Complementary FIR Filter Pair for Distributed Impairment Compensation of WDM Fiber Transmission

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***Abstract*—A method of designing a complementary filter pair is proposed to reduce error accumulation in the split-step backward propagation for distributed impairment compensation of wavelength-division-multiplexing transmission. The complementary filter pair is designed so that the individual errors of the two filters cancel each other. A 12 × 100 Gb/s 16-ary quadrature amplitude modulation transmission system in nonzero dispersion-shifted fiber is simulated using such a filter pair. The required filter length is halved by the complementary filter pair design for a transmission distance of 1200 km.**

***Index Terms*—Complementary filter pair, impairment compensation, optical communication.**

I. Introduction

COHERENT detection combined with digital filtering can be used for real-time electronic dispersion compensation (DC). A 40-Gb/s receiver based on a digital signal processor (DSP) for DC has been reported [1]. A universal scheme of electronic impairment compensation via split-step backward propagation was recently proposed [2], where DC and nonlinearity compensation (NLC) for wavelength-division-multiplexing (WDM) transmission was investigated.

Digital backward propagation by means of the split-step method is based on the division of the total transmission distance into short segments. A large number of segments are required for high bandwidth systems, where the effect of the walk-off between channels has to be properly described. In a real-time implementation, dispersion is compensated on every segment by using finite impulse response (FIR) filters. For a large number of segments, the frequency response of each FIR filter has to be very accurate in order to minimize the error accumulation. Such accuracy is translated into a large filter length and hence, into a large computational load. In this letter, a method of designing a complementary filter pair is proposed for the first time. The individual errors in the frequency response of the two filters cancel each other. As a result, a larger individual filter error can be tolerated and the required filter length is significantly reduced.

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II. Complementary Filter Pair Design

The nonlinear Schrödinger equation for the backward propagation can be written as $\partial A/\partial z = (\hat{D}^{-1} + \hat{N}^{-1})A$, where the DC operator and NLC operator are given, respectively, by

$$\hat{D}^{-1} = \frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t^3}, \quad \hat{N}^{-1} = -i\gamma|A|^2. \quad (1)$$

α, β_2, β_3, γ, t and A are the absorption coefficient, second- and third-order dispersion, nonlinear parameter, retarded time, and field amplitude, respectively [3]. At a sampling frequency of ω_s, the desired frequency response of the FIR filter for DC is a periodic function which, in one period, is given by

$$H_{\rm DC}(\omega) = \exp\left[\left(\frac{\alpha}{2} + i\frac{\beta_2\omega^2}{2} + i\frac{\beta_3\omega^3}{6}\right)h\right] \quad \left(\frac{-\omega_s}{2} < \omega < \frac{\omega_s}{2}\right) \quad (2)$$

where h is the step size which should be small enough to avoid significant walk-off between the edging WDM channels [4], resulting in a limited maximum phase shift of $H_{\rm DC}(\omega)$.

Due to the finite length of the FIR filter, there is an error between the desired response and the filter response [5]. This error accumulates because of the multistep character of the backward propagation. The waveform distortion due to the accumulated error in the DC operations introduces additional errors to the NLC operations. The FIR filter can be optimized by applying appropriate windows, such as Tukey windows in both frequency and time domains [6], [7]. However, optimization can only reduce the integral squared error (ISE) of an individual filter to a value correlated to the filter length.

To reduce error accumulation, the window parameters can be adjusted so that the errors of two filters cancel each other. To illustrate this method, let us analyze a zero-phase-shift all-pass filter when rectangular windows are applied. The finite length of the filter corresponds to a time-domain uniform window. Such a window has a frequency response given by

$$W(\omega) = \sin\left(\frac{2m+1}{2}\omega T\right) / \sin\left(\frac{\omega T}{2}\right) \quad (3)$$

where $2m+1$ is the filter length (tap number) and T the sampling interval. The side lobes in (3), which give rise to the rippling in the filter response, are characterized by oscillations with zero crossings every $\Omega = 2\pi/[(2m+1)T]$ and decreasing amplitudes [5]. With a rectangular window applied in the frequency

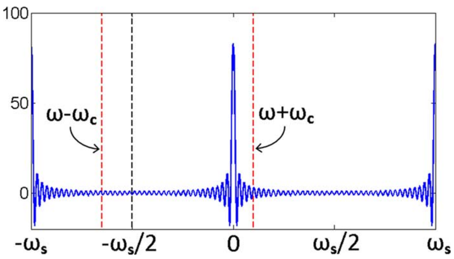

Fig. 1. Graph of $W(\omega)$ (solid line) and the domains of integration (between the dashed lines).

domain, the desired frequency response of the all-pass filter becomes

$$H_D(\omega) = \begin{cases} 1, & (-\omega_c < \omega < \omega_c) \\ 0, & (\text{elsewhere}) \end{cases} \quad (4)$$

where ω_c is the cut-off frequency. Note that there is a margin between the signal bandwidth B and the sampling frequency, so $B/2 < \omega_c < \omega_s/2$. The frequency response of the resulting filter is the convolution of $H_D(\omega)$ with $W(\omega)$. For $-\omega_c < \omega < 0$, the filter response can be expressed as

$$H_f(\omega) = \frac{1}{2\pi} \int_{-\frac{\omega_s}{2}}^{\omega+\omega_c} W(\omega) d\omega + \frac{1}{2\pi} \int_{\omega-\omega_c}^{-\frac{\omega_s}{2}} W(\omega) d\omega. \quad (5)$$

The graph of $W(\omega)$ and the domains of integration are illustrated in Fig. 1, where it is shown that the integral from $\omega - \omega_c$ to $-\omega_s/2$ is much smaller than the integral from $-\omega_s/2$ to $\omega + \omega_c$. Hence the second term on the right hand side of (5) can be neglected. The nth maximum point of $H_f(\omega)$ counting from $\omega = -\omega_c$ is located at $\omega = -\omega_c + (2n-1)\Omega$, whereas the nth minimum point is at $\omega = -\omega_c + 2n\Omega$. Likewise, for $0 < \omega < \omega_c$, the filter response can be expressed as

$$H_f(\omega) = \frac{1}{2\pi} \int_{\omega-\omega_c}^{\frac{\omega_s}{2}} W(\omega) d\omega + \frac{1}{2\pi} \int_{\frac{\omega_s}{2}}^{\omega+\omega_c} W(\omega) d\omega \quad (6)$$

where, again, the second term can be neglected. The nth maximum point of $H_f(\omega)$ counting from $\omega = \omega_c$ is at $\omega = \omega_c - (2n-1)\Omega$, whereas the nth minimum point is at $\omega = \omega_c - 2n\Omega$.

The error is positive at the maximum points and negative at the minimum points. By increasing or decreasing the cut-off frequency ω_c by Ω, the maxima of $H_f(\omega)$ can be shifted to the location of the original minimum points. Within the signal bandwidth, the shifted and original filters have approximately opposite errors which cancel each other.

When Tukey windows have been applied to optimize an FIR filter, the ripples can be shifted by adjusting the roll-off parameter of the frequency-domain Tukey window. Additionally, modifying the time-domain Tukey window can change its frequency response, and consequently change the shape and position of the ripples in the filter response. Hence a global search was performed for all window parameters of both filters to minimize the ISE of the filter pair.

Fig. 2 shows the accumulated error of two identical filters optimized with Tukey windows and the error of a complementary filter pair designed for the simulated WDM system. The filter

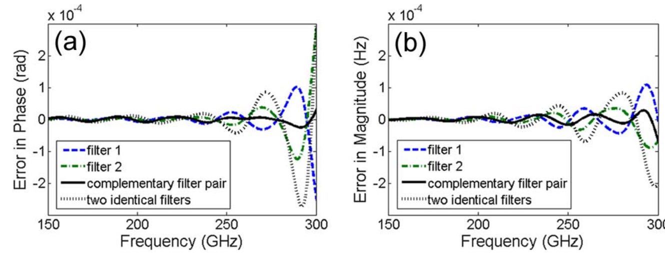

Fig. 2. (a) Phase error and (b) magnitude error of the filter responses. The dotted lines are the accumulated errors of two identical filters. The solid lines are the total errors of the complementary filter pair. The dashed lines are the errors of two individual filters of the complementary filter pair.

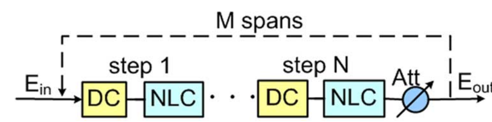

Fig. 3. Block diagram of the split-step backward propagation.

length is 83. The roll-off parameters of the time-domain Tukey windows of Filters 1 and 2 are 0.360 and 0.422, respectively. The roll-off parameters of the frequency-domain Tukey windows are 0.122 and 0.146, respectively. It is shown that the two complementary filters have approximately the opposite error in both phase and amplitude within the signal bandwidth, which is -300 to 300 GHz. Note that the errors are negligible from -150 to 150 GHz. The ISE of the filter pair is 1.06×10^{-10}. The total ISE of two identical filters is 3.94×10^{-9}.

III. Simulation Results

Forward transmission of 12 WDM channels, with a channel spacing of 50 GHz, is simulated with the VPI Transmission Maker. In the 16-ary quadrature amplitude modulation (16-QAM) transmitter of each channel, the laser output is split into in-phase (I) and quadrature (Q) components and modulated with Mach–Zehnder modulators at 100 Gb/s. The loss, dispersion, dispersion slope, and nonlinearity of the nonzero dispersion-shifted fiber (NZ-DSF) are 0.2 dB/km, 4.4 ps/km/nm, 0.045 ps/km/nm^2, and 1.46 W/km, respectively. The span length is 100 km and the erbium-doped fiber amplifier noise figure is 5 dB. At the receiver, the optical signals are mixed with phase-locked local oscillators in a 90° hybrid. The laser linewidth of the transmitter and the local oscillator is 500 kHz. The I and Q components of each channel are detected at a sampling rate of 25 GSa/s. Then the signal is up-sampled to 800 GSa/s by DSP so that the total optical field can be reconstructed for backward propagation. Phase estimation is performed after backward propagation and demultiplexing [8], [9].

The block diagram of the backward propagation is shown in Fig. 3, where an FIR filter and an exponential operator are used for DC and NLC in each step, respectively [2]. The two filters of each complementary filter pair are used in adjacent steps. When the step size is limited by the nonlinearity, a symmetric structure allows a larger step size so that the total number of operations could be reduced [2]. However, in this WDM system using NZ-DSF, the step size is limited by the walk-off due to dispersion. Hence we use a structure with one DC operator and one

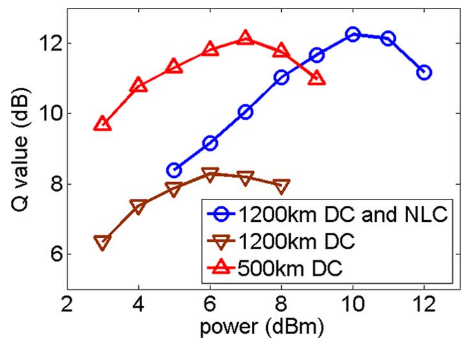

Fig. 4. Q-value of the seventh channel versus total launching power.

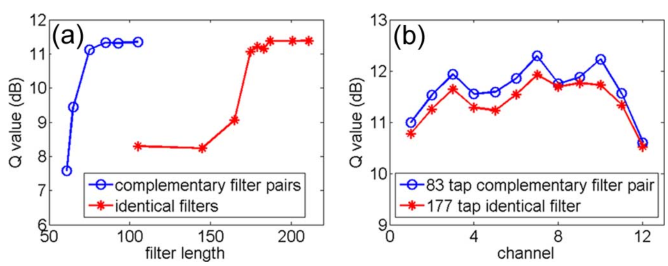

Fig. 5. (a) Mean Q-value of the 12 channels versus filter length and (b) maximum Q-value of each channel.

NLC operator per step which requires fewer operations. Additionally, fewer filters are used in this structure, leading to less error accumulation.

In order to determine the optimum step size, backward propagation was performed using the split-step Fourier method, from which a maximum and stationary Q-value was obtained for $h \leq$ 200 m. As a tradeoff between the computational load and the Q-value penalty, the step size was chosen to be 500 m corresponding to a Q-value penalty of 0.5 dB. For $h = 500$ m, an optimum filter length of 83 taps was obtained according to the design method described in Section II. This filter length ensures a Q-value penalty due to filter error of less than 0.1 dB.

The system performance as a function of launching power is shown in Fig. 4. With NLC, the Q-value is increased by 4 dB in comparison with the Q-value without NLC for a 1200-km link. The transmission distance can be increased from 500 to 1200 km by NLC preserving the same Q-value.

The mean Q-value of the 12 channels as a function of the filter length is shown in Fig. 5(a), where the Q-value increases to a maximum and stationary value when the effect of the filter error becomes negligible. Fig. 5(b) shows the Q-values of all the channels after 1200-km transmission and impairment compensation. It is shown that the performance of the complementary filter pairs with a filter length of 83 is better than the performance of the identical filters with a filter length of 177. The complementary filter pair design reduces the required filter length and, consequently, the total computational load by a factor of 2.

IV. Discussion and Conclusion

The computational requirement is a crucial consideration for DSP implementation. The computation can be parallelized so that the DSP can operate at a speed lower than the sampling rate [2]. Ignoring the computation and latency caused by up-sampling, demultiplexing, and phase estimation, the number of multiply-accumulate (MAC) units is

$$N_{\mathrm{MAC}} = N_b \times N_s \times (4N_t + 16) \tag{7}$$

where N_b is the number of the parallelization branches, N_s is the number of steps, and N_t is the filter length. Each FIR filter requires $4N_t$ multiplications and $4N_t - 2$ summations. Each NLC operation includes 16 multiplications and 7 summations. The latency of the backward propagation is

$$T_L = N_s\left(\lceil \log_2 N_t \rceil + 9\right) \times T/2 \tag{8}$$

where $\lceil x \rceil$ stands for the ceiling function of x. It is assumed that each multiplication or summation requires half a clock cycle $(T/2)$. The computation time for DC and NLC in one step are $(\lceil \log_2 N_t \rceil + 2) \times T/2$ and $7 \times T/2$, respectively [2].

By assuming a DSP speed of 25 GHz, the impairment compensation of a 1200-km link requires 22.3 M of MAC units. The computational efficiency is 464 kMAC/bit, which is defined as the total MAC operation rate divided by the total bit rate. The latency is 10.2 μs which is small in comparison with the transmission latency of 6 ms in the fiber.

The robustness of the backward propagation method against fiber parameter uncertainty is also of practical interest. Preliminary simulations indicate that it is quite robust. For example, there is negligible penalty for 1% fluctuation in fiber dispersion, if an adaptive filter is used at the receiver to compensate for the residual dispersion. The backward propagation method has been demonstrated experimentally in which the exact fiber parameters were unknown [10].

According to simulation results, to reduce the filter error accumulation in the distributed impairment postcompensation, a method of designing a complementary filter pair is proposed. For a 12 × 100 Gb/s 16-QAM/WDM system using NZ-DSF, the computational load is halved by the complementary filter pair design. The transmission distance is increased from 500 to 1200 km by NLC preserving the same Q-value.

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