Focal Length Invariance of Perceptual Image Quality for Long Range Imaging Applications

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Abstract: For long range imaging applications, perceptual image quality will not vary with focal length. Theoretical justification and Modulation Transfer Function (MTF) evidence are presented.
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When imaging a specific object, or if assumptions are made about a scene, MTF-based metrics [1-3] can be evaluated equivalently in the object plane, image plane, or retinal plane. However, when an optical system is being evaluated for its capability to produce high quality imagery, it is likely that scene information is unknown since the device may be used for a variety of applications. In this case it is imperative to use the object plane MTF so that changes in image usefulness resulting from varying object distances are accounted for.

The traditional view of the image formation process is the image plane convolution of the point spread function with the geometrically magnified object. A less conventional but equally valid view of imaging is the object plane convolution of the projected PSF with the object, followed by a geometric magnification of the result. The irradiance distribution in the object plane is then the unmagnified representation of the image. This latter approach, although yielding the same irradiance distribution, is the correct one to use for systems operating at non-unity transverse optical magnifications [4]. This approach also naturally leads to using an object plane MTF in calculating metrics.

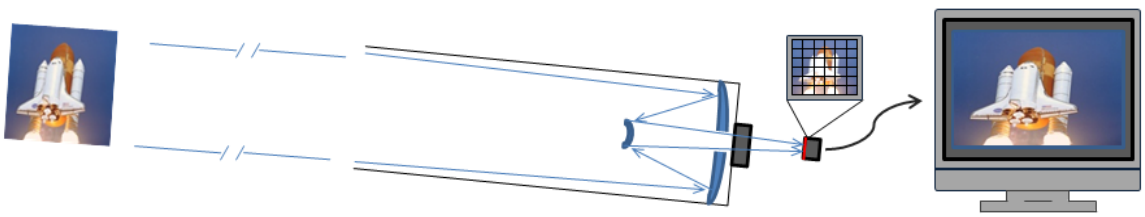

Figure 1. A long-range imaging device creates a non-ideal replication of an object on an image plane (red line) which is displayed visually with some display magnification. The display is then viewed by an observer from a fixed viewing distance.

Transfer and threshold function curves in the image plane can be transferred to the object plane using the optical magnification (m) of the system. Although it is most accurate to use the absolute value of m, the absolute value sign has been dropped since the sign of the magnification has no real meaning in transfer functions.

$$MTF_{obj}(u_x, u_y) = MTF_{img}\left(\frac{u_x}{m}, \frac{u_y}{m}\right) \tag{1}$$

The viewing distance (d) allows retinal frequencies to be converted to spatial frequencies at the display ($u_{disp\ x}$, $u_{disp\ y}$) which can then be related to image spatial frequencies by a factor of the display magnification (M).

$$CTF_{obj}(u_x, u_y) = CTF_{disp}\left(\frac{u_{disp\ x}}{mM}, \frac{u_{disp\ y}}{mM}\right) \tag{2}$$

The following assumptions are made regarding the relative usefulness of two images: (*i*) for two images with equal feature sizes, the image with the smallest visible feature is the image with greater usefulness, and (*ii*) if two images are not displayed with equal size, the relative usefulness of the two images cannot in general be determined.

2. Theory

It is well known from linear systems theory [5] that the complex pupil function (p), the point spread function (PSF), and the modulation transfer function (MTF) are all related by Fourier Transform operations (equation 3).

$$p(x,y) \xrightarrow[u_x = \frac{x}{\lambda f},\ u_y = \frac{y}{\lambda f},]{|\text{Fourier Transform}|^2} PSF(x_o,y_o) \xrightarrow{|\text{Fourier Transform}|} MTF(u_x,u_y) \quad (3)$$

The complex pupil function is composed of an aperture transmittance function t(x,y) and a phase, both of which are normalized to the pupil diameter (D). Writing the complex pupil function in terms of absolute variables rather than normalized variables yields

$$p(x,y)=t\left(\frac{x}{D},\frac{y}{D}\right)\exp\left(j\frac{2\pi}{\lambda}W\left(\frac{x}{D},\frac{y}{D}\right)\right). \quad (4)$$

Working in the object plane, the ASF is scaled by the inverse of the optical magnification. When transformed, the similarity theorem dictates that the Fourier Transform of the complex pupil function (the amplitude spread function (ASF)) will be a function (F) of the variables Du_x m and Du_y m.

$$ASF(u_x,u_y)=F(mDu_x, mDu_y). \quad (5)$$

Solving the thin lens law for image distance, substituting into the expression for magnification and making a long-range imaging approximation yields such that the object distance (R) is much greater than the focal length (f),

$$m=-\frac{fR}{R(R\text{-}f)} = -\frac{f}{(R\text{-}f)} \cong -\frac{f}{R}. \quad (6)$$

Given the appropriate substitutions for spatial frequencies (equation 3), and for some constant C, the PSF in the object plane is then a function of the form

$$PSF(x_o,y_o)=C\left|F\left(\frac{mDx}{\lambda f},\frac{mDy}{\lambda f}\right)\right|^2 = C\left|F\left(\frac{fDx}{R\lambda f},\frac{fDy}{R\lambda f}\right)\right|^2 = C\left|F\left(\frac{Dx}{R\lambda},\frac{Dy}{R\lambda}\right)\right|^2. \quad (7)$$

Transforming to obtain the MTF yields a function (G) of variables $R\lambda u_x/D$ and $R\lambda u_y/D$, the result being independent of focal length.

$$MTF_{obj}(u_x,u_y)= G\left(\frac{f\lambda u_x}{mD},\frac{f\lambda u_y}{mD}\right) = G\left(\frac{R\lambda u_x}{D},\frac{R\lambda u_y}{D}\right) \quad (8)$$

Figure 2(a) illustrates the MTF and CTF curves for two different focal lengths and fixed display magnification (M). Equation (8) predicts identical MTF curves for the two focal lengths. However, since optical magnification (m) varies with focal length, equation (2) predicts two different CTF curves result, yielding a difference in perceptual image quality for the two focal lengths.

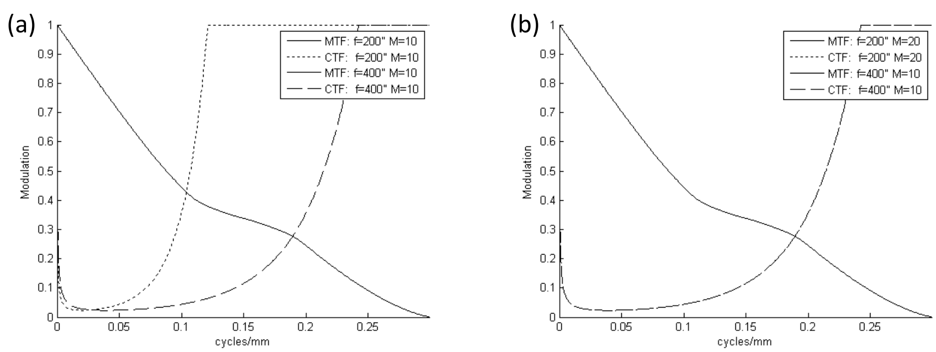

Figure 2. Object plane CTF and MTF curves. MTF curves for f=200", and f=400" are identical. (a) CTF curves vary for fixed display magnification (M). (b) CTF curves are identical when the display magnification (M) is varied in reciprocal proportion to the change in optical magnification (m).

As implied by equation (2), if the display magnification (M) is varied in reciprocal proportion to the change in optical magnification (m), then the CTF becomes invariant under focal length changes, illustrated in figure 2(b). This results in images of identical perceptual image quality.

3. Experimental Validation

To validate the claimed focal invariance, two perceptual tests were conducted using simulated imagery. Test 1 was the classic forced choice paired image comparison test [6] in which observers were asked to choose one image from each pair of images that had greater "usefulness". Test 2 was a variation in which observers were not forced to chose one image over the other, but were allowed the option of "equal usefulness." Ten observers familiar with launch imagery participated.

Table 1. Results of paired image comparison (Test 1) and the variation allowing "equal usefulness" option (Test 2). For each image pair, the reference image was simulated using a focal length of 50 in.

	Test 1	Test 2		
f [in]	Ref. % Sel.	Ref. % Sel.	Ref. % Sel.	% Sel. Equal
200	50	30	20	50
400	30	30	30	40
800	30	30	30	40
1000	50	20	10	70

4. Results

Results of the perceptual testing are listed in Table 1. The uncertainty in the results is considered to be a difference of ± selection by one observer since this is the discrete minimum "measurement interval". This yields an error of ±10%. In Test 1, results of 50±10% indicate the reference image to be perceptually equal to the test image. Although this is not strictly met for 2 of the four points, the data does not contradict the claim of focal invariance since the data does not have a functional form indicative of just noticeable differences occurring with changes in f. Note that slight variations of brightness between test and reference images are likely the cause of the problematic data points.

In Test 2, perceptually equivalent test and reference images is more complicated to determine. Here, some fraction of observers chose equal quality directly for two images. The remainder either chose the test or reference as having greater usefulness. For those not choosing equal usefulness, an equal fractional selection of the two images then indicates perceptually equivalent images. From Table 1, fractional selection of reference and test are equal within uncertainty, validating the claim.

5. Summary

Although focal length has long been considered important in designing imaging systems, it has no effect on image content for long range imaging. As the focal length increases, the PSF size decreases but is compensated by an increase in magnification yielding an object plane MTF invariant with focal length. With a shorter focal length, the image formed will be smaller, causing the usefulness to be perceived as lower. A subsequent increase in display magnification to compensate for the change in optical magnification restores the perceived usefulness of the image. Note however, that the conclusions are not necessarily valid in the presence of non-ideal display magnification processes, noise or inadequately sampled images.

7. References

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