

# Alignment induced aberration fields of next generation telescopes

Tobias Schmid<sup>\*a</sup>, Kevin Thompson<sup>\*b</sup>, Jannick Rolland<sup>\*a</sup>

<sup>a</sup>CREOL, University of Central Florida, 4000 Central Blvd., Orlando, FL, USA 32816,

<sup>b</sup>Optical Research Associates, 3280 E. Foothill Blvd, Pasadena, CA, USA 91107

## ABSTRACT

There is a long list of new ground-based optical telescopes being considered around the world. While many are conventional Cassegrain and Ritchey-Chretien designs, some are from a family of three mirror anastigmatic (TMA) telescopes that are configured with an offset field (but still obscured) that trace back to designs developed in the 1970s for military applications. The nodal theory of aberrations, developed in the late 1970s, provides valuable insights into the response of TMA telescopes to alignment errors. Here it is shown for the first time that the alignment limiting aberration in any TMA telescope is a 3<sup>rd</sup> order astigmatism term with a new field dependence, termed **field-asymmetric, field-linear 3<sup>rd</sup> order astigmatism**. It is also shown that a TMA telescope under assembly that is only measured to have excellent/perfect performance on-axis is not aligned in any significant way. This is because the new astigmatic term is always zero on-axis, even though it is large over the field of view. Knowledge of this intrinsic misalignment aberration field for any TMA telescope aids greatly in ensuring it can be aligned successfully. The James Webb Space Telescope (JWST), is used an example of a relevant TMA system.

Keywords: telescopes, aberration, alignment, three mirror anastigmat

## 1. INTRODUCTION

Astronomers have been aligning large, two-mirror telescopes for over 100 years. Until recently, all astronomical telescopes have been two mirror forms with either a parabolic (Cassegrain) or a hyperbolic (Ritchey-Chretien) primary mirror. The field of view of the Cassegrain form is limited by uncorrected 3<sup>rd</sup> order coma, which is an aberration that increases linearly with field of view. The field of view of the Ritchey-Chretien form is limited by uncorrected astigmatism, which is an aberration that increases quadratically with field of view.

It is well known from experience (and as explained by nodal aberration theory) that if either of these forms is in a misaligned state, it displays on-axis coma. It is common practice to align these telescopes by tilting and/or decentering the secondary until the coma is removed. In the last decade, it has been recognized that this approach can result in degraded edge of field performance, as recently explained using nodal aberration theory by Schmid and Thompson<sup>1</sup> paralleling the experimental work of McLeod<sup>2</sup>. The work of McLeod brings full closure to the alignment of two mirror astronomical telescopes.

For three mirror anastigmatic (TMA) telescopes, the aberration field response to misalignments is also predictable irrespective of the details of the telescope, as was the case for two mirror telescopes. In this case though, there is very little industry experience, especially in astronomical community. Here, nodal theory, which predicts the response of telescope systems to misalignments, can be used to demonstrate what aberration fields can be anticipated to dominate during the alignment of a TMA telescope.

Significantly, while the appearance of coma on-axis is still a key characteristic of a misaligned TMA the second key and common characteristic is the appearance of astigmatism with a new, unique aberration field dependence, **field-asymmetric, field-linear 3<sup>rd</sup> order astigmatism**. Most importantly, this astigmatism typically remains centered on the optical axis. As a result, if alignment is attempted using only on-axis measurements as it is with two mirror telescopes, it is nearly guaranteed that the TMA will remain in an unacceptably misaligned set. Alignment of a TMA telescope requires measurement at multiple field points, some or all of which are at or near the edge of the format. In fact, it is of

no value to measure on-axis performance of these systems as the coma contribution is constant over the field and can be measured anywhere in the field of view.

## 2. ABERRATION FIELD CHARACTERISTICS IN MISALIGNED TELESCOPES

The key to developing insights into the response of a large telescope to alignment errors is to have a foundation for what forms the aberrations take when perturbations are introduced. It has been shown<sup>3</sup> that there are no new aberrations when a perturbation breaks the rotational symmetry of an optical system. What does change is the field dependence of the individual aberration terms. For an astronomical TMA telescope we can concentrate on the third order aberrations that degrade the quality of the image when the telescope is in a misaligned state. These include 3<sup>rd</sup> order coma and 3<sup>rd</sup> order astigmatism. More specifically, TMA telescopes by definition have a corrected 3<sup>rd</sup> order coma and a corrected 3<sup>rd</sup> order astigmatic field. This condition results in a special case condition for both aberrations. The material that follows presents the special case where the 3<sup>rd</sup> order coma and astigmatism are corrected in the aligned telescope design.

## 3. AN EXAMPLE TMA TELESCOPE: THE JWST

It is convenient to use an actual optical design to illustrate the response to misalignments. A TMA telescope currently under development with a published prescription is the James Webb Space Telescope (JWST). A layout of the JWST based on the data found in SPIE 5487, 2004 is shown below.

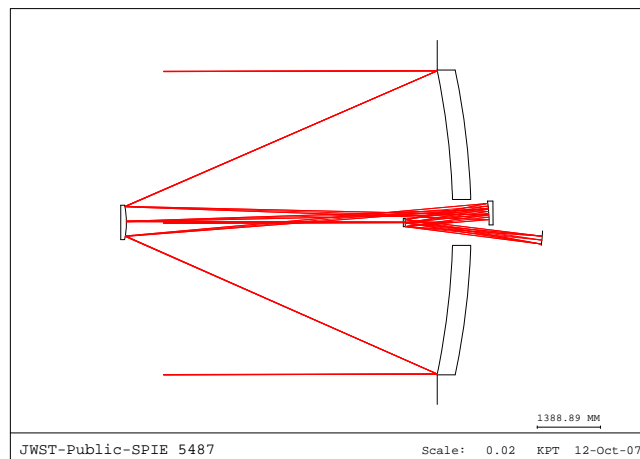


Fig. 1 Schematic Layout of the James Webb Space Telescope Design

This is a Three Mirror Anastigmat (TMA) design form, which is corrected for spherical aberration, coma, and astigmatism through 3<sup>rd</sup> order. While the form has obscuration, a field bias is used to allow the addition of a third powered mirror into the optical train (in addition to a fourth flat pointing mirror). The form is corrected for all third order aberrations (on a curved surface). The performance limiting aberration for the nominal (i.e. perfectly aligned) optical system is found to be 5<sup>th</sup> order astigmatism, as shown in the field curve illustrated in Figure 2.

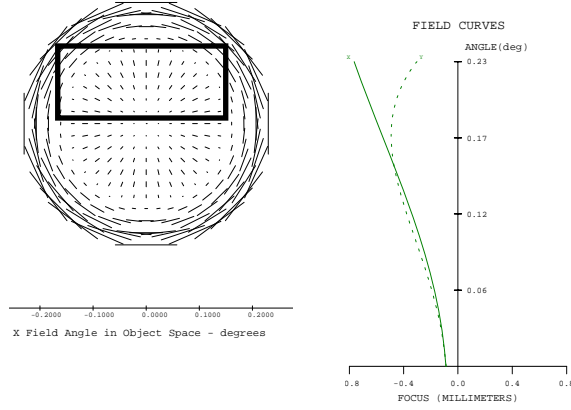


Fig. 2 The limiting aberration of the JWST, 5<sup>th</sup> order astigmatism. The rectangle represents the field of view that will be used.

#### 4. APPLICATION OF NODAL THEORY TO DERIVE THE DOMINANT, CHARACTERISTIC RESPONSE OF ANY TMA TELESCOPE TO MISALIGNMENTS

Given that 3<sup>rd</sup> order coma and 3<sup>rd</sup> order astigmatism are the aberrations of interest, Equation 4.3 of Reference 3 can be used to initiate the derivation of the dominant aberrations of a misaligned TMA. Specifically, the wave aberration through 3<sup>rd</sup> order for a misaligned TMA telescope is

$$\begin{aligned}
 W = & \Delta W_{20}(\vec{\rho} \cdot \vec{\rho}) + \Delta W_{11}(\vec{H} \cdot \vec{\rho}) + \sum_j W_{040_j}(\vec{\rho} \cdot \vec{\rho})^2 \\
 & + \sum_j W_{131_j}[(\vec{H} - \vec{\sigma}_j) \cdot \vec{\rho}](\vec{\rho} \cdot \vec{\rho}) \\
 & + \sum_j W_{222_j}[(\vec{H} - \vec{\sigma}_j) \cdot \vec{\rho}]^2 + \sum_j W_{220_j}[(\vec{H} - \vec{\sigma}_j) \cdot (\vec{H} - \vec{\sigma}_j)](\vec{\rho} \cdot \vec{\rho}) \\
 & + \sum_j W_{311_j}[(\vec{H} - \vec{\sigma}_j) \cdot (\vec{H} - \vec{\sigma}_j)][(\vec{H} - \vec{\sigma}_j) \cdot \vec{\rho}]
 \end{aligned} \quad (1)$$

where all notation is described in Reference 3. Concentrating first on the coma term ( $W_{131}$ ), starting from Equation 4.5 of Reference 3,

$$\begin{aligned}
 W = & \sum_j W_{131_j}[(\vec{H} - \vec{\sigma}_j) \cdot \vec{\rho}](\vec{\rho} \cdot \vec{\rho}) \\
 = & [(\sum_j W_{131_j} \vec{H}) - (\sum_j W_{131_j} \vec{\sigma}_j)] \cdot \vec{\rho}](\vec{\rho} \cdot \vec{\rho})
 \end{aligned} \quad (2)$$

Here, the first summation results in simply the contribution of the rotationally symmetric system, which for a TMA telescope is zero,

$$\sum_j W_{131_j} \vec{H} = W_{131} \vec{H} = 0 \quad (\text{any TMA telescope}) \quad (3)$$

The second summation is the sum of the surface contribution displacement vectors in the image plane each weighted by the corresponding surface contribution to the wave aberration for 3<sup>rd</sup> order coma<sup>3</sup>. This summation results in a net, unnormalized vector in the image plane,

$$\vec{A}_{131} \equiv \sum_j W_{131_j} \vec{\sigma}_j \quad (4)$$

This leads immediately to the 3<sup>rd</sup> order coma aberration in any misaligned TMA of

$$W = W_{131}[\vec{A}_{131} \cdot \vec{\rho}](\vec{\rho} \cdot \vec{\rho}) \quad (5)$$

This is a 3<sup>rd</sup> order coma aberration that is constant over the field of view. This aberration is also a characteristic of a two-mirror Ritchey-Chretien telescope, or, any optical system with overall correction of 3<sup>rd</sup> order coma.

Following a similar derivation for 3<sup>rd</sup> order astigmatism ( $W_{222}$ ), starting from Equation 4.14 of Reference 3

$$W = \frac{1}{2}[\sum_j W_{222j} \vec{H}^2 - 2\vec{H}(\sum_j W_{222j} \vec{\sigma}_j) + \sum_j W_{222j} \vec{\sigma}_j^2] \cdot \vec{\rho}^2 \quad (6)$$

Again, for a TMA telescope, which is corrected also for 3<sup>rd</sup> order astigmatism, the initial term is zero, which then gives

$$W = \frac{1}{2}[-2\vec{H}(\sum_j W_{222j} \vec{\sigma}_j) + \sum_j W_{222j} \vec{\sigma}_j^2] \cdot \vec{\rho}^2 \quad (\text{any TMA telescope}) \quad (7)$$

As was done for coma, define two unnormalized displacement vectors,

$$\vec{A}_{222} \equiv \sum_j W_{222j} \vec{\sigma}_j \quad (8)$$

$$\vec{B}_{222}^2 \equiv \sum_j W_{222j} \vec{\sigma}_j^2 \quad (9)$$

which allows writing the characteristic field dependence for astigmatism in a misaligned TMA as

$$W = \frac{1}{2}W_{222}[-2\vec{H}\vec{A}_{222} + \vec{B}_{222}^2] \cdot \vec{\rho}^2 \quad (10)$$

Unlike coma, which in a misaligned TMA is constant over field, the astigmatism for a misaligned TMA has both a linear with field component and a constant with field component.

Combining Equations 5 with 10, the general aberration state for a misaligned TMA telescope is

$$W \equiv W_{131}[\vec{A}_{131} \cdot \vec{\rho}](\vec{\rho} \cdot \vec{\rho}) + \frac{1}{2}W_{222}[-2\vec{H}\vec{A}_{222} + \vec{B}_{222}^2] \cdot \vec{\rho}^2 \quad (11)$$

These two components are illustrated in Figure 3.

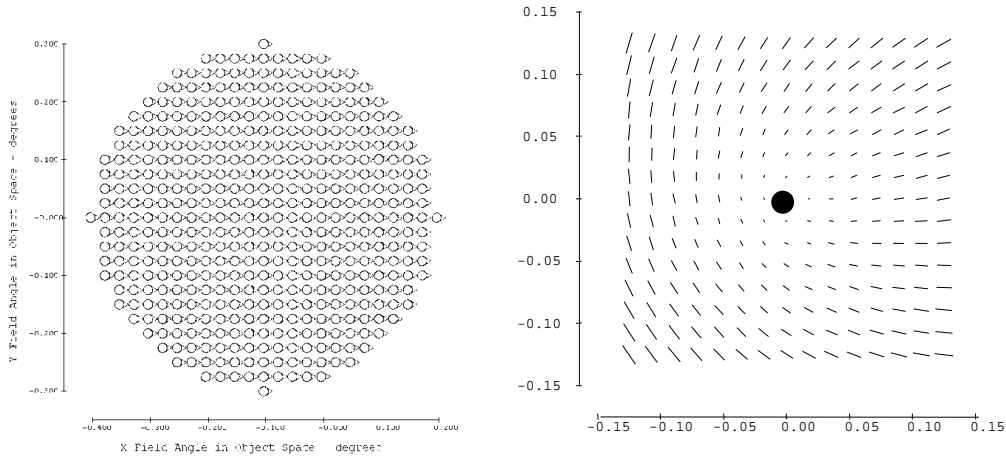


Figure 3. The dominant residual aberrations of a misaligned TMA telescope, (a) 3<sup>rd</sup> order coma that is constant over field, (b) 3<sup>rd</sup> order astigmatism that this is field-asymmetric and field-linear.

## 5. IMPLICATIONS OF THE DOMINANT, CHARACTERISTIC MISALIGNED TMA ABERRATION FIELD RESPONSE FUNCTIONS FOR ALIGNMENT

It is well known that a misaligned two-mirror telescope displays axial coma and for the case of a Ritchey-Chretien design, which is nominally corrected for 3<sup>rd</sup> order coma, this coma is in fact constant over the field of view<sup>1</sup>. This coma is often corrected by a tilt or decenter of the secondary mirror. It can be shown through nodal theory or with other method<sup>4</sup> that there is what is termed a coma-free pivot point, which is an external rotation point along the optical axis where if the secondary rotates about this point the coma is unaffected but magnitude of misalignment astigmatism can be changed.<sup>1</sup> This same condition applies to a TMA telescope and, because in addition a TMA telescope is in fact corrected for 3<sup>rd</sup> order astigmatism in the aligned state, these telescopes are substantially more sensitive to residual alignment induced 3<sup>rd</sup> order astigmatism. In general then, if the misalignment induced coma is in fact corrected by the tilt and/or decenter of one element, this correction will not in all the but the rarest of cases control the misalignment induced astigmatism. In this case then, when the misalignment induced coma has been removed, the residual misalignment induced aberration will be given by

$$W = \frac{1}{2} W_{222} [-2\vec{H}\vec{A}_{222} + \vec{B}_{222}^2] \cdot \vec{\rho}^2 \quad (12)$$

which is illustrated in Figure 4.

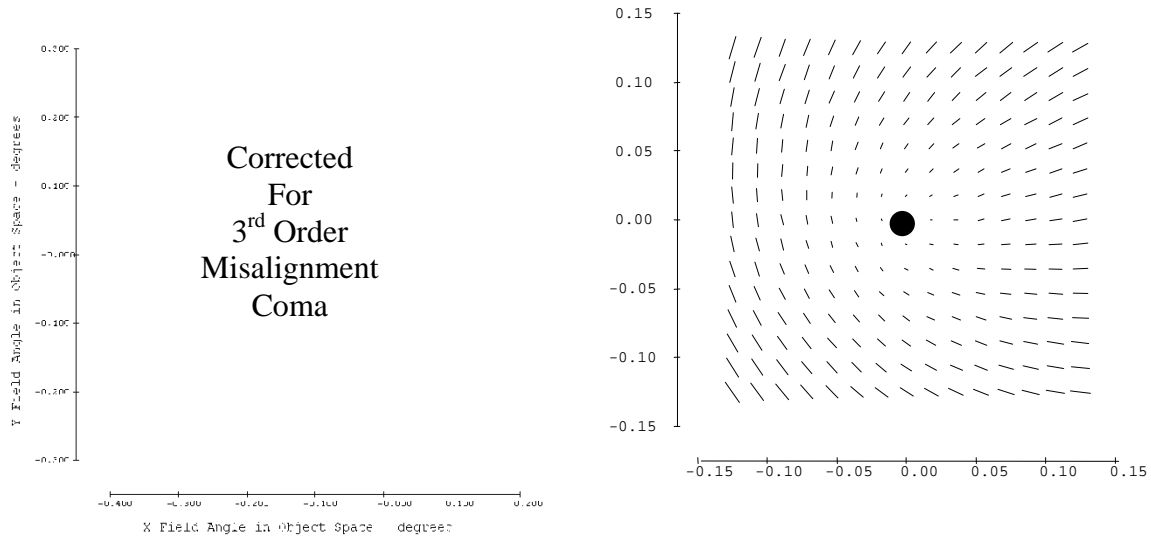


Figure 4. The dominant misalignment aberration of a misaligned TMA telescope with corrected on-axis performance is **field-asymmetric, field-linear 3<sup>rd</sup> order astigmatism**.

A critical point here is that the location for the zero for the field-asymmetric, field-linear astigmatism will, in the absence of astigmatic figure error on any of the mirrors, always reside at the center of the field of view. This can be seen from equation 11, which shows that the solution for the field point with zero misalignment astigmatism is given by,

$$\vec{H} = -1/2(\vec{B}_{222}^2 / \vec{A}_{222}) \quad (13)$$

Looking back to equation 11, the term  $\vec{B}_{222}^2$  is an astigmatism term that is constant over the field. This term arises not from misalignment tilts and decenters (which are small and therefore negligible when squared) but rather it arises instead from astigmatic figure errors, if at all. Therefore, there are two cases of interest

$\vec{B}_{222}^2 = 0$	No figure error	$\vec{H} = 0$
$\vec{B}_{222}^2 \neq 0$	Astigmatic figure error	$\vec{H} \neq 0$

Table 1. The two important cases for zero 3<sup>rd</sup> order astigmatism in a misaligned TMA.

This point is brought out to emphasize that the alignment of TMA telescopes cannot be accomplished using on-axis performance data alone. More important, and the key point from this paper

**The measurement of a corrected on-axis image in a TMA telescope in NO WAY ensures that the telescope is aligned**

A second useful conclusion is that if there is on-axis astigmatism measured in a TMA telescope, it is due not to alignment, but rather to figure error in the telescope optics and significantly, it cannot be corrected by alignment methods. It must be corrected by removing the figure error.

## 6. CONCLUSIONS

This paper has demonstrated how nodal aberration theory can be used to rapidly determine the aberrations and aberration field dependencies for misalignment induced aberrations of any three mirror anastigmatic TMA telescope, including as a specific example the James Webb Space Telescope (JWST).

Specifically, it is shown that a misaligned TMA telescope will suffer from two residual 3<sup>rd</sup> order aberrations. These will be

**3<sup>rd</sup> order coma, which is constant magnitude and orientation over the field**  
**3<sup>rd</sup> order astigmatism, which is field-asymmetric in orientation and increases linearly with field**

As illustrated in Figure 3.

The most significant result is that during assembly a TMA telescope that is aligned to provided diffraction-limited performance on-axis, based on on-axis measurements alone, will not be in an aligned state. There will be significant **field-asymmetric, field-linear 3<sup>rd</sup> order astigmatism**. This is a new aberration form reported here for the first time (shown in Figure 4). In particular, knowing that there is a specific, intrinsic field-asymmetric behavior is an important result for engineers attempting to interpret off-axis performance measurements, especially those based on Zernike component reductions of measured wavefront data. This result can be used to substantial advantage in the alignment of the JWST, or any TMA.

A second important result is that if a TMA telescope is aligned to remove axial coma and under this condition if astigmatism is measured on-axis this astigmatism is due to astigmatic mirror figure error and is not due to misalignment. Importantly, the on-axis astigmatism can only be removed by correcting the mirror figure error; it cannot be corrected, or even reduced through alignment.

## REFERENCES

- [1] T. Schmid, K.P. Thompson, and J.P. Rolland, "Alignment of Two Mirror Astronomical Telescopes", SPIE Conference on Astronomy and Astronomical Instruments, Marseilles, 2008.
- [2] McLeod, "Collimation of Fast Wide-Field Telescopes," Publ. Astron. Soc. Pac. 108, 217-219 (1996).
- [3] K.P. Thompson, "Description of the third-order optical aberrations of near-circular pupil optical systems without symmetry," *JOSA. A*, Vol 22, 1389-1401, 2005.

- [4] M. Bottema and R.A. Woodruff, "Image quality in telescopes with image motion compensation by secondary mirror control", *Applied Optics*, Vol. 11, pp2965-2970, 1972.