Coupling between energy and phase in hollow-core fiber based *f*-to-2*f* interferometers

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Abstract: The dependence of the carrier-envelope (CE) phase of the pulses from a hollow-core fiber on the input laser energy was studied using two *f*-to-2*f* interferometers. The CE phase in the in-loop *f*-to-2*f* interferometer was measured with the octave spanning white-light spectrum from the hollow-core fiber, whereas the out-of-loop interferometer was based on a sapphire plate. By modulating the input power of the in-loop interferometer and measuring the out-of-loop CE phase at the same time, the coupling coefficient between the measured CE phase and the laser energy for the hollow-core fiber was determined to be 128 mrad per 1% energy change.

OCIS codes: (320.7100) Ultrafast measurement; (320.7110) Ultrafast Nonlinear optics; (320.7140) Ultrafast processes in fibers.

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1. Introduction

Carrier-envelope (CE) phase controlled, high power few-cycle pulses are an indispensable tool for generating single isolated attosecond pulses and studying other high field physics processes [1,2]. To produce such pulses, CE phase stabilized multi-cycle pulses from a chirped-pulse amplifier (CPA) are focused into a hollow-core fiber filled with a noble gas to broaden the spectrum. Either chirped mirrors or adaptive phase modulators are then applied to compress the pulses down to a few cycles [3-6].

The CE phase of few-cycle high power laser pulses can be determined by measuring the asymmetry of the angular distribution of electrons in above-threshold ionization [2]. However, it is difficult to apply this technique to multi-cycle pulses. Instead, it is common to lock the CE phase of CPA systems by using *f*-to-2*f* interferometers [7, 8], assuming the phase shift determined from the interferometer fringe is the CE phase shift. In this work, we followed this tradition. So far, the CE phase stability of CPA systems has been extensively investigated, but very few studies have been conducted to investigate the effects of the hollow-core fiber on the CE phase stability of the compressed pulses.

For CE phase measurement after a hollow-core fiber, the required octave spanning spectrum for the *f*-to-2*f* measurement can be obtained by focusing the output pulses from the hollow-core fiber into a sapphire plate if their bandwidth is narrower than an octave [9]. When the bandwidth of the white-light from the hollow-core fiber covers an octave, it can be used directly to perform *f*-to-2*f* measurements [10]. This approach is more appealing because of the simplicity and the removal of the CE phase noise introduced by the sapphire plate.

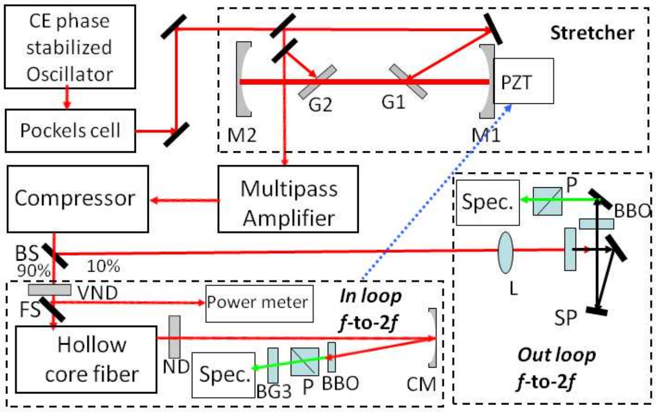

Fig. 1. Experimental setup for determining the energy to CE phase coupling. VND: variable neutral density filter; L: focusing lens; SP: Sapphire plate; BBO: frequency doubling crystal; P: polarizers; FS: fused silica; Spec.: spectrometer and computer. In stretcher, G1 and G2: gratings; PZT: piezoelectric transducer; M1and M2: mirrors; BS: beam splitter.

Previously, the CE phase error introduced by the nonlinear phase noise in microstructure fiber was studied, and the power dependence of the refraction index was determined to be the main factor [11]. In the hollow core fiber, recent experimental_studies indicated that the CE phase noise after the fiber could be significantly larger than that of the input pulses [9, 10]. In ref. 9, when the CE phase was stabilized before a neon-filled hollow-core fiber within 189 mrad RMS, its standard deviation measured by a sapphire plate based *f*-to-2*f* interferometer

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after the neon-filled hollow-core fiber was 370 mrad. In another report, when the CE phase was measured to be 490 mrad RMS before the hollow-core fiber, 610 mrad RMS of the CE phase was measured after the fiber using the octave-spanning spectrum of the white-light pulses [10]. The increase of the CE phase noise is likely caused by the laser power fluctuation inside the hollow-core fiber or the sapphire-based interferometers. However, it was not clear which of the two are more susceptible to the power instability. For sapphire-based *f*-to-2*f* interferometers, previous studies showed that 1% energy change can cause 160 mrad of CE phase measurement error [12, 13] and a two step model was proposed to explain the coupling between the power fluctuation and the CE phase error [14]. In this letter, we study the influence of the laser energy fluctuation on the precision of the *f*-to-2*f* measurements using the octave-spanning white-light from the hollow-core fiber.

2. Experiment

The experiment was carried out with the Kansas Light Source, which is a Ti:Sapphire chirped pulse amplification laser system equipped with a grating-based stretcher and compressor, as shown in Fig. 1. The oscillator CE offset frequency f_0 was stabilized. After amplification, more than 2 mJ, 1 kHz, 30 fs pulses with a beam diameter of 1 cm from the laser were focused by a *f*=1.5 m dielectric mirror and coupled into a 0.9 meter long, 400 μm inner core diameter hollow-core fiber filled with 2 bar of neon gas. With such high energy seeding pulses, strong self-phase modulation produced white-light pulses with 1.2 mJ energy, whose spectrum covered more than one octave from 400nm to 1000nm as shown in Fig. 2. In order to improve the accuracy of CE phase control, the laser power before the hollow-core fiber was stabilized to around 0.5% RMS [13].

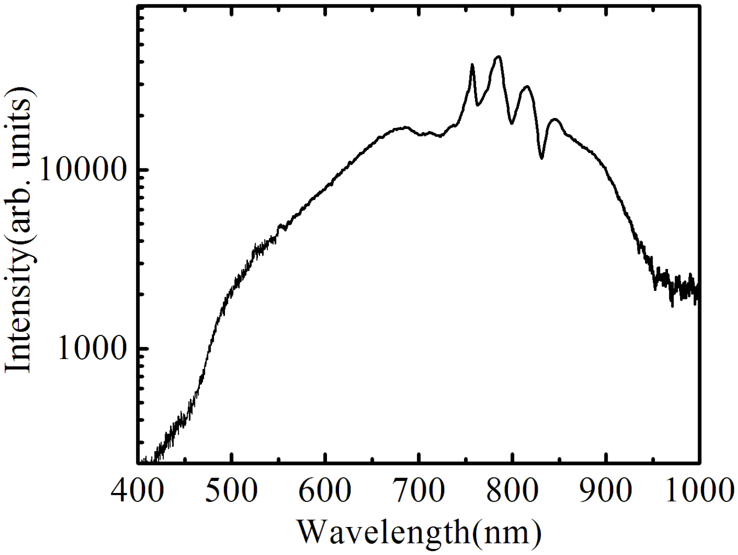

Fig. 2. The output spectrum of the octave-spanning white-light from the hollow-core fiber with 2 mJ input and 2 bar Ne pressure.

The effect of the input laser energy stability on the carrier-envelope phase of the pulses from a hollow-core fiber was studied using two *f*-to-2*f* interferometers. The CE phase in the first *f*-to-2*f* interferometer was measured with the octave-spanning white-light spectrum from the hollow-core fiber. The beam from the fiber was collimated by *f*=1.5 m silver mirror and attenuated by a reflective fused silica neutral density (ND) filter. The 1 cm diameter beam was focused into a 100 μm-thick Barium Borate crystal (BBO) with an *f*=50 cm cylindrical mirror for second harmonic generation (SHG). To produce the *f*-to-2*f* interference fringes with the best contrast, the BBO phase matching angle was set at 26 degrees, which corresponds to a maximum Type I SHG efficiency around 900 nm of the fundamental wavelength. A polarizer

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was used to project the second harmonic and the fundamental field onto the same axis to facilitate interference.

Since the white-light was focused into the BBO without temporal compression by chirped mirrors, the large group delay between the fundamental light centered around 450nm and the second harmonic produced sufficient fringes for extracting the CE phase. A spectrometer (Ocean Optics HR2000+, 380nm-580nm) with a resolution of 0.11 nm was used to record the fringes over a 50 ms exposure time. To avoid the saturation of the spectrometer CCD caused by the strong fundamental light, a BG3 filter was used to block the fundamental wavelength within the above range, but longer than 500nm. The CE phase drift was extracted from the fringes by the standard algorithm of Fourier transform spectral interferometry (FTSI). The second *f*-to-2*f* interferometer was based on a sapphire plate [12].

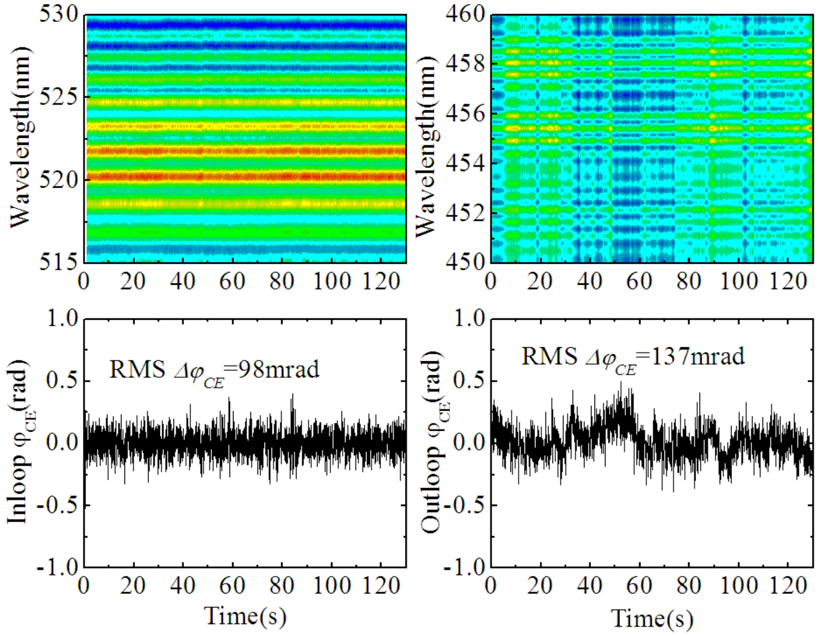

Fig. 3. In-loop CE phase stabilized by a sapphire plate based *f*-to-2*f* interferometer *(left)* and out-of-loop CE phase measured by a hollow-core fiber based *f*-to-2*f* interferometer *(right)*.

First the sapphire plate based *f*-to-2*f* interferometer was used to lock the CE phase before the hollow-core fiber [15, 16], and the hollow-core fiber based *f*-to-2*f* interferometer was used to check the CE phase of the pulses after the fiber. As shown in Fig. 3, when the CE phase was locked within an accuracy of 98 mrad before the hollow-core fiber, the CE phase fluctuation after the hollow-core fiber was 137 mrad. Compared with the previous results of 370 mrad after the hollow-core fiber [9], the CE phase stability has been improved by more than a factor of two. We believe that the main reason is the removal of the sapphire plate in the CE phase measurement after the hollow-core fiber. The power fluctuation after the hollow-core fiber is much larger than that of the input. In this situation, if the sapphire plate was applied after the hollow-core fiber, the white-light generation would increase the error in the CE phase measurement [12].

We then locked the CE phase by measuring the phase shift with the white-light from the hollow-core fiber and feedback controlling the grating-based stretcher. As shown in Fig. 4, the CE phase was locked within an in-loop accuracy of 94 mrad RMS. At the same time, the out-of-loop sapphire plate based *f*-to-2*f* interferometer measurement showed a CE phase fluctuation of 134 mrad before the fiber. These two experiments indicated that the hollow-core fiber does not introduce significant amounts of CE phase drift when the power of the input pulses is stabilized to 0.5% RMS. They also indicated that the change in grating separation in the CPA stretcher for stabilizing the CE phase did not affect the pulse propagation in the fiber, which was expected because the variation of effective grating separation is less than 1 micrometer [15].

To quantitatively measure the laser energy to CE phase coupling coefficient for the hollow-core fiber based interferometer, a variable reflective fused silica ND filter driven by an electric motor was placed before the focusing mirror of the hollow-core fiber to modulate the input power, as shown in Fig. 1. The front surface of the fused silica glass filter is coated with a thin metal film to attenuate the power by reflection. Since the glass does not absorb the 800 nm light, the thermal lensing effect is very small and can be ignored. Previously this method has been used to measure the power to CE phase coupling coefficient for a sapphire plate *f*-to-2*f* interferometer [12]. When the ND filter was driven periodically within the range of 5 degrees, the power was modulated within the range of 10%. 8% of the modulated beam was reflected by a thin fused silica plate to a power meter. In order to keep more than one octave span of the white-light spectrum, the neon pressure inside the hollow core fiber was increased to 2.3 bar to compensate for the power loss caused by the fused silica plate. Under this new pressure of 2.3 bar, we repeated the experiment discussed above to confirm that, regardless of the configuration of the two *f*-to-2*f* interferometers as control loop and measurement loop, the same CE phase stability could be obtained. Finally, the hollow-core fiber *f*-to-2*f* interferometer was used to stabilize the CE phase after the fiber, while the sapphire plate *f*-to-2*f* interferometer was used to measure the out-of-loop CE phase where the laser power was not modulated.

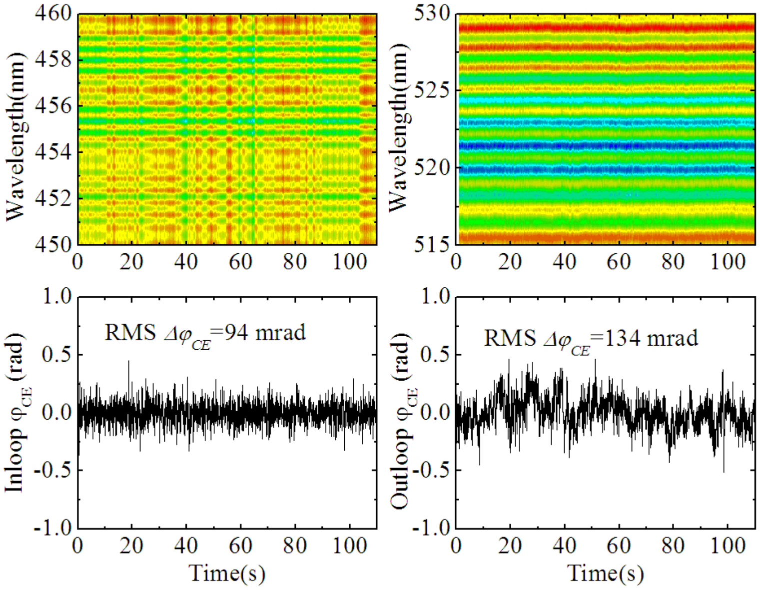

Fig. 4. In-loop CE phase stabilized by a hollow-core fiber based *f*-to-2*f* interferometer *(left)* and Out-of-loop CE phase measured by a sapphire plate based *f*-to-2*f* interferometer *(right)*.

Although the interference fringe intensity was modulated periodically due to the power modulation, the in-loop measurement shows a CE phase fluctuation of 109 mrad_in Fig. 5(a), which is similar to the case without power modulation. Figure 5(b) shows the anti-correlation between the in-loop power modulations and out-of-loop CE phase measurement. To simplify the analysis, it is assumed that the out-of-loop CE phase drift is only caused by modulation of the in-loop input power. A least-square linear fit in Fig. 5(c) shows the 1% power fluctuation introduced a 128 mrad CE phase error. This value is smaller than the 160 mrad for sapphire plate based *f*-to-2*f* interferometers [12], which can be related to the fact that the white-light generation in the hollow-core fiber is produced by the self-phase modulation all the way through the 0.9 m long gas-filled waveguide rather than the strong self-focusing process combined with filamentation in a 2.3 mm sapphire plate [14].

3. Discussion

Due to the dispersion the CE phase changes during the process of propagation, as expressed:

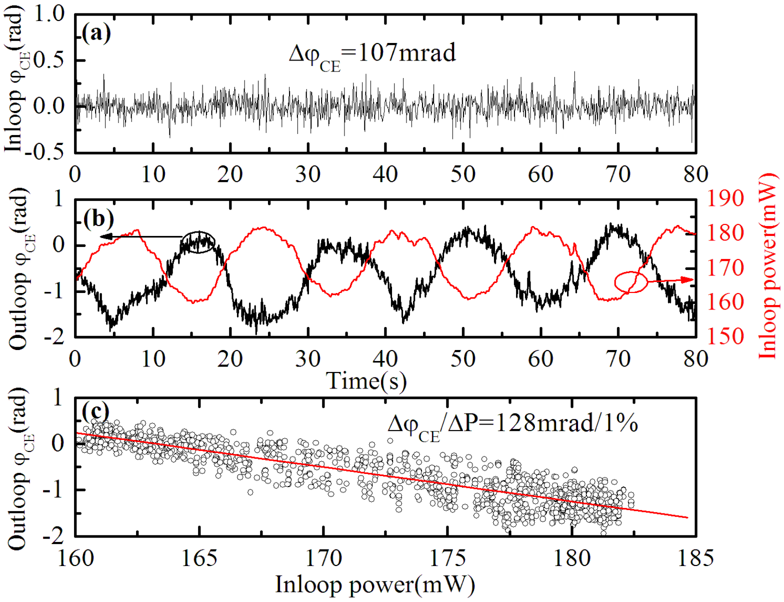

Fig. 5. (a) In-loop CE phase locked by a hollow-core fiber based *f*-to-2*f* interferometer; (b) Out-of-loop CE phase measured by a sapphire plate based *f*-to-2*f* interferometer and the in-loop power modulation; (c) CE phase change to laser power coupling coefficient by a least-square fitting.

$$\Delta\phi_{CE} = 2\pi L\frac{dn}{d\lambda} \quad , \tag{1}$$

where *L* is the length of the hollow-core fiber and *n* is the total refractive index. Ignoring the wave guide dispersion, the total refractive index *n* of the hollow-core fiber can be expressed as:

$$n = n_0 + n_2 I \quad , \tag{2}$$

where n_0 is the linear refractive index of the noble gas medium and *I* is the intensity of the incident wave. Therefore, the total change of CE phase can be expressed as:

$$\Delta\phi_{CE} = 2\pi L(\frac{dn_0}{d\lambda} + \frac{dn_2}{d\lambda}I)\,. \tag{3}$$

The nonlinear part can be written as [17]:

$$n_2 = \frac{3}{2n_0^2\varepsilon_0 c}\chi^{(3)}, \tag{4}$$

where $\chi^{(3)}$ is the nonlinear susceptibility, ε_0 is the dielectric constant and *c* is the vacuum speed of light.

When the power of the input laser pulse fluctuates, the value of the second term in Eq. (3) is influenced and causes the intrinsic CE phase error, which can be expressed:

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$$\begin{aligned}\delta\Delta\phi_{CE} &= 2\pi L\frac{dn_2}{d\lambda}\Delta I = 2\pi L\Delta I\left(-\frac{3\chi^{(3)}}{n_0^3\varepsilon_0 c}\frac{dn_0}{d\lambda}+\frac{3}{2n_0^2\varepsilon_0 c}\frac{d\chi^{(3)}}{d\lambda}\right)\\ &= 2\pi L\Delta I n_2\left(-2\frac{dn_0}{n_0 d\lambda}+\frac{d\chi^{(3)}}{\chi^{(3)}d\lambda}\right)\end{aligned} \tag{5}$$

For the first term of Eq. (5), when the optical frequency ω is much lower than resonance frequency ω_0 of neon gas (21.56 eV), the linear refraction index can be derived from the harmonic oscillator model [18]:

$$n(\omega)=\sqrt{1+\frac{Nq_e^2}{\varepsilon_0 m_e}\left(\frac{1}{\omega_0^2-\omega^2}\right)}. \tag{6}$$

Here, q_e and m_e are the charge and mass of the electron, respectively, and N is the number density of the ideal gas. For the second term of Eq. (5), under the same off-resonant condition, the nonlinear susceptibility can be approximately expressed as [19, 20]:

$$\chi^{(3)}(\omega)\approx\frac{\omega_0-\omega_a}{\omega_0-\omega}\chi^{(3)}(\omega_a), \tag{7}$$

where $\chi^{(3)}(\omega_a)$ is the nonlinear susceptibility measured at frequency ω_a. Here we took $\chi^{(3)}(\omega_a)$ for neon gas from Ref. [19], $\chi^{(3)}(1055nm)=6.2\times10^{-28}m^2/V^2$ under 1 atm pressure and at a temperature of 0° C.

Considering the 2 mJ input pulse energy W, pulse duration τ of 30 fs, and hollow-core fiber inner core radius r of 200 µm, the peak intensity is calculated:

$$I=\frac{W}{\pi r^2\tau}=\frac{2\times10^{-3}J}{\pi\times(200\times10^{-6}m)^2\times30\times10^{-15}s}=5.3\times10^{17}W/m^2. \tag{8}$$

When the laser has 1% power fluctuation $\Delta I = I/100 = 5.3\times10^{15}W/m^2$, under the fiber pressure of 2.3 bar and room temperature of 17° C, the intrinsic CE phase error due to intensity fluctuation is calculated from Eq. (5) to be $\delta\Delta\phi_{CE}\approx1.2mrad$, which is much smaller than the measured coupling coefficient and can be ignored.

Although the CE phase error due to the fluctuation of nonlinear dispersion is very small in the hollow core fiber, the measured CE phase, which is the phase delay between the blue component centered at $\lambda_1=455nm$ and the frequency doubled component originally centered at $\lambda_2=910nm$ nm, fluctuates when the input pulse energy changes [8, 14]. To be more specific, the phases of the λ_1 and λ_2 components can be expressed as:

$$\phi_1(L_{pro})=\phi_{NL}^1+\phi_L^1(L_{pro})+\phi_{CE}, \tag{9}$$

$$\phi_2(L_{pro})=\phi_{NL}^2+\phi_L^2(L_{pro})+\phi_{CE}. \tag{10}$$

Here, ϕ_{NL} is the nonlinear phase shift introduced by the self-phase modulation process and ϕ_L is the linear phase shift introduced by the propagation after self-phase modulation and is a function of the propagation distance L_{pro}. L_{pro} is thus equal to $L-l$, where l is the self phase modulation distance for generating the spectrum with one octave span.

When the 910 *nm* component is frequency doubled in the BBO crystal, its phase becomes:

$$\varphi_2(L_{pro}) = 2\phi_2(L_{pro}) = 2\phi_{NL}^2 + 2\phi_L^2(L_{pro}) + 2\phi_{CE}\,. \qquad (11)$$

The measured CE phase is then expressed as:

$$\Phi = \varphi_2(L_{pro}) - \phi_1(L_{pro}) = \left[2\phi_{NL}^2 - \phi_{NL}^1\right] + \left[2\phi_L^2(L_{pro}) - \phi_L^1(L_{pro})\right] + \phi_{CE}\,. \qquad (12)$$

When the intensity of the input laser pulse into the hollow-core fiber fluctuates by 1%, the nonlinear phase caused by self phase modulation, given by $\int_0^l 2\pi I/\lambda dz$, also fluctuates. In order to compensate the intensity fluctuation and maintain an octave spanning spectrum, the required self phase modulation distance l also must fluctuate by 1%, which in turn changes the value of the measured CE phase Φ due to the fluctuation of the propagation distance ΔL_{pro}.

Since the octave spanning spectrum is generated at the end of the hollow-core fiber, the self phase modulation distance is close to the total distance of the fiber $l \approx L$. Therefore the fluctuation of the propagation distance is:

$$\Delta L_{pro} = \Delta l = l/100 \approx L/100 = 9mm\,, \qquad (13)$$

which gives the CE phase measurement error of:

$$\Delta\Phi = 2\pi\Delta L_{pro}\left[2\frac{n(\lambda_2)}{\lambda_2} - \frac{n(\lambda_1)}{\lambda_1}\right] = 2\pi\Delta L\left[\frac{n(\lambda_2)}{\lambda_1} - \frac{n(\lambda_1)}{\lambda_1}\right] \approx 120\ mrad\ . \qquad (14)$$

In the above discussion if the dispersion of the pulse is also considered, the peak intensity of the pulse drops as it propagates. Therefore, the self phase modulation strength decreases as the pulse becomes longer. To compensate the 1% intensity fluctuation at the entrance of the fiber, the self phase modulation distance *l* changes more than 1% because the pulse duration becomes longer during the propagation process. Considering this factor along with the previous discussion, the prediction our model matches very closely to the measured value of 128 mrad per 1% power fluctuation.

4. Conclusion

In conclusion, we found that the hollow-core fiber based *f*-to-2*f* interferometer provided a higher accuracy of CE phase measurement than a sapphire plate based interferometer placed after the fiber. The laser energy to CE phase coupling for the hollow-core fiber based *f*-to-2*f* interferometer was measured. It shows that 128 mrad of CE phase drift in the measurement was introduced by a 1% of laser energy change under our experimental conditions. Furthermore, a simple model was proposed to explain the coupling mechanism. Determining the coupling coefficient is important for specifying the power stability of CPA lasers for seeding the hollow-core fibers. Understanding the CE phase properties of such fibers is crucial for attosecond pulse generation, ATI and other experiments at the frontiers of ultrafast science.

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