Determining the phase-energy coupling coefficient in carrier-envelope phase measurements

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For *f*-to-2*f* interferometers based on white-light generation in sapphire plates, the accuracy of the carrier-envelope (CE) phase measurement and stabilization is affected by the laser energy fluctuation. The coupling coefficient between the CE phase and the laser energy has been determined by modulating the pulse energy in an in-loop *f*-to-2*f* interferometer while measuring the CE phase variation with an out-loop interferometer. When the total spectral phase measured by the in-loop interferometer was locked, a 1% change in laser energy caused a 160 mrad shift in the CE phase of the output pulses. © 2007 Optical Society of America

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A laser pulse can be described by a time-dependent electric field $E(t)=E_0(t)\cos(\omega_c t+\beta(t)t+\varphi_{\mathrm{CE}})$, where the carrier-envelope (CE) phase, φ_{CE}, specifies the offset between the peak of the envelope amplitude, $E_0(t)$, and the oscillation peak of the carrier wave with frequency ω_c, $\beta(t)$, represents possible chirp in the pulse. When the duration of high-power laser pulses approaches a single optical cycle, the CE phase of the pulses starts to play a major role in laser–matter interactions.[1] For pulses from laser amplifiers, the CE phase change from one laser shot to the next can be measured optically by using *f*-to-2*f* interferometers.[2,3] The measured phase variation between successive pulses can be used as a feedback control signal to stabilize the CE phase of the amplified pulses,[3–5] provided that the oscillator phase is also locked.[6] It has been expected that the amplifier CE phase measurement and stabilization are influenced by laser pulse energy fluctuations.[3,7] In this Letter, the coupling coefficient between the CE phase and the laser energy, $C_{\mathrm{PE}}=\Delta\varphi_{\mathrm{CE}}/(\Delta\epsilon/\epsilon)$, is determined, where $\Delta\varphi_{\mathrm{CE}}$ is the CE phase change of the output pulse caused by a relative laser energy change $\Delta\epsilon/\epsilon$ in the in-loop *f*-to-2*f* interferometer.

The experiments were done using the Kansas Light Source laser system, which is equipped with grating-based stretchers and compressors.[8] The laser generated 2.5 mJ, 25 fs pulses at 1 kHz repetition rate. The setup for determining the coupling coefficient between the CE phase and the laser energy included two *f*-to-2*f* interferometers as shown in Fig. 1. During the experiment, the oscillator CE offset frequency f_0 was stabilized to a quarter of the oscillator repetition rate f_{rep} ($f_{\mathrm{rep}}=80$ MHz).[9] The in-loop interferometer was used to stabilize the CE phase of the amplified pulses by controlling the effective grating separation in the stretcher.[4,5] It is well known that the CE phase shift measured by the in-loop interferometer does not represent the true value of the output pulse CE phase shift. This is because the *f*-to-2*f* interferometer cannot distinguish the CE phase shift from the change of the retrieved total phase due to laser energy fluctuation and other factors.[10] Thus, an out-loop *f*-to-2*f* interferometer was used for the CE phase measurements. To reduce the effects of laser energy fluctuation on the CE phase measurements and stabilization, the pulse energy from the CPA amplifier was stabilized. The energy stability was better than 0.1% rms within a 0–5 Hz bandwidth.

In both *f*-to-2*f* interferometers, 25 fs pulses with <1 μJ energy were split off from the laser output and focused into a 2.3 mm thick sapphire plate to generate white light with a spectrum covering an octave range. At the input surface of the plate, the total spectral phase of the laser pulse can be expressed as $\Phi(\omega)_{\mathrm{L}}=\phi(\omega)_{\mathrm{L}}+\varphi_{\mathrm{CE}}$, where $\Phi(\omega)_{\mathrm{L}}$ is the conventional spectral phase. The laser energy was fine tuned with a variable neutral density filter until a single stable filament was formed in the sapphire plate. At the exit of the plate, the total phase of the white light can be described as $\Phi(\omega)_{\mathrm{WL}}=\phi(\omega)_{\mathrm{WL}}+\varphi_{\mathrm{CE}}^{\mathrm{WL}}\cdot\varphi_{\mathrm{CE}}^{\mathrm{WL}}$ is the CE phase of the white-light pulse, which is related to the CE phase of the laser pulses by the expression $\varphi_{\mathrm{CE}}^{\mathrm{WL}}=\varphi_{\mathrm{CE}}+\delta\varphi_{\mathrm{CE}}$. To perform the relative CE phase

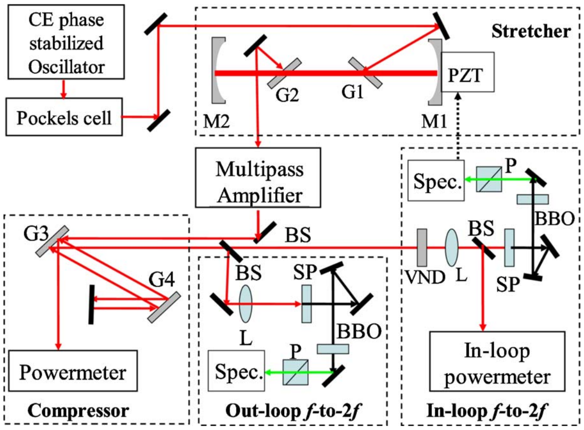

Fig. 1. (Color online) Experimental setup for determining the phase-energy coupling factor. In the *f*-to-2*f* interferometers: VND, variable neutral density filter; L, focusing lens; SP, sapphire plate; BBO, frequency-doubling crystal; P, polarizer; Spec., spectrometer and computer. In the stretcher and compressor: G1–G4, gratings; PZT, piezoelectric transducer; Ml and M2, mirrors; BS, beam splitter.

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measurement, the IR portion of the spectrum centered at 1000 nm was frequency doubled in a BBO crystal. The green portion of the white light around 500 nm with power spectrum $I_{\mathrm{WL}}(\omega)$ and the second harmonic of the IR with power spectrum $I_{\mathrm{SHG}}(\omega)$ were projected onto the same polarization direction by a polarizer. The transmitted pulses were sent to a spectrometer to measure the interference of the two pulses in the spectral domain.[2,3]

In the frequency domain, the measured spectral interferogram is

$$S(\omega) = I_{\mathrm{WL}}(\omega) + I_{\mathrm{SHG}}(\omega) + 2\sqrt{I_{\mathrm{WL}}(\omega) I_{\mathrm{SHG}}(\omega)} \times \cos[\phi_{\mathrm{SHG}}(\omega) - \phi_{\mathrm{WL}}(\omega) - \omega\tau_0 + \varphi_{\mathrm{CE}}^{\mathrm{WL}}], \quad (1)$$

where τ_0 is the lag of the green pulse relative to the IR pulse, which is caused primarily by the dispersion of the sapphire plate, and $\phi_{\mathrm{WL}}(\omega)$ and $\phi_{\mathrm{SHG}}(\omega)$ are the spectral phases of the green pulse and the frequency-doubled IR pulse, respectively. From the interference pattern, the total phase, $\Phi(\omega) = \phi_{\mathrm{SHG}}(\omega) - \phi_{\mathrm{WL}}(\omega) - \omega\tau_0 + \delta\varphi_{\mathrm{CE}} + \varphi_{\mathrm{CE}}$, can be retrieved using Fourier transforms and filtering techniques.[11,12] In our measurements, the average value of the relative total phase $\Delta\Phi(\omega)$ over a 30 nm spectral range was stabilized by the feedback loop. The values of the first four terms in the total phase depend on the laser intensity in the sapphire plate.

In the past, the dependence of two terms in the total phase, the white-light partial phase, $\Phi_{\mathrm{p}}(\omega) = -\phi(\omega)_{\mathrm{WL}} + \delta\varphi_{\mathrm{CE}}$, on the laser energy in a 60 nm spectral range around 800 nm was studied by Baltuška *et al.* using a linear interferometer.[3] They found that a 1% change of laser energy caused an 84 mrad variation of $\Phi_{\mathrm{p}}(\omega)$. As was pointed out, since the measurements were not done in the green and IR portions of the white light, and also because the effects of the laser power fluctuation on $\phi_{\mathrm{SHG}}(\omega)$ and τ_0 were not included, the results give only the lower limit of the phase-energy coupling coefficient. In our work, the dependence of the CE phase of the amplified pulses on the laser energy in the in-loop interferometer was studied using two *f*-to-2*f* interferometers. The laser energy in the sapphire plate of the out-loop interferometer was fixed, whereas the laser energy in the in-loop interferometer was modulated by a motor-controlled neutral density filter. The triangle-shaped in-loop pulse-energy modulation is shown in Fig. 2(a). When the energy modulation was introduced, a triangle-shaped phase modulation was observed by the out-loop *f*-to-2*f* interferometer, as shown in Fig. 2(b). The energy stability ($<0.1\%$) of the in-loop interferometer and the total in-loop phase stability (250 mrad rms) are shown in Fig. 2(c).

To simplify the analysis, the pulse energy from the amplifier is assumed to be constant. It is also assumed that the total phase in the in-loop *f*-to-2*f* interferometer is perfectly stabilized. Under these assumptions, the CE phase of the laser pulses is modulated by the laser energy modulation introduced in the in-loop *f*-to-2*f*. The magnitude of the CE phase modulation of the output pulse is

$$\Delta\varphi_{\mathrm{CE}} = \Delta\Phi(\omega) - \Delta[\phi_{\mathrm{SHG}}(\omega) - \phi_{\mathrm{WL}}(\omega) - \omega\tau_0 + \delta\varphi_{\mathrm{CE}}]. \quad (2)$$

This CE phase modulation was carried by the pulses propagating to the out-loop *f*-to-2*f* and was measured there. The dependence of the relative CE phase measured by the out-loop *f*-to-2*f* interferometer on the in-loop laser energy is shown in Fig. 3(a), together with a least-squares fit to a linear function. The slope of the fitted line corresponds to $C_{\mathrm{PE}} = 160$ mrad per 1% of energy variation. The value is two times that measured by Baltuška *et al.*, who used linear interferometers. We believe this new value is more accurate for two reasons. First, the effects of laser energy variation on all four terms in the total phase are included in our experiments, whereas the contribution of the delay time and the second-harmonic generation pro-

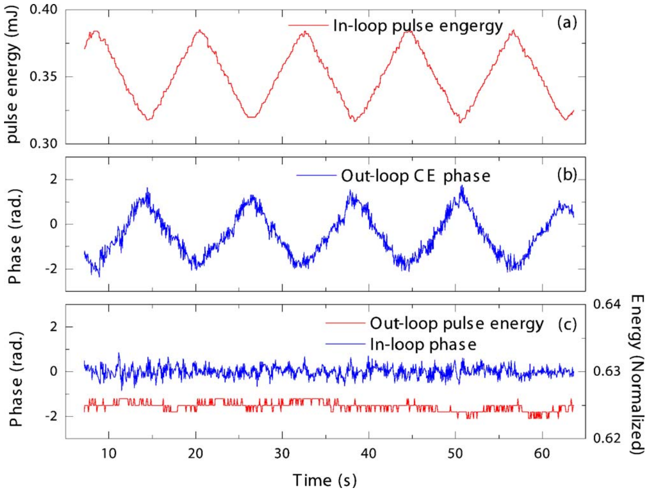

Fig. 2. (Color online) Temporal evolution of measured phase and laser energy. (a) Modulated in-loop pulse energy, (b) measured out-loop phase, and (c) the in-loop phase and out-loop energy.

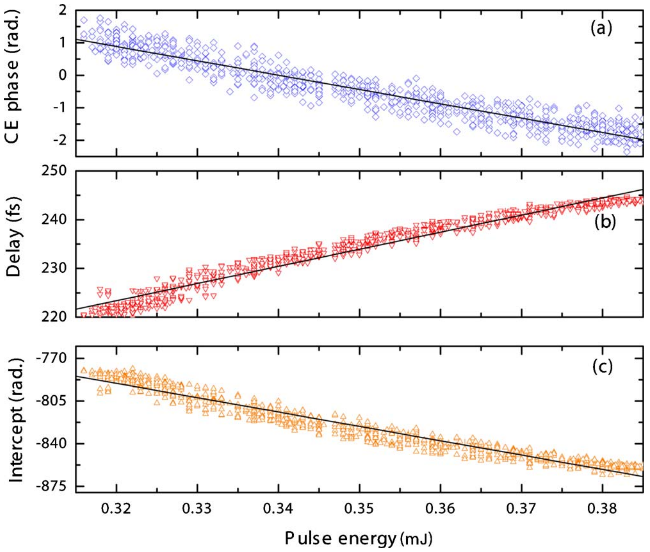

Fig. 3. (Color online) Retrieved experimental results versus the laser energy. (a) Relative CE phase, (b) delay time, and (c) the residual intercept after the subtraction of the CE phase.

cess were not included in the work of Baltuška *et al.* Second, our measurements were done at the green portion (~500 nm) where the *f*-to-2*f* interferometers operate, whereas the previous value was obtained at a different wavelength (800 nm). We pointed out that the phase-energy coupling coefficient value presented here could be different for other *f*-to-2*f* interferometers. However, the method we have demonstrated can be applied to other systems to find the coefficient.

To study the contribution of the delay time to CE phase measurement, the values of τ_0 and the phase terms $\phi_{\mathrm{SHG}}(\omega)-\phi_{\mathrm{WL}}(\omega)+\varphi_{\mathrm{CE}}^{\mathrm{WL}}$ were extracted from the measured $\Phi(\omega)$. When the power spectrum of the interferogram was Fourier transformed, there were two sidebands. The positive frequency sideband, which corresponded to the total phase described by $\Phi(\omega)=\phi_{\mathrm{SHG}}(\omega)-\phi_{\mathrm{WL}}(\omega)+\omega\tau_0+\varphi_{\mathrm{CE}}+\delta\varphi_{\mathrm{CE}}$, was filtered out in our calculation. The measured $\Phi(\omega)$ was close to a linear function of the angular frequency ω. The delay time τ_0 was determined by the slope of the least-squares fitting, while the $\phi_{\mathrm{SHG}}(\omega)-\phi_{\mathrm{WL}}(\omega)+\varphi_{\mathrm{CE}}^{\mathrm{WL}}$ was the intercept of the fitted line to the vertical axis.[7] The dependence of the delay time on the laser energy and the linear fit are plotted in Fig. 3(b). It can be deduced from the fitting that a 1% energy increase caused an extra 1.23 fs delay, which corresponded to 4.45 rad of phase upshift at the green wavelength. This number is surprisingly much larger than the 160 mrad total phase change. In Fig. 3(c), the dependence of $\phi_{\mathrm{SHG}}(\omega)-\phi_{\mathrm{WL}}(\omega)+\delta\varphi_{\mathrm{CE}}$ on laser energy and a linear fit are plotted. The graph shows that the phase sum is downshifted. The amount of downshift is close to the upshift due to the delay change. Clearly it is the cancellation between these two effects that leads to a much smaller intensity-dependent total phase variation compared with the phase change due to either of them.

Under well-controlled environmental conditions (temperature $20\pm0.5\,^{\circ}\mathrm{C}$ and relative humidity $5\pm5\%$), the laser energy fluctuation of typical diode-pumped kilohertz femtosecond laser systems is 1.5% rms. The long-term energy drift is also close to this value. For arc-lamp-pumped amplifiers, the energy fluctuation and drift are even larger. When the phase of the laser is stabilized, the in-loop measurement of the phase does not provide the true phase quality. The actual phase noise is 250 mrad rms or higher even if the in-loop measurement shows perfect phase locking. To improve the phase locking, the energy fluctuation in the in-loop *f*-to-2*f* must be reduced.

In conclusion, the CE phase to laser energy coupling coefficient was determined experimentally by modulating the pulse energy in the in-loop *f*-to-2*f* interferometer while measuring the CE phase variation with an out-loop interferometer. The value $C_{\mathrm{PE}}=160\,\mathrm{mrad}/1\%$ of energy change. This result is important for the measurement and stabilization of the CE phase of high-power laser pulses.

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