated in Fig. 3(b).

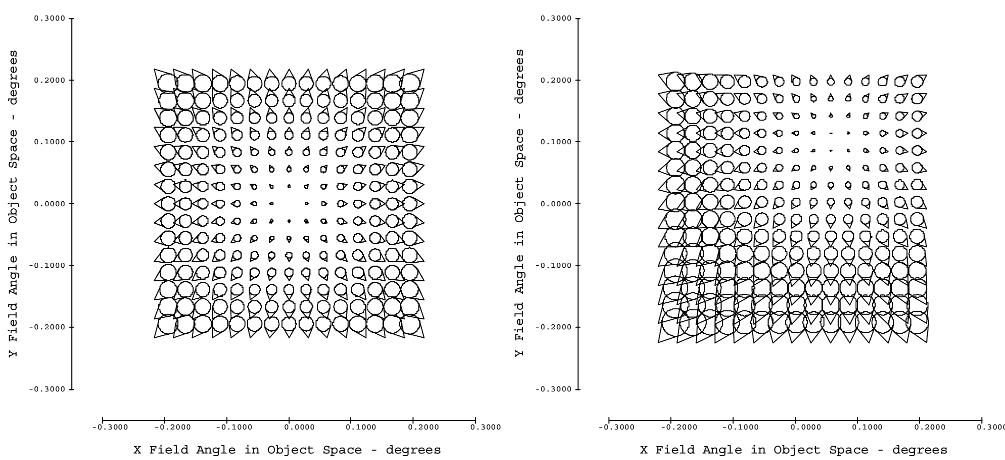

Fig. 2. The 3rd order coma of: (a) an aligned Cassegrain, 3rd order field linear coma (b) a misaligned Cassegrain, offset 3rd order field linear coma.

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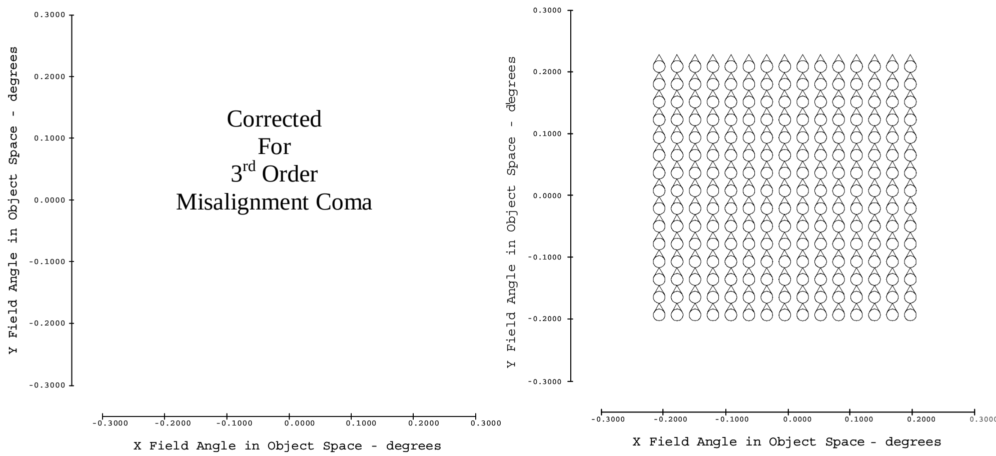

Fig. 3. The 3rd order coma of: (a) an aligned Ritchey-Chretien, no 3rd order coma (b) a misaligned Ritchey-Chretien, 3rd order coma that is constant in magnitude and orientation over the field of view.

While the correction of 3rd order misalignment coma is routine, the removal of alignment astigmatism in Ritchey-Chretien telescopes is less routine. McLeod describes an implementation of an alignment method that includes astigmatism for the telescope sited on Mount Hopkins [1]. S. Kim et. al recently reported on a method based on data reduction and numerical methods but without a tie to aberration theory [2]. T. Schmid et. al have reported on the application of nodal aberration theory to this problem and demonstrated that this theory completely describes the results of McLeod [3].

With the advent of the James Webb Space Telescope (JWST) program, which depending on its date of deployment will represent one of the earliest broad-based uses of three mirror anastigmatic (TMA) telescopes and instruments in astronomy (the JWST plus its currently planned instruments contain at least 9 TMAs), it has become urgent to provide a theory of misalignment aberrations for this class of telescopes. To-date, the alignment planning has been based on simulation of the Zernike reduction of the wavefront over a grid of field points [4]. Currently, the program is working to simplify the alignment process [5], and having knowledge of the aberrations of a misaligned TMA can aid this process significantly. In this paper, it is shown that for TMA telescopes, the aberration field response to misalignments is also predictable irrespective of the details of the telescope, as was the case for two-mirror telescopes. Here, 3rd order nodal aberration theory [6], which predicts the response of telescope systems to misalignments, can be used to demonstrate which aberration fields dominate during the alignment of a TMA telescope.

A key concept here is to realize that in a TMA telescope (or a Ritchey-Chretien) there are an infinite number of grossly misaligned states that would have no 3rd order coma but substantial misalignment astigmatism. In a TMA telescope, there are two alignment contributions for any of the three mirrors: the decenter of the aspheric departure of the mirror from the optical axis, which creates aberration that is proportional to the asphericity of the mirror, and the decenter of the center of curvature of the mirror, which can result from either a decenter or a tilt of the mirror, which results in the center of curvature moving off the optical axis and thus dominantly creates aberration that is proportional to the power of the mirror. These are independent parameters that change the amount of coma and as a result a mirror that has a fixed decenter can always be tilted to remove misalignment coma and the resulting system consisting of three tilted and decentered mirrors will have no 3rd order coma. Here the decenter induced coma is balancing the tilt induced coma so that the resulting position of the mirror is in nearly all cases a misaligned state. Particularly in a TMA telescope, the removal of misalignment coma does not result in the removal of misalignment astigmatism (although it is important to note that misalignment astigmatism, when present, is always zero on-axis) and there are an infinite number of misaligned states for the TMA that will have no coma but substantial misalignment astigmatism.

Significantly, while the appearance of 3rd order coma on-axis is still a key characteristic of a misaligned TMA, the second key and common characteristic is the appearance of 3rd order astigmatism with a new, unique aberration field dependence; field-asymmetric, field-linear, 3rd order astigmatism. Most importantly, this astigmatism typically remains centered on the optical axis and is therefore zero "on-axis". Here, "on-axis" is defined as the location of the center of the Gaussian image plane on the detector, which is often displaced from the center of the image space detector by the boresight error, which is also induced by tilt and decenter. Many telescope alignment plans contain a separate test plan and control loop to eliminate the boresight error using degrees of freedom that may or may not also affect the misalignment aberrations (most often it does). This paper assumes that is step has been performed first. As a result, when alignment is conducted using only on-axis measurements, as it is with two mirror telescopes, while misalignment coma will be removed, it is nearly guaranteed that a TMA will remain in an unacceptably misaligned state due to field-asymmetric, field-linear, 3rd order astigmatism. Note that this new aberration field dependence is a special case of binodal astigmatism that arises when the nominal telescope design is corrected for third order astigmatism. While not exactly accurate, conceptual, this new field dependence can be thought of as binodal astigmatism where one node has moved well outside of the field of view. Alignment of a TMA telescope requires measurement at multiple field points, some or all of which are at or near the edge of the format. In fact, it is of no added value to measure the on-axis performance of these systems using an interferometer that provides a Zernike decomposition of the wavefront as the coma contribution is constant over the field and can be measured anywhere in the field of view. It would be true if a focal plane based test is used, working on-axis would allow viewing the misalignment coma in isolation.

The key to developing insights into the response of a large telescope to alignment errors is to have a foundation for what forms the aberrations take when perturbations are introduced. It has been shown that there are no new aberrations when a perturbation breaks the rotational symmetry of an optical system [6]. What does change is the field dependence of the individual aberration terms. For an astronomical TMA telescope, we can concentrate on the third order aberrations that degrade the quality of the image when the telescope is in a misaligned state. These include 3rd order coma and 3rd order astigmatism. More specifically, TMA telescopes by definition have a corrected 3rd order coma and a corrected 3rd order astigmatic field. This condition results in a special case condition for both aberrations. The material that follows presents the special case where the 3rd order coma and astigmatism are corrected in the aligned telescope design (e.g. TMA forms). In addition, to the 3rd order field aberrations (coma and astigmatism), there is a need to control axial spacings to avoid the introduction of 3rd order spherical aberration. This is a well established practice in optical system assembly and will not be discussed here.

2. Application of nodal theory to derive the dominant characteristic response of any TMA telescope to

Given that 3rd order coma and 3rd order astigmatism are the aberrations that will limit the performance of TMA-based astronomical telescopes in a misaligned state, Equation 4.3 of Reference 6 can be used to initiate the derivation of the dominant aberrations of a misaligned TMA. Specifically, the wave aberration expansion through 3rd order for a misaligned TMA telescope (or any misaligned optical system for that matter) is

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$$
\begin{aligned}
W = &\overset{\text{Focus}}{\Delta W_{20}(\vec{\rho}\cdot\vec{\rho})} + \overset{\text{Tilt}}{\Delta W_{11}(\vec{H}\cdot\vec{\rho})} + \overset{3^{rd}\ \text{Spherical}}{\sum_j W_{040_j}(\vec{\rho}\cdot\vec{\rho})^2} \\
&+ \overset{3^{rd}\ \text{Coma}}{\sum_j W_{131_j}[(\vec{H}-\vec{\sigma}_j)\cdot\vec{\rho}](\vec{\rho}\cdot\vec{\rho})} \\
&+ \overset{3^{rd}\ \text{Astigmatism}}{\sum_j W_{222_j}[(\vec{H}-\vec{\sigma}_j)\cdot\vec{\rho}]^2} + \overset{3^{rd}\ \text{Field Curvature}}{\sum_j W_{220_j}[(\vec{H}-\vec{\sigma}_j)\cdot(\vec{H}-\vec{\sigma}_j)](\vec{\rho}\cdot\vec{\rho})} \\
&+ \overset{3^{rd}\ \text{Distortion}}{\sum_j W_{311_j}[(\vec{H}-\vec{\sigma}_j)\cdot(\vec{H}-\vec{\sigma}_j)][(\vec{H}-\vec{\sigma}_j)\cdot\vec{\rho}]}\ ,
\end{aligned} \tag{1}
$$

where the subscript j is the surface number, W_{klm} are the wave aberration coefficients, $\vec{H}$ is the vector that locates the image point of interest in the focal plane, $\vec{\rho}$ is the aperture vector in the exit pupil, and $\vec{\sigma}_j$ is the surface by surface location of the center of the aberration field for each surface, which is a vector residing in the Gaussian image plane [6], [7].

2.1 3rd Order Misalignment Coma in a TMA Telescope

Concentrating first on the coma term (W_{131}), starting from Equation 4.5 of Reference 6,

$$
\begin{aligned}
W &= \sum_j W_{131_j}[(\vec{H}-\vec{\sigma}_j)\cdot\vec{\rho}](\vec{\rho}\cdot\vec{\rho}) \\
&= [((\sum_j W_{131_j}\vec{H}) - (\sum_j W_{131_j}\vec{\sigma}_j))\cdot\vec{\rho}](\vec{\rho}\cdot\vec{\rho})\ .
\end{aligned} \tag{2}
$$

Here, the first summation results in the contribution of the rotationally symmetric system, which for a TMA telescope is zero,

$$
\sum_j W_{131_j}\vec{H} = W_{131}\vec{H} = 0\ . \tag{3}
$$

The second summation is the sum of the surface contribution displacement vectors in the image plane, $\boldsymbol{\sigma_j}$ each weighted by the corresponding surface contribution to the wave aberration for 3rd order coma, W_{131j}, which typically will contain a spherical and an aspheric contribution [6]. This summation results in a net, unnormalized vector in the image plane,

$$
\vec{A}_{131} \equiv \sum_j W_{131_j}\vec{\sigma}_j\ , \tag{4}
$$

which parameterizes the magnitude and the orientation of the coma field. This leads immediately to the equation that describes the misalignment induced 3rd order coma aberration in TMA telescopes,

$$
W = -(\vec{A}_{131}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})\ . \tag{5}
$$

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This is a 3rd order coma aberration that is constant over the field of view as illustrated in Fig. 3. This aberration is also a characteristic of a two-mirror Ritchey-Chretien telescope, or, any optical system with overall correction of 3rd order coma.

2.1 3rd Order Misalignment Astigmatism in TMA Telescopes

Following a similar derivation for 3rd order astigmatism (W_{222}), start with Equation 4.14 of Reference 6,

$$W = \frac{1}{2}[\sum_j W_{222_j} \vec{H}^2 - 2\vec{H}(\sum_j W_{222_j} \vec{\sigma}_j) + \sum_j W_{222_j} \vec{\sigma}_j{}^2] \cdot \vec{\rho}^2 . \qquad (6)$$

Again, for a TMA telescope, which is by design corrected also for 3rd order astigmatism, the initial term is zero,

$$\sum_j W_{222_j} \vec{H}^2 = W_{222} \vec{H}^2 = 0 , \qquad \text{(any TMA telescope)} \qquad (7)$$

which then gives

$$W = \frac{1}{2}[-2\vec{H}(\sum_j W_{222_j} \vec{\sigma}_j) + \sum_j W_{222_j} \vec{\sigma}_j{}^2] \cdot \vec{\rho}^2 , \quad \text{(any TMA telescope)} \qquad (8)$$

Here, it is important to recall that this equation in particular involves extensive use of an uncommon operation, vector multiplication [6]. As was done for coma, we now define two unnormalized displacement vectors,

$$\vec{A}_{222} \equiv \sum_j W_{222_j} \vec{\sigma}_j \qquad (9)$$

and

$$\vec{B}^2_{222} \equiv \sum_j W_{222_j} \vec{\sigma}_j{}^2 , \qquad (10)$$

which allows writing the characteristic field dependence for astigmatism in a misaligned TMA as

$$W = -\frac{1}{2}(2\vec{H}\vec{A}_{222} + \vec{B}^2_{222}) \cdot \vec{\rho}^2 . \qquad (11)$$

Unlike coma, which in a misaligned TMA is constant over field, the astigmatism for a misaligned TMA has both a linear with field component and a constant with field component. In general, for misalignment level perturbations, $\vec{B}^2_{222}$, which is proportional to the misalignment squared is negligible. As a result, it will be dropped going forward, in this paper.

Combining Equations (5) with (11), the general aberration state for a misaligned TMA telescope is given as

$$\boxed{W = -(\vec{A}_{131} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) - (\vec{H}\vec{A}_{222}) \cdot \vec{\rho}^2}. \ (\vec{B}^2_{222} \cong 0) \qquad (12)$$

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where the notation $\vec{H}\vec{A}_{222}$ and $\vec{B}_{222}^{2}$ denotes vector multiplication [6]. These two components are illustrated in Fig. 4. A terminology for the astigmatic term, which is isolated here for the first time as a dominant characteristic of misaligned TMA telescopes, is **field-asymmetric, field-linear, 3rd order astigmatism**. The reason this is a field asymmetric aberration is because while $\vec{H}$ would create a rotationally symmetric behavior the vector $\vec{A}_{222}$ has a constant magnitude and orientation (similar to the $\vec{A}_{131}$ which created constant coma). The product of these two vectors, one of which is a constant in the field, results in an aberration that is field asymmetric.

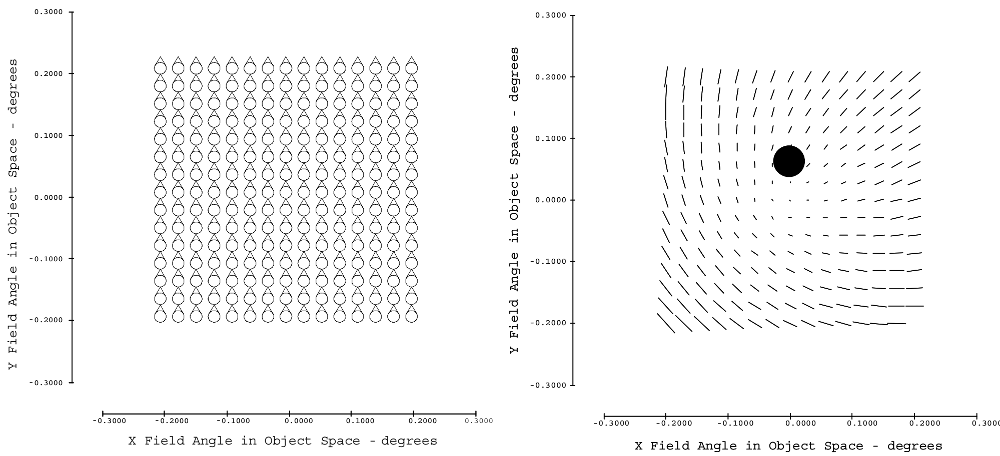

Fig. 4. The dominant residual aberrations of a misaligned TMA telescope, (a) 3rd order coma that is constant over field, (b) 3rd order astigmatism that is field-asymmetric and field-linear.

4. Interpretation of the dominant characteristic misaligned TMA aberration field response functions

It is well known that a misaligned two-mirror telescope displays axial coma and for the case of a Ritchey-Chretien design, which is nominally corrected for 3rd order coma, this coma is in fact constant over the field of view [6]. This misalignment coma is often corrected by a tilt and/or decenter of the secondary mirror based on measurement of the wavefront or other performance criteria, on-axis. It can be shown through nodal theory or with other methods [3] that there is what is termed a coma-free pivot point, which is an external rotation point along the optical axis where if the secondary mirror in a Ritchey Chretien telescope is rotated about this point the coma is unaffected but the magnitude of misalignment astigmatism can be changed (i.e. reduced) [3]. This same condition applies to a TMA telescope and, because, in addition, a TMA telescope is in fact corrected for 3rd order astigmatism in the aligned state, a TMA telescope is substantially more sensitive to residual alignment induced 3rd order astigmatism than a corresponding Ritchey Chretien two-mirror form.

In general, if the misalignment induced coma is corrected by the tilt and/or decenter of one of the TMA mirrors, this correction will result in no aberration on-axis, but it will not in all but the rarest of cases control the misalignment induced astigmatism. In this case, when the misalignment induced coma has been removed, the residual misalignment induced aberration in a TMA telescope will be given by

$$. \quad W = -[\vec{H}\vec{A}_{222}]\cdot\vec{\rho}^{2} \qquad \vec{A}_{131} = \sum_{j} W_{131_j}\vec{\sigma}_{j} = 0 \tag{13}$$

which is illustrated in Fig. 5.

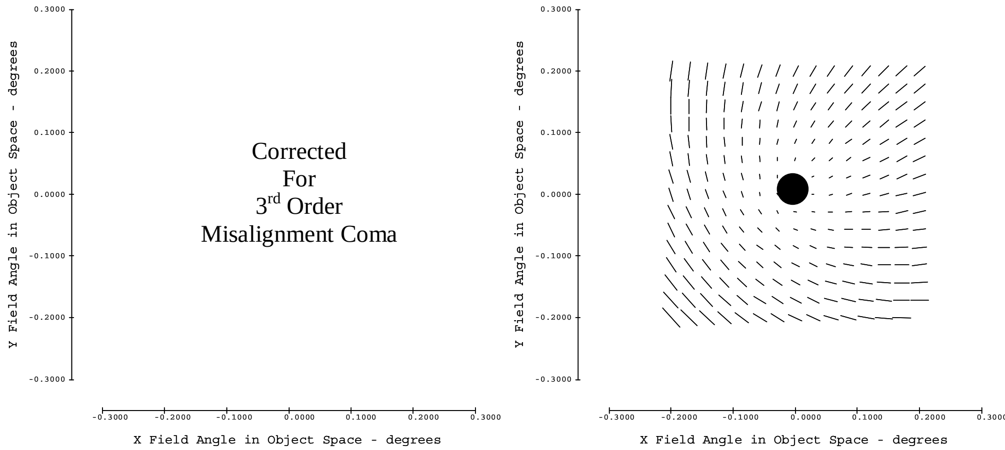

Fig. 5. The dominant misalignment aberration of TMA telescopes with corrected on-axis performance is field-asymmetric, field-linear 3rd order astigmatism, as illustrated in b).

A critical point here is that the location for the point of zero misalignment induced aberration for the field-asymmetric, field-linear, 3rd order astigmatism will, in the absence of astigmatic figure error on any of the mirrors, always reside at the center of the field of view. The impact of mirror figure error, which enters through the term $\vec{B}^{2}_{222}$, will be the subject of another paper, but is an important caveat to be aware of in the context of these conclusions.

This point is brought out to emphasize that the alignment of TMA telescopes cannot be accomplished using on-axis performance data alone. More important, and the key point from this paper: the measurement of a corrected on-axis image in a TMA telescope in no way ensures that the telescope is aligned.

5. Conclusions

This paper has demonstrated how nodal aberration theory can be used to rapidly isolate and describe the aberration field dependencies that arise in the presence of misalignments in any three mirror anastigmatic (TMA) telescope.

Specifically, it is shown that a misaligned TMA telescope will have two residual 3rd order aberrations, both of which have a field dependence that is not found in a rotationally symmetric, aligned optical system. These will be

> 3rd order coma, which is constant in magnitude and orientation over the field
>
> 3rd order astigmatism, which is field-asymmetric in orientation and increases linearly with field

These two aberrations are illustrated in Fig. 4.

The most significant result is that during assembly a TMA telescope that is aligned to provide diffraction-limited performance on-axis, based on on-axis measurements alone, will not be in any way necessarily in an aligned state. Most likely, there will be significant field-asymmetric, field-linear, 3rd order astigmatism. While linear astigmatism developed from power series expansions is commonly reported, the specific asymmetric orientation that is

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report here is a new aberration type isolated here for the first time (shown in Fig. 5(b)). In particular, knowing that there is a specific, intrinsic field-asymmetric behavior is an important result for engineers attempting to interpret off-axis performance measurements, especially those based on Zernike component reductions of measured wavefront data over a limited set of measurement points in the field of view. This result can be used to substantial advantage in the alignment of any TMA.

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