Mechanism of phase-energy coupling in *f*-to-2*f* interferometry

Chengquan Li, Eric Moon, Hiroki Mashiko, He Wang, Christopher M. Nakamura, Jason Tackett, and Zenghu Chang*

J. R. Macdonald Laboratory, Department of Physics, Kansas State University, Manhattan, Kansas 66506, USA

*Corresponding author: chang@phys.ksu.edu

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White-light generation has been used widely in single-shot f-to-2f interferometers for stabilizing the carrier-envelope (CE) phase of laser amplifiers. The accuracy of the relative phase values measured by such an interferometer is affected by fluctuations in the laser pulse energy. A simple two-step model is proposed to explain the mechanism that couples the laser energy and the CE phase. The model explains the experimentally observed dependence of the group delay between the f and the 2f pulses on the laser energy, as well as the CE phase shift caused by the pulse energy variation. © 2009 Optical Society of America

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With the rapid development of ultrashort laser pulse generation, the carrier-envelope (CE) phase acts a very important role in recent researches of strong field processes. Accurate measurement and precision control of the CE phase is important for attosecond vacuum-ultraviolet pulse generation and other atomic physics studies [1,2]. The electric field of this ultrashort laser pulse can be described by $E(t) = E_0(t)\cos(\omega_0 t + \beta(t) + \varphi_{CE})$, where the CE phase,$\varphi_{CE}$, specifies the offset between the peak of the amplitude envelope $E_0(t)$ and the closest oscillation peak of the carrier wave with frequency ω_0, and $\beta(t)$ represents possible chirp in the pulse. Conventionally, for high-power pulses from laser amplifiers, the shot-to-shot CE phase change can be measured optically by using f-to-2f interferometry [3,4]. The measured phase variation between successive pulses can be used as a feedback control signal to correct the CE phase drift of the amplified pulses [4]. Laser energy fluctuation was found to be one of the primary sources of error in CE phase measurements by f-to-2f interferometry [4–6]. In our experiments reported in [6], an in-loop f-to-2f interferometer was used to correct the CE phase drift of the pulses from a grating-based chirped pulse amplifier [7]. To determine the dependence of the CE phase on the laser energy, the pulse energy in the in-loop f-to-2f interferometer was varied with a variable neutral density (VND) filter while the relative CE phase caused by the energy fluctuation was measured by the out-loop f-to-2f interferometer [6]. Figure 1(a) shows the experimental setup of our f-to-2f interferometer, which is very similar to commercial units used in many laboratories [4,8]. Laser beams with energies $<1\,\mu$J were focused into a sapphire plate to generate white light by filamentation. The spectrum of this white light covered an octave. In performing the relative CE phase measurement, the IR portion of the spectrum (the f pulse) was centered at 1064 nm and was frequency doubled in a BBO crystal. The second harmonic of the IR, with power spectrum $I_{SHG}(\omega)$ and the green portion of the white light near 532 nm, with power spectrum $I_G(\omega)$ (the 2f pulse), was projected onto the same polarization direction by a polarizer. The transmitted pulses were then sent to a spectrometer to measure the interference of the two pulses in the spectral domain [3,4] to determine the shot-to-shot CE phase variation.

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The measured spectral interferogram is

$$S(\omega) = I_{\mathrm{SHG}}(\omega) + I_G(\omega) + 2\sqrt{I_{\mathrm{SHG}}(\omega) I_G(\omega)} \cos[\phi_{\mathrm{SHG}}(\omega) - \phi_G(\omega)], \quad (1)$$

where $\phi_{\mathrm{SHG}}(\omega)$ and $\phi_G(\omega)$ are the total spectral phases, which include the CE phase of the frequency-doubled IR pulse and the green pulse, respectively. From the interference signal, the total phase difference, $\Phi(\omega) = \phi_{\mathrm{SHG}}(\omega) - \phi_G(\omega)$, can be retrieved using Fourier transforms and filtering techniques [9,10]. When the laser energy is kept constant, the change of CE phase can be obtained by measuring the pulse-to-pulse variation of $\Phi(\omega)$ at a given frequency ω_G [3,4]. In reality the pulse-to-pulse laser power fluctuation affects the accuracy of the relative CE phase measurement.

Here a two-step model is proposed to explain the coupling between the laser energy fluctuation and the CE phase changes, as shown in the inset of Fig. 1(a): first the self-focus process inside the sapphire plate and second the white light propagated through the sapphire. We found out that this model is also very helpful in analysis of self-focusing phenomena by ultrashort laser pulse as well as the CE phase stabilization.

The generation of white light by forming a filament in a sapphire plate involves complicated nonlinear processes, which have been studied by solving the nonlinear Schrödinger equation numerically [11,12]. Our model is analytical for the energy range considered. When the laser peak power, P, is higher than the critical value, $P_C = \pi(0.61)^2\lambda_0^2/8(n_0 n_2)$, a filament is formed inside the sapphire plate [12,13]. Here $\lambda_0 = 0.79\,\mu$m is the center wavelength of our laser system, $n_0(\lambda_0) = 1.76$ and $n_2 = 2.9 \times 10^{-16}\,\mathrm{cm}^2/\mathrm{W}$ are the linear and nonlinear indices of refraction of sapphire. The calculated critical power is $P_C = 1.79\,\mathrm{MW}$. As shown in Fig. 1(a), for an input laser diameter D, the focal spot radius at the input of the sapphire plate is $\omega_0 = \lambda_0 f/D$. In our experimental setup, the focal lengths of the lenses are $f = 70\,\mathrm{mm}$ in the in-loop interferometers and $f = 75\,\mathrm{mm}$ in the out-loop interferometers, and the diameter $D = 5\,\mathrm{mm}$. The laser energy was fine-tuned with a VND filter until a single stable filament was formed in the sapphire plate. The laser beam size decreases as it propagates inside the plate due to Kerr self-focusing until defocusing caused by laser-produced plasma balances the self-focusing [12]. The self-focusing distance is [13]

$$z_{\mathrm{sf}}(\varepsilon) = \frac{2n_0 w_0^2}{\lambda_0} \frac{1}{\sqrt{P/P_c - 1}} = \frac{2n_0 w_0^2}{\lambda_0} \frac{1}{\sqrt{\varepsilon/(\tau_p P_c) - 1}}, \quad (2)$$

where ε is the pulse energy. τ_P is the input pulse duration, which is 35 fs for our laser system. The

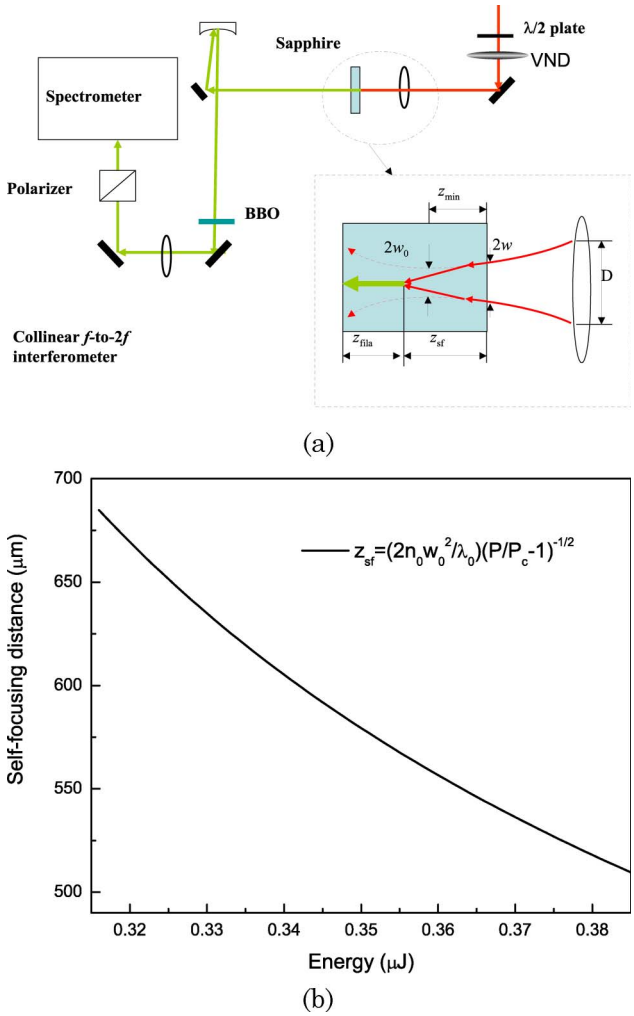

Fig. 1. (Color online) (a) Collinear f-to-$2f$ interferometer. Inset: Formation of a single filament by self-focusing. D, diameter of the laser beam; f, focal length of the focusing lens; ω_0, radius of the focal spot; z_{sf}, self-focusing distance; z_{fila}, length of the filament. (b) Calculated dependence of the self-focusing distance on the laser energy.

calculated relative self-focusing distance $\Delta z_{\mathrm{fila}}(\varepsilon)$ as a function of the laser energy is shown in Fig. 1(b). The filament length is $z_{\mathrm{fila}}(\varepsilon) = L - z_{\mathrm{sf}}(\varepsilon)$. $L = 2.3\,\mathrm{mm}$ is the thickness of the sapphire plate.

Other nonlinear processes, such as self-phase modulation and self-steepening, also occur as the beam contracts to a filament. They broaden the pulse spectrum as the pulse propagates along the z direction. The origin of z is the input surface of the plate. The spectrum width at $z = z_{\mathrm{sf}}$ is $\Delta\lambda \approx \phi_{\mathrm{spm}}\Delta\lambda_0$ due to self-phase modulation alone [14]. The input spectral width is $\Delta\lambda_0 \approx 35\,\mathrm{nm}$. The maximum nonlinear phase shift is

$$\phi_{\mathrm{spm}} = -\int_0^{z_{\mathrm{sf}}} \frac{2\pi}{\lambda_0} n_2 \frac{2\varepsilon}{\tau_p \pi w^2(z)} \mathrm{d}z \approx -\frac{2\pi^2}{\lambda_0^2} \frac{n_2 \varepsilon}{\tau_p}, \quad (3)$$

where $\omega(z)$ is the beam waist. Under our experimental conditions, the nonlinear phase shift is $\phi_{\mathrm{spm}} \approx 10$.

The combination of the self-phase modulation and self-steepening processes during the self-focusing produce the required IR and green components in the wings of the white-light spectrum for the f-to-$2f$ measurements.

Accounting for both linear and nonlinear dispersions, the spectral phases of the green and IR pulses at the beginning of the filament are

$$\phi_{G,\mathrm{sf}}(\omega) = \varphi_{\mathrm{CE}} + \Delta\varphi_{n_0} + \Delta\varphi_{n_2} + \phi_{\mathrm{spm}} \approx \phi_{G.\mathrm{sf}}(\omega_G), \tag{4}$$

$$\phi_{\mathrm{IR,sf}}(\omega) = \varphi_{\mathrm{CE}} + \Delta\varphi_{n_o} + \Delta\varphi_{n_2} + \phi_{\mathrm{spm}} \approx \phi_{\mathrm{IR,sf}}(\omega_{\mathrm{IR}}), \tag{5}$$

where ω_{IR} and $\omega_G = 2\omega_{\mathrm{IR}}$ are the center angular frequencies of the input pulse (IR and green pulses), ϕ_{CE} is the CE phase at the input surface of the sapphire plate. The CE phase shift caused by the linear dispersion is $\Delta\varphi_{n0} = \omega_0\Delta\tau_0$, where $\Delta\tau_0$ is the difference between the group and the phase delay at the input laser frequency ω_0. The contribution from the nonlinear dispersion is $\Delta\varphi_{n_2} = -\phi_{\mathrm{spm}}\omega_0(\mathrm{d}n_2/\mathrm{d}\omega)_{\omega_0}/n_2$. For sapphire the parameters are $\omega_0(\mathrm{d}n_2/\mathrm{d}\omega)_{\omega_0} = 8\times10^{-17}\,\mathrm{cm}^2/\mathrm{W}$, and $n_2 = 2.9\times10^{-16}\,\mathrm{cm}^2/\mathrm{W}$ at 800 nm [15].

For simplicity it is assumed that the spectral phase difference $\phi_{\mathrm{SHG}}(\omega) - \phi_G(\omega)$ is only affected by linear dispersion in the second stage while the IR and green pulses are propagated in the filament. This is justified because the peak power decreases as the pulse duration increases in the filament. Adding the linear dispersion in the filament, the two phases become

$$\phi_G(\omega) = \phi_{G,\mathrm{sf}} - [\beta_G + \beta'_G(\omega - \omega_G)]z_{\mathrm{fila}}, \tag{6}$$

$$\phi_{\mathrm{IR}}(\omega) = \phi_{\mathrm{IR,sf}} - [\beta_{\mathrm{IR}} + \beta'_{\mathrm{IR}}(\omega - \omega_{\mathrm{IR}})]z_{\mathrm{fila}}, \tag{7}$$

where $\beta_G z_{\mathrm{fila}} = z_{\mathrm{fila}} n(\omega_G)\omega_G/c$ and $\beta_{\mathrm{IR}} z_{\mathrm{fila}} = z_{\mathrm{fila}} n(\omega_{\mathrm{IR}})\omega_{\mathrm{IR}}/c$ are from the phase delay of the IR and green carrier waves, c is the speed of light in vacuum, and $\beta'_G z_{\mathrm{fila}} = [\mathrm{d}\beta/\mathrm{d}\omega]|_{\omega_G} z_{\mathrm{fila}}$ and $\beta'_{\mathrm{IR}} z_{\mathrm{fila}} = [\mathrm{d}\beta/\mathrm{d}\omega]|_{\omega_{\mathrm{IR}}} z_{\mathrm{fila}}$ are the group delays of the IR and green pulses. Assuming perfect phase matching during the second harmonic generation (SHG) of the IR pulse, the phase of the pulse after the frequency conversion is

$$\phi_{\mathrm{SHG}}(\omega) = 2\phi_{\mathrm{IR,sf}}(\omega_{\mathrm{IR}}) - [2\beta_{\mathrm{IR}} + \beta'_{\mathrm{IR}}(\omega - \omega_G)]z_{\mathrm{fila}}. \tag{8}$$

Finally the phase difference between the SHG pulse and the green pulses is

$$\Phi(\omega) = \varphi_{\mathrm{CE}} + \Delta\varphi_{n_2} + \phi_{\mathrm{spm}} + \omega_0\Delta\tau_0 + \omega_G\Delta\tau_{\mathrm{ph}} - (\omega_G - \omega)\Delta\tau_g. \tag{9}$$

The phase delay between the two pulses is $\Delta\tau_{\mathrm{ph}} = z_{\mathrm{fila}}(\beta_G/\omega_G - \beta_{\mathrm{IR}}/\omega_{\mathrm{IR}})$, and the group delay between them is $\Delta\tau_g = z_{\mathrm{fila}}[\beta'_G - \beta'_{\mathrm{IR}}]$. All the linear and nonlinear terms can be expressed as explicit functions of the laser pulse energy. Thus Eq. (9) can be used to estimate the phase errors introduced by energy fluctuation. When the laser pulse energy is perfectly stable, the CE phase difference between two adjacent pulses, labeled i and j, can be measured accurately by the change of $\Phi(\omega)$, i.e., $\Delta\varphi_{\mathrm{CE}} = \varphi_{\mathrm{ce},j} - \varphi_{\mathrm{ce},i} = \Delta\Phi(\omega) = \Phi_j(\omega) - \Phi_i(\omega)$. This is the foundation of the relative CE phase measurement by f-to-$2f$ interferometers.

When the f-to-$2f$ interferometer is used to correct the CE phase drift of the pulses from the laser amplifier, the phase difference $\Delta\Phi(\omega)$ is set to zero. Previously the variation of the time delay between the SHG and the green pulses has been identified as the major contribution to the CE phase measurement error [5,15]. The time delay is the group delay difference,$\Delta\tau_g$, in Eq. (9) [3]. The dependence of the time delay on laser energy has been measured experimentally from the slope of the $\Phi(\omega)$ plot. Our measured results in [6] are shown in Fig. 2(a). It can be fitted well with the calculated results from $\Delta(\Delta\tau_g) = \Delta z_{\mathrm{fila}}[\beta'(\omega_G) - \beta'(\omega_{\mathrm{IR}})]$. Two fitting parameters are used. The first is the power at which the filament is formed for the IR and the green pulses. The former and latter are produced in the leading and trailing edges of the driving pulses, respectively [12]. According to the moving focus model, the filament staring point moves with the instantaneous laser power. In the calculation the power is chosen as ~32% of the peak power of the input pulse. The second is the spot size w_0, which is fitted as $9.68 \pm 0.4\,\mu$m. The fitting is necessary because we are using a simple model to describe a very complicated nonlinear propagation process. The same parameters are used in Fig. 1(b). For CE phase measurement and stabilization, $\Phi(\omega)$ is measured at $\omega \approx \omega_G$, thus $(\omega_G - \omega)\Delta\tau_g \approx 0$. Therefore the time delay fluctuation does not directly affect the CE phase measurement. This is an interesting result of our model. When $\Phi(\omega_G)$ is stabilized for CE phase locking, the calculated product $\omega_G\Delta\tau_g$ should agree with the intercept of the measured $\Phi(\omega)$ plot. This is indeed the case, as shown in Fig. 2(b). The curvature of the measured time delay and intercept can be explained by our model. It is caused by the square-root dependence of the self-focusing distance and the filament length on the laser power as expressed by Eq. (2).

Our model indicates that the phase errors are caused by the power-fluctuation-induced pulse-to-pulse variation of the quantity: $\Delta\varphi_{\mathrm{err}} = \Delta(\Delta\varphi_{n_2} + \phi_{\mathrm{spm}} + \omega_0\Delta\tau_0 + \omega_G\Delta\tau_{\mathrm{ph}})$. As the laser energy increases, $\omega_G\Delta\tau_{\mathrm{ph}}$ increases as the lengthening of z_{fila}, whereas $\omega_0\Delta\tau$ decreases as the shortening of z_{sf}, which cancels the effects of $\omega_G\Delta\tau_{\mathrm{ph}}$ to a large degree, as our calculation shows. The nonlinear term ϕ_{spm} also counters the effects of $\omega_G\Delta\tau_{\mathrm{ph}}$. However,

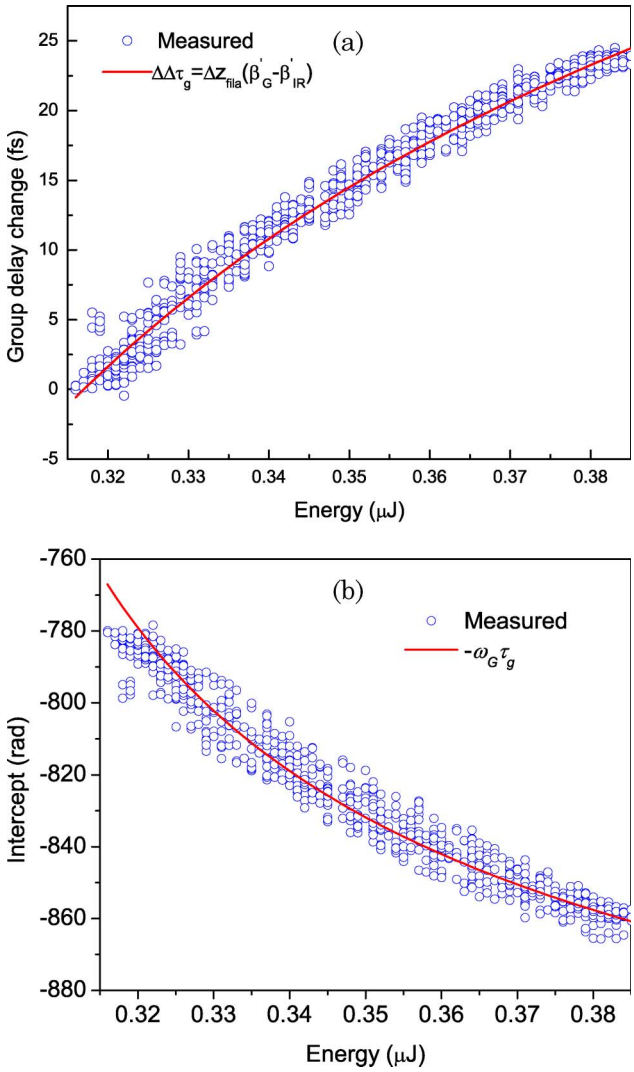

Fig. 2. (Color online) Comparison between the experimental and the calculated results. (a) Relative group delay between the green pulses and the infrared pulses as a function of laser energy. (b) Intercept of the $\Phi(\omega)$ plot, i.e., $\Phi(0)$ as a function of laser energy.

the overall result is that $\Delta\varphi_{\mathrm{err}}$ increases with laser energy. This explains the measured decrease of $\Delta\varphi_{\mathrm{CE}}=-\Delta\varphi_{\mathrm{err}}$ with laser energy when $\Phi(\omega_G)$ was locked to zero, as shown in Fig. 3. Here the center wavelength of the pulse for calculating $\omega_0\Delta\tau$ is chosen as 750 nm, which is another fitting parameter of this model. Previously the value of ϕ_{spm} had been determined using linear interferometry [4]. The value is less than the $\Delta\varphi_{\mathrm{err}}$ determined by our experiment. The difference can now be understood, because ϕ_{spm} is only a portion of the phase error in the f-to-$2f$ measurements.

This model can be used to choose parameters in the setup to minimize the phase errors caused by the power fluctuation. Since

$$\frac{\mathrm{d}\Delta\varphi_{\mathrm{err}}}{\mathrm{d}\varepsilon}\propto\frac{\mathrm{d}z_{\mathrm{sf}}}{\mathrm{d}\varepsilon}\propto\frac{w_0^2}{P^{3/2}},\qquad(10)$$

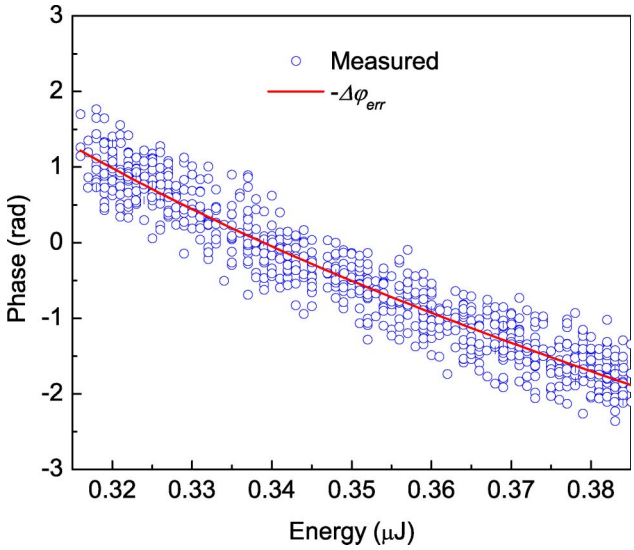

Fig. 3. (Color online) Comparison between the experimental and the calculated results of the CE phase shift as a result of the laser energy change.

it is clear that smaller focal spot and higher input power should be used. The decrease of the slope in Fig. 3 clearly shows the benefit of the higher input power; however, the maximum power is set by the formation of multiple filaments. It was previously

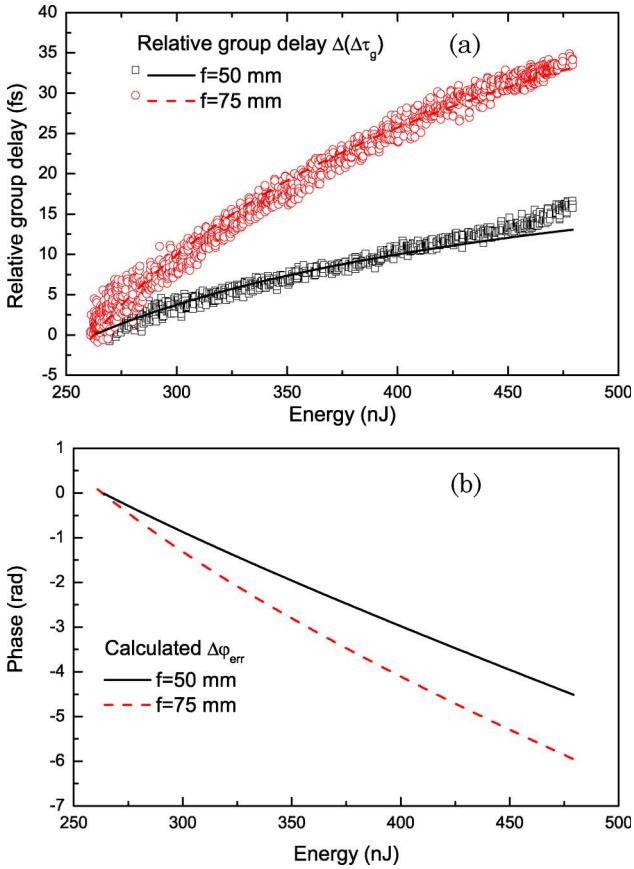

Fig. 4. (Color online) (a) Relative group delay with different focus length. (b) Calculated relative CE phase.

reported that the maximum power should be less than 1 μJ to avoid the formation of multiple filaments [4]. A smaller spot size can be achieved by using increasingly tight focusing. The $f/\# = f/D$, however, cannot be too small, or optical breakdown occurs, preventing filament formation and damaging the sapphire plate [16]. For water the minimum $f/\#$ for generating a filament without causing breakdown is ~12. For sapphire we found that $f/\# = 10$ can still produce a single stable filament while still causing no observable damage. Figure 4 shows the experimental results of relative delay time with different focus length. It is clearly shown that the shorter focus length, the smaller the group delay changes and the smaller the relative CE phase range.

In conclusion, a two-step, analytical model was introduced to understand the effects of laser energy fluctuation on relative CE phase measurements using f-to-2f interferometers based on white-light generation in sapphire plates. The pulse-to-pulse variation of the self-focusing distance and filament length in the sapphire were found to be the cause of the phase error. Both linear and nonlinear effects must be taken into account to explain the measured results. The model suggests that the dynamics of nonlinear pulse propagation in solids can also be studied using f-to-2f interferometry.

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C. Li is currently with Coherent Inc., Santa Clara, California 95054. Z. Chang's e-mail address is chang@phys.ksu.edu.

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