New capabilities for predicting image degradation from optical surface metrology data

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ABSTRACT

Image degradation due to scattered radiation form residual optical fabrication errors is a serious problem in many short wavelengths imaging system. Most currently-available image analysis codes require the bidirectional scattering distribution function (BSDF) data as an input in order to calculate the image quality from such systems. This BSDF data is difficult to measure and rarely available for the operational wavelengths of interest. Since the smooth-surface approximation is often not satisfied at these short wavelengths, the classical Rayleigh-Rice expression that indicates the BSDF is directly proportional to the surface PSD cannot be used to calculate BSDFs from surface metrology data for even slightly rough surfaces. An FFTLog numerical Hankel transform algorithm enables the practical use of the computationally intensive Generalized Harvey-Shack surface scatter theory to calculate BRDFs for increasingly short wavelengths that violate the smooth surface approximation implicit in the Rayleigh-Rice surface scatter theory. A generalized Peterson analytical scatter model is then used to make accurate image quality predictions. The generalized Peterson model is numerically validated by both ASAP and ZEMAX.

Keywords: Scattering, image degradation

1. INTRODUCTION

In the 21st century, surface scatter phenomena has been one of the important issues in diverse areas of science, engineering and even computer graphics. In particular, image degradation due to surface scatter from residual optical fabrication errors remains a serious problem for short wavelength (X-ray/EUV) applications. Estimating image degradation from the surface roughness metrology data has been not easy problem because there are theoretical, technical and computational difficulties. Predicting the bidirectional scattering distribution function (BSDF) for moderately rough surface has been one of the difficult problems in the field of surface scattering, and estimating image quality from the BSDF is computationally intensive. Recently, the Generalized Harvey Shack (GHS) surface scatter theory[1,2] has been suggested as a method to predict the BSDF for moderately rough surfaces. In Section 2 of this paper we discuss the use of surface metrology data in the form of the surface power spectral density (PSD) function as a starting point for calculating image degradation due to surface scatter phenomena. The GHS surface scatter theory and the use of an FFTLog algorithm[3] to enable the necessary computations for a surface PSD with a large dynamic range in spatial frequency is described in Section 3. In Section 4, Peterson's analytic scatter model[4] which is a simple approximation to the complex scattered image is introduced and extended to enable calculations for moderately rough surfaces. In Section 5, tracing millions of rays using commercially-available optical analysis codes is performed and compared to the extended Peterson's analytic model for an example of a two-mirror EUV telescope.

2. SURFACE PSD FROM METROLOGY DATA

The surface power spectral density (PSD) function is the key of predicting the BSDF. Nowadays, there are several available measuring equipments such as interferometers, micro phase-measuring interferometers and atomic force microscopes, which give us statistical information of a surface profile. However, each instrument has its own band limit in frequency space. Moreover, undesired factors such as air fluctuation electrical fluctuation, heating and dust

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interfere with measurement. From previous researches, it is generally accepted that PSD of a well-polished mirror has the form of an abc-function or K-correlation function[5,6]. If the surface has isotropic homogeneous roughness, its two-dimensional form is given by

$$PSD(f)_{2-D} = K\frac{AB}{[1+(B\,f)^2]^{(C+1)/2}},\ \ K = \frac{1}{2\sqrt{\pi}}\frac{\Gamma\left(\left(C+1\right)/2\right)}{\Gamma\left(C/2\right)}. \tag{1}$$

Based on this assumption, the surface PSD function can be obtained by fitting an abc-function or combination of several abc-functions to the measured metrology data. Even though it cannot be always better, function fitting provides us lots of numerical freedom when dealing with surface characteristics numerically, and it is important because BSDF is obtained numerically for most realistic optical surfaces.

3. BRDF PREDICTION USING PSD

3.1 Generalized Harvey-Shack theory

From the paper written by Church[7] in 1979, numerous scattering theories which relate microscopic surface characteristics and macroscopic scattering behavior have been developed. However, as mentioned by Elfouhaily[8] and other authors, there is no absolute theory explaining all the scattering phenomena properly. Nevertheless, they typically fall into two categories: perturbation techniques such as the Rayleigh-Rice theory,[9] and variations of the Beckmann-Kirchhoff theory[10]. The Rayleigh-Rice method is particularly used when the surface roughness of optical surface is smooth, and Beckmann-Kirchhoff method is believed to be useful when predicting scattering behavior in the case of small incident and scatter angles regardless of roughness of the optical surface. However, there is no satisfactory theory describing scattering behavior for arbitrary incident and scatter angles and for both small and rough optical surfaces. With the arising need of dealing with extreme ultraviolet or X-ray light, predicting wide angle scattering behavior by moderately rough surfaces becomes an important issue. Krywonos suggested in his dissertation a new approximation to the scattering behavior called Generalized Harvey-Shack (GHS) theory which appears to accurately describe the distribution of scattered light for moderately rough surfaces and for both smooth and rough optical surfaces. It thus exhibits the advantages of the classical Rayleigh-Rice and Beckmann-Kirchhoff theories without the disadvantages of either.

3.2 Numerical Consideration

According to the GHS surface scatter theory[1], in the case when every polarization effects are ignored, and when specular ray is excluded, and when irradiance of incident light is unit, bidirectional reflectance distribution function (BRDF) can be written by

$$BRDF = F\left[H\left(\hat{x},\hat{y};\gamma_i,\gamma_s\right)\cdot\exp\left(-i2\pi\beta_0\hat{y}\right)\right]. \tag{2}$$

where F stands for fourier transform operator, H is surface transfer function, $\gamma_i = \cos\theta_i$, $\gamma_s = \cos\theta_s$ and $\beta_0 = -\sin\theta_i$ where θ_i, θ_s are angles of incident and scattered ray. Statistical microscopic characteristics of an optical surface is described in terms of autocovariance function (ACV), and the relationship between ACV and the PSD is

$$ACV_s\left(\hat{x},\hat{y}\right) = F\left\lfloor PSD\left(f_x, f_y\right)\right\rfloor. \tag{3}$$

where f_x, f_y are spatial frequencies and $\hat{x},\hat{y}$ are wavelength normalized rectangular coordinate of space. The surface transfer function H is given by

$$H\left(\hat{x},\hat{y};\gamma_i,\gamma_s\right) = \exp\left\{-\left[2\pi\hat{\sigma}_s\left(\gamma_i+\gamma_s\right)\right]^2\cdot\left[1-ACV_s\left(\hat{x},\hat{y}\right)/\hat{\sigma}_{rel}^2\right]\right\}. \tag{4}$$

where $\hat{\sigma}_{rel}$ is relevant rms surface roughness[5]. If the optical surface has isotropic roughness and the autocovariance function is rotationally symmetric, equation (1) and (3) can be reduced to

$$BRDF = 2\pi \int_0^\infty \exp\left\{ -\left[2\pi\hat{\sigma}_s \left(\gamma_i + \gamma_s \right) \right]^2 \cdot \left[1 - C_s(\hat{r}) / \hat{\sigma}_s^2 \right] \right\} \cdot \exp\left(-i 2\pi \beta_0 \hat{y} \right) J_0\left(2\pi\beta_s \right) \hat{r} \cdot d\hat{r} \,. \qquad (5)$$

where $\hat{r}$ is wavelength normalized polar coordinate of space and J_0 stands for Bessel function of the first kind. Unfortunately, when PSD function has a form of abc-function, there is no analytic solution to the equation (5), but still it is possible to calculate it numerically. Typical behavior of $C_s(\hat{x}, \hat{y})$ which is Fourier transform of abc-function is slowly varying in linear space. On the other hand, Bessel function in equation (5) varies rapidly in that space. These characteristics require extremely large size of sampling domain with tight spacing, which makes numerical calculation practically impossible in linear space. Talman suggested an algorithm called FFTLog which is taking Hankel transform of arbitrary function numerically which varies smoothly in logarithmic space. The fact that we know the behavior of Bessel function in logarithmic space and the assumption that given function is smooth in logarithmic space enable us to calculate Hankel transform in logarithmic space with small number of sampling points. In mathematical point of view, equation (4) is merely integration, but it can be also calculated by taking Hankel transform substituting corresponding γ_s for every β_s value. By applying FFTLog algorithm to the equation (5), BRDF for moderately rough surface can be obtained numerically and it is validated in the other research[5].

4. ESTIMATION OF IMAGE DEGRADATION BY BRDF

4.1 Point Spread Function by Commercial Software

Most commercially-available image analysis codes have ability to calculate scattering behavior. Typically, they calculate point spread function (PSF) by using geometrical configuration of an optical system with given BRDF. They provide some scattering model, but scattered light does not always follow those models. Some codes provide an option that user inputs tabular BRDF which is not necessarily to have same behavior to one of their models, which enables users to modeling actual scattering. However, there are lots of what users have to keep in mind when they use commercial software. Aside from using complex options which algorithm is not fully described in its manual, many codes do not provide logarithmic spaced detector. BRDF behavior is typically smooth in logarithmic space, so users have to be cautious to observe image degradation by scattering in linearly spaced detector. Next, enough number of rays for analysis has to be used. There is no firm criterion for how many rays must be traced for obtaining accurate scatter analysis. In most cases, the more rays traced, the better. But this requires much time. Sometimes, it could be an over-night job. Also, the number of rays is limited by memory capacity of user's computer. Using an insufficient number of rays produces wrong results or undesired fluctuations. Although the software may provide smoothing methods, it can sometimes distort the resulting prediction. Using appropriate smoothing algorithm or filter is user's own obligation. However, currently it is the only way to calculate PSF by scattering for the case of large angle of scattering or for the case of low f-number optical system or combination of both two cases.

4.2 Peterson Analytical Scatter Model

However, many optical systems have a relatively high f-number and there are practical cases of interest that exhibit only small angle scattering (such as most telescopes). In these cases, instead of calculating ray tracing numerous times, a simple approximation suggested by Peterson can be applied. Using the Lagrange invariant of 1st-order imaging theory and the brightness theorem, the irradiance by scattered light in the focal plane of an imaging system from the j-th element for an incoherent point source is given by

$$E_{sj}(r) = E_{ent}\pi(na)^2 T \frac{s_{ent}^2}{s_j^2} BSDF\left((na)\frac{r}{s_j}\right). \qquad (6)$$

where r is the radial distance from the optical axis to geometrical image in the image plane of the telescope, (na) is the numerical aperture of the system, T is the system transmittance, s_{ent} is the radius of the entrance pupil, s_j is the radius of the beam on the jth element, and E_{ent} is the irradiance in the entrance pupil of the system. Note that this formulation is based upon both a smooth-surface and a paraxial assumption.

4.3 Extended Peterson Analytical Scatter Model to the Rough Surface

For a two-mirror telescope with rough surfaces (relative to the wavelength), the scattered-scattered radiation dominates the resulting point spread function. In such a case, equation (6) is not enough to describe the scattering phenomena because it does not consider scattering of scattered light. For simplicity, let's take a two mirror telescope which has effective focal length of f, and which has relevant rms surface roughness of σ_1, and σ_2 for the first and the second mirror. The fraction of the total reflected radiant power contained in the scattered halo is given by[10]

$$A_j = \exp\left[-(4\pi\cos\theta_i\sigma_j/\lambda)^2\right] \quad (j = 1,2). \qquad (7)$$

where θ_i is the angle of incidence. The total integrate scatter (TIS) value is

$$B_j = 1 - A_j \quad (j = 1,2). \qquad (8)$$

PSF consists of four components, which are direct-direct (dd), scattered-direct (sd), direct-scattered (ds), scattered-scattered (ss)

$$PSF(r) = PSF_{dd}(r) + PSF_{sd}(r) + PSF_{ds}(r) + PSF_{ss}(r). \qquad (9)$$

And the fractional energy contained by these four components are A_1B_1, A_2B_1, A_1B_2, and A_2B_2 respectively. $PSF_{dd}(r)$ may be a conventional airy function if aberrations are ignored. Since the total radiant power reaching the focal plane is given by $P_T = E_{ent}\pi s_{end}^2 T$, and $(na) = 1/(2F^{\#}) = s_{ent}/f$ for large f-number system, the next two components of PSD normalized by the total radiant power using equation (6) become

$$PSF_{sd}(r) = A_2B_1 \cdot \left(\frac{1}{f}\right)^2 BSDF_1\left(r/f\right). \qquad (10)$$

$$PSF_{ds}(r) = A_1B_2 \cdot \left(\frac{1}{f}\right)^2 \left(\frac{s_p}{s_s}\right)^2 BSDF_s\left((s_{ent}/s_2)(r/f)\right). \qquad (11)$$

Scattered light by the first surface can be considered as a light emitted by extended source located at the first mirror with the irradiance distribution of $BSDF_1$. Thus, the PSF by scattered-scattered light is a scattered image of that virtual extended source by second surface. This process is mathematically exactly same as convolution, which is written by

$$PSF_{ss}(r) = \frac{A_2B_2}{A_2B_1 \cdot A_1B_2} \cdot PSF_{sd}(r) \otimes PSF_{ds}(r). \qquad (12)$$

where $\otimes$ stands for convolution operator. For two mirror telescope, image of point source is blurred by the sum of these four PSD components and in the situation when none of B_j values are too small to be ignored, the overall image quality might be degraded by scattering. Generally, if an imaging system has n elements, the total number of terms of PSD is 2^n, and they are obtained in the same manner described in this section. Note that smooth surface approximation is removed by considering scattered-scattered light, but this formulation is still under the assumption of paraxial or small angle approximation.

5. APPLICATION TO TWO MIRROR TELESCOPE

5.1 Surface PSD from the Metrology Data

As an example of the process to obtain PSF from the measured metrology data, a two-mirror telescope is considered here. The two mirrors are assumed well polished and to have almost same isotropic homogeneous roughness profiles. Figure 1 illustrates measured surface PSD data of one mirror surface using interferometer, phase-measuring interferometers and atomic force microscope, successively. In the diagram, there are four bands which corresponds to the frequency domain that each measuring method works properly. The behavior of measured PSD data looks abnormal at the every edge of the bands. It is hard to believe that these sudden behaviors at every band edges come from real surface characteristics. To remove these artifacts and to have numerical flexibility, one abc-function is fitted to the metrology data as shown in Figure 1.

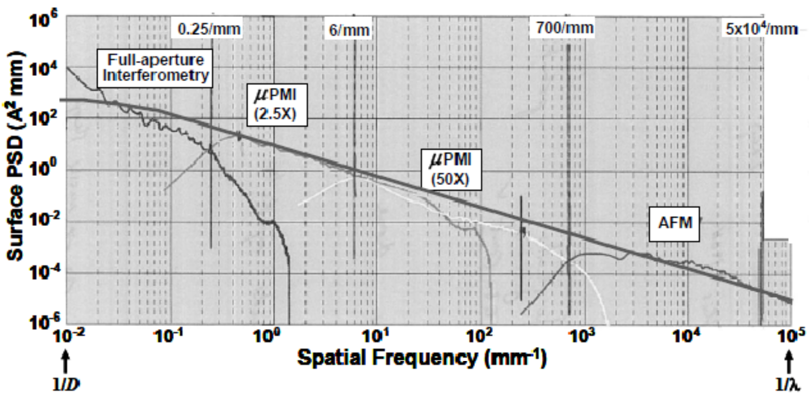

Figure 1. Measured surface metrology data and the abc-function (red) which is fitted to the data.

5.2 BRDF Prediction Using PSD

The working wavelength band of light source for the telescope spreads between 93.9 Å and 1000 Å. Using the PSD function obtained in previous section, wavelength dependent rms surface roughness are values of 6.8171Å at λ = 93.9 Å, 6.6470Å at λ = 195.2Å and 6.5698Å at λ = 1000Å. Corresponding TIS values are 0.5650 at λ = 93.9 Å, 0.1721Å at λ = 195.2Å and 0.0068Å at λ = 1000Å. Except the case for the wavelength of 1000Å, the surface cannot be considered as a smooth surface. Figure 2 shows the predicted BRDF from GHS theory for the wavelength of 93.9 Å, 131.2 Å, 171.1 Å, 195.1 Å, 284.2Å, 303.8Å, 500Å and 1000Å at normal angle of incidence.

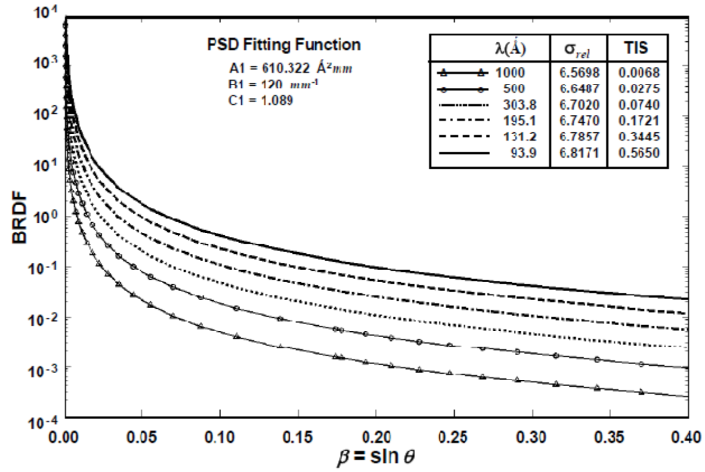

Figure 2. Predicted BRDF by GHS for a moderately rough surface

5.3 Estimation of Image Degradation by BRDF

The geometrical configuration of the two-mirror telescope is shown in figure 3. F-number of the telescope is approximately value of 9 with the effective focal length of 1730mm, so the paraxial approximation can be applied. And the assumption that BRDF for the incident rays which is slightly deviated from normal angle of incident is almost the same as BRDF for the normal incident ray is placed. Then the analysis discussed in section 4.3 can be directly applied.

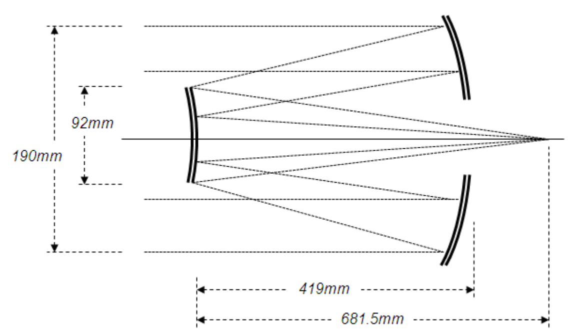

Figure 3. Geometrical configuration of the two-mirror telescope.

For the shortest wavelength of 93.9 Å, the respective radiant power contained in the four components described in section 4.3 of the total PSF are $A_1A_2 = 0.1892$, $A_1B_2 = 0.2458$, $B_1A_2 = 0.2458$ and $B_1B_2 = 0.3192$. It is worth noting that less than 20% of the total energy reaching the focal plane will reside in the specular beam, or image core. Furthermore, almost 32% of the energy will reside in the scattered-scattered component. Using equation (10), (11), and (12), profiles of the scatter-direct, direct-scatter and scatter-scatter components and sum of them for the shortest wavelength are plotted in Figure 4. The optical system does not have aberration so the direct-direct component would be classical airy function with annular aperture but it is not considered in the Figure 4. From the fact that its irradiance function decreases as an inverse power law with a slope of -3, whereas other components obey an inverse power law with a slope of approximately -2, one can expect that the contribution of direct-direct component to the irradiance distribution is weak. Therefore, the scattered-scattered light is indeed the dominant component of the irradiance distribution at this very short wavelength of 93.9 Å. Clearly there is a strong need to be able to perform accurate image quality predictions as degraded by surface scatter effects from real metrology data throughout the optical fabrication process.

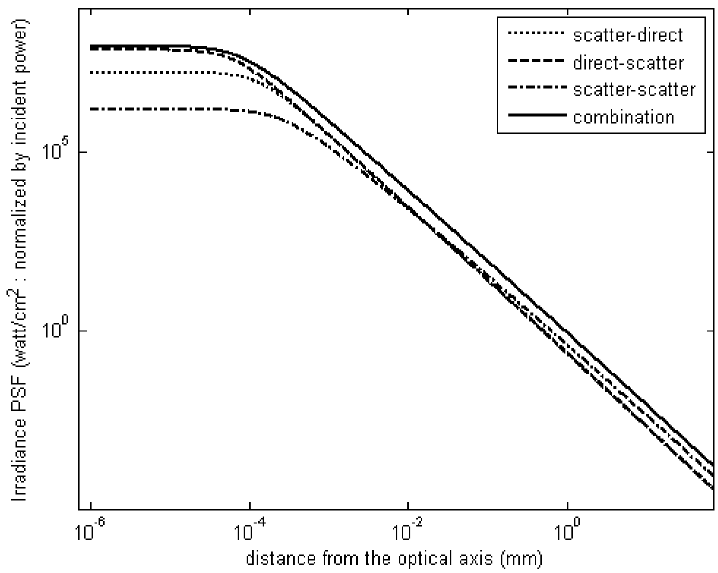

Figure 4. Three components of PSD and sum of them.

5.4 Comparison with ZEMAX and ASAP Image Quality Predictions

Figure 5 illustrates a direct comparison of the irradiance in the image plane of the two-mirror telescope as predicted by the approximation method introduced in section 4.3 and calculated from both the well-known ZEMAX and ASAP commercial software. The software calculation is done by ray-tracing millions of rays and taking several hours, whereas the results of extended Peterson analytic scatter model is obtained by numerical calculation which takes a few seconds with a personal computer. The irradiance distribution by ZEMAX (dashed line) is smoothened by rotationally averaging irradiance values in order to reduce fluctuation, whereas any smoothening algorithm as applied to the ASAP results (dotted line) reveals fluctuations which is normally apparent when scattering calculation is performed using optical software. The three approaches are in excellent agreement for all wavelengths of interest.

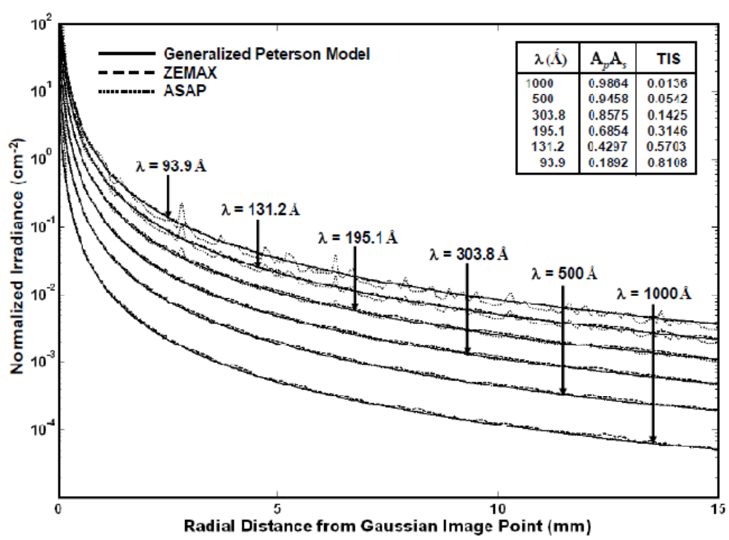

Figure 5. Irradiance distribution predicted by three different approaches: analytic approximation method (solid line), ZEMAX (dashed line) and ASAP (dotted line).

6. CONCLUSION

We have discussed the full process to obtain irradiance distribution at focal plane from the measured surface metrology data. Reasonable treatment of PSD function was suggested and GHS theory was introduced, which is a promising approach to calculate BRDF for moderately rough surfaces. Instead of ray-tracing based scattering analysis using commercial software, generalized Peterson approximation was suggested. The approach was demonstrated for a two-mirror telescope and validated by comparison of image quality predictions with the computationally-intensive calculations provided by the well-known ZEMAX and ASAP codes. The whole process presented here is expected to be valuable particularly for short-wavelength applications where image degradation due to surface scatter is severe.

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