Predicting Image Quality from Optical Surface Metrology Data

Plus

Generalizing Gary Peterson's Elegant Analytical Scatter Model

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Abstract

Image degradation due to scattered radiation from residual optical fabrication errors is a serious problem in many short wavelength (X-ray/EUV) imaging systems. Most currently-available image analysis codes (ZEMAX, ASAP, FRED, etc.) require the scatter behavior (BRDF data) as input in order to calculate the image quality from such systems. This BRDF data is difficult to measure and rarely available for the operational wavelengths of interest. Since the smooth-surface approximation is often not satisfied at these short wavelengths, the classical Rayleigh-Rice expression that indicates the BRDF is directly proportional to the surface PSD cannot be used to calculate BRDFs from surface metrology data for even slightly rough surfaces. An FFTLog numerical Hankel transform algorithm enables the practical use of the computationally intensive Generalized Harvey-Shack (GHS) surface scatter theory to calculate BRDFs for increasingly short wavelengths that violate the smooth surface approximation implicit in the Rayleigh-Rice surface scatter theory. A generalized Peterson analytical scatter model is then used to make accurate image quality predictions. The generalized Peterson model is numerically validated by both ASAP and ZEMAX.

Statement of the Problem

For short wavelength applications, surface scatter effects from residual optical fabrication errors frequently limit the performance of imaging systems rather than geometrical aberrations or diffraction effects!

- Optical fabrication tolerances necessary to satisfy specific image quality requirements must be derived:

  o Calculate the BRDF from assumed metrology data.

  o Calculate the image degradation caused by that BRDF.

  (ZEMAX ASAP FRED)

- Optical surfaces aren't always "smooth" relative to the operational wavelength; hence, surface scatter theories using smooth surface approximations or perturbation techniques (Rayleigh-Rice) are _not_ valid.

- A new generalized surface scatter theory (GHS) is valid for both rough surfaces and non-paraxial incident and scattering angles.*

- The large dynamic range in the relevant spatial frequencies has caused severe computational problems in implementing the new generalized scatter theory.

* J. Harvey, A. Krywonos, and J. Stover, *Unified Scatter Model for Rough surfaces at Large Incident and Scattered Angles*, Proc. SPIE 6672-12 (2007). 3

Outline

- **Brief Review of Surface Scatter Theory.**

- **Total Integrated Scatter (TIS) for Moderately Rough Surfaces.**

- **Demonstration of the Generalized Harvey-Shack Scatter Theory.**
  - **Very Computationally Intense Calculations.**
  - **Problem: Large dynamic Range of Relevant Spatial Frequencies.**
  - **Solution: FFTLog Numerical Hankel Transform Algorithm.** **NEW**

- **Example of measured metrology data from optical surface.**

- **BRDFs from Surface PSDs for increasingly short wavelengths (that violate the smooth surface approximation).**

- **Generalized Peterson Analytical Scattering Model.**
  - **Dealing with the "Scattered-Scattered" Light.**
  - **Numerical Validation with ASAP and ZEMAX.**

- **Results and Conclusions.**

Rayleigh-Rice Surface Scatter Theory*

The Rayleigh-Rice scattering theory is based on the vector perturbation approach first done by Rayleigh for gratings. Using a perturbation of the surface height and solving for the exact boundary conditions at the scattering surface leads to an infinite number of equations and unknowns which is only practically solved when the roughness is small.

The Rayleigh-Rice surface scatter theory agrees well with experimental wide angle scatter measurements with large incident angles for smooth surfaces, however not all surfaces of interest satisfy this smooth-surface requirement.

The scattered intensity (normalized by incident power) is given by

$$\frac{(dP/d\Omega_s)}{P_i} = \left(\frac{16\pi^2}{\lambda^4}\right)\cos\theta_i \ \cos^2\theta_s \ Q \ S(f_x, f_y)$$

(1)

$S(f_x, f_y)$ is the two-sided, two-dimensional surface *PSD* function expressed in terms of the sample spatial frequencies

$$f_x = \frac{\sin\theta_s \cos\phi_s - \sin\theta_i}{\lambda} \ \text{and} \ f_y = \frac{\sin\theta_s \sin\phi_s}{\lambda}$$

Q is the polarization dependent reflectance of the surface. For *TE* polarization and measurements in the plane of incidence Q is given exactly by the geometric mean of the sample specular reflectances at θ_i and θ_s

$$Q = [R_s(\theta_i) \ R_s(\theta_s)]^{1/2}$$

* S. O. Rice, "Reflection of Electromagnetic Waves from Slightly Rough Surfaces", Commun. Pure Appl. Math, 4, p 351 (1951). 5

Beckmann-Kirchhoff Scattering Theory*

Beckmann used a Kirchhoff diffraction approach to solving the surface scatter problem. Instead of solving for the exact boundary conditions, he approximated the fields and normal derivatives at a point on the scattering surface with the fields and normal derivatives that would exist on a plane tangent to that point. This allows the Beckmann-Kirchhoff theory to be used for rougher surfaces, however it also requires the radius of curvature of the surface features to be much larger than the wavelength. In addition, the Beckmann-Kirchhoff theory contains a built-in paraxial limitation.

Beckmann only provides a closed-form solution for the case of moderately rough and very rough surfaces when the surface can be described by a Gaussian autocovariance function. In that case, the scattered intensity is given by

$$I(\theta,\phi) = \frac{\pi \ell_c^2 F^2 \exp(-g)}{A} \sum_{m=1}^{\infty} \frac{g^m}{m!\, m} \exp\left[-\frac{v_{xy}^2 \ell_c^2}{4m}\right] \quad (2)$$

For smooth and moderately rough surfaces

$$I(\theta,\phi) = \frac{\pi F^2 \ell_c^2}{A^2 g} \exp\left(-\frac{v_{xy}^2 \ell_c^2}{4g}\right) \quad (3)$$

For very rough surfaces

where

$$F = \left[\left(\frac{1}{\cos\theta_o}\right)\frac{1+\cos(\theta_o+\theta_s)}{\cos\theta_o+\cos\theta_s}\right]^2$$

$$v_{xy} = k\sqrt{\sin^2\theta_s + \sin^2\theta_o}$$

$$g = \left(\frac{2\pi\sigma}{\lambda}\right)^2 (\cos\theta_o + \cos\theta_s)$$

A = Illuminated Surface Area

ℓ_c = Correlation Length

* P. Beckmann and A. Spizzichino, *Scattering of Electromagnetic Waves from Rough Surfaces*, Pergamon Press, New York, 1963.
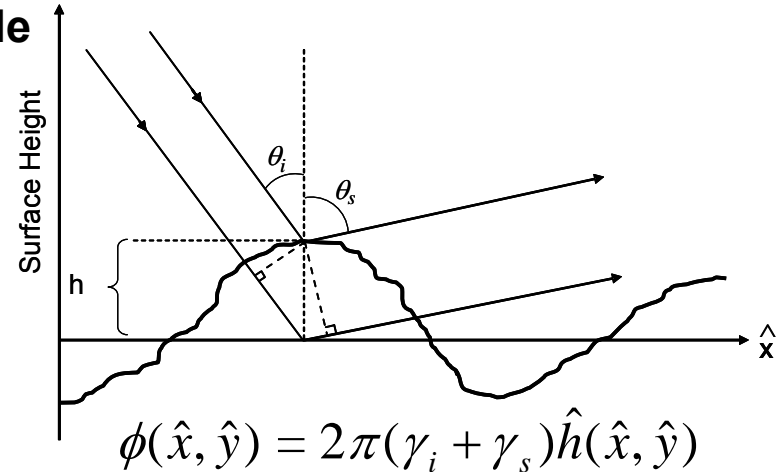
Generalized Harvey-Shack Surface Scatter Theory*
(Arbitrarily Rough Surfaces, Large Incident and Scatter Angles)

Limitations of Original Harvey-Shack Theory

- Scalar theory (no polarization effects).

- Does not account for redistribution of energy from evanescent to propagating waves.

- Surface transfer function has a built-in paraxial limitation.

Phase Variation Depends on Scattering Angle

$$\phi(\hat{x}, \hat{y}) = 2\pi(\gamma_i + \gamma_s)\hat{h}(\hat{x}, \hat{y})$$

New Surface Transfer Function

$$H_s(\hat{x}, \hat{y}; \gamma_i, \gamma_s) = \exp\left\{-\left[2\pi\hat{\sigma}_s(\gamma_i + \gamma_s)\right]^2 \left[1 - C_s(\hat{x}, \hat{y})/\sigma_s^2\right]\right\}$$

(4)

$$\gamma_i = \cos\theta_i$$

$$\gamma_s = \sqrt{1 - \alpha_s^2 - \beta_s^2} = \cos\theta_s$$

$$C_s(\hat{x}, \hat{y}) \equiv \text{Surface Autocovariance Function}$$

$$TIS = 1 - \exp\left\{-\left[2\pi\hat{\sigma}_{rel}(\gamma_i + \gamma_s)\right]^2\right\}$$

$$BRDF = Q\, \mathcal{F}\{H(\hat{x}, \hat{y}; \gamma_i, \gamma_s)\}$$

(5)

New Generalized Harvey-Shack Theory

The system is no longer shift invariant (requires a different transfer function for each incident and scattering angle).

This is similar to imaging systems with field-dependent aberrations, where a different MTF is necessary for each field angle.

This new surface scatter model has been quazi-vectorized by merely substituting the polarization reflectance factor, Q, for the reflectance, R, in the scalar treatment.

* A. Krywonos, *Predicting Surface Scatter using a Linear Systems Formulation of Non-paraxial Scalar diffraction*, PhD Dissertation, UCF (2006).
** J. Harvey, et.al., *Unified Scatter Model for Rough surfaces at Large Incident and Scattered Angles*, Proc. SPIE 6672-12 (2007).

Smooth-surface Approximation to GHS Theory
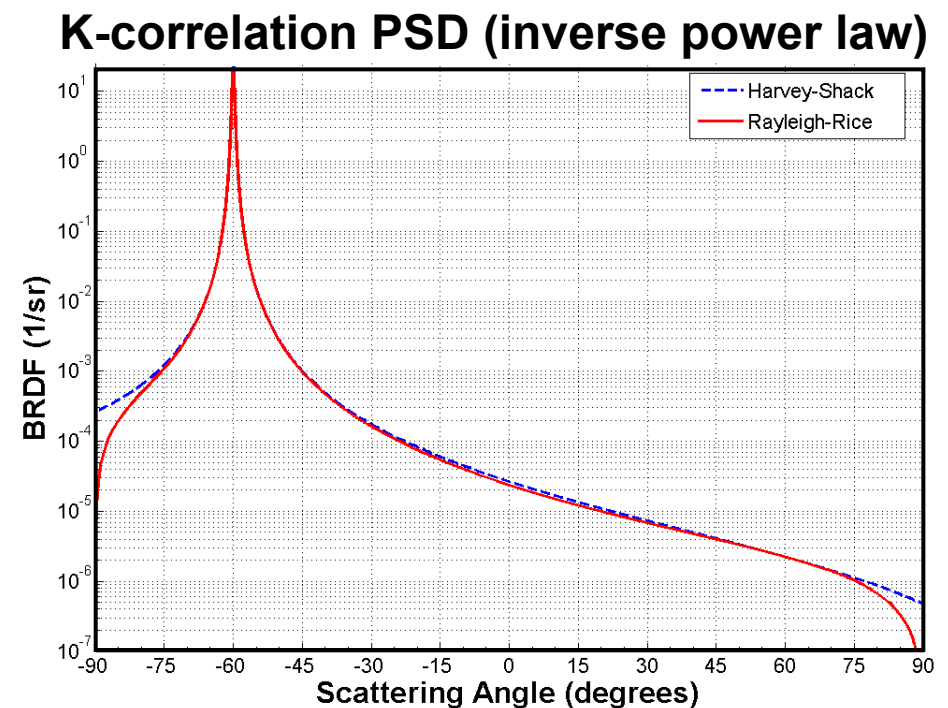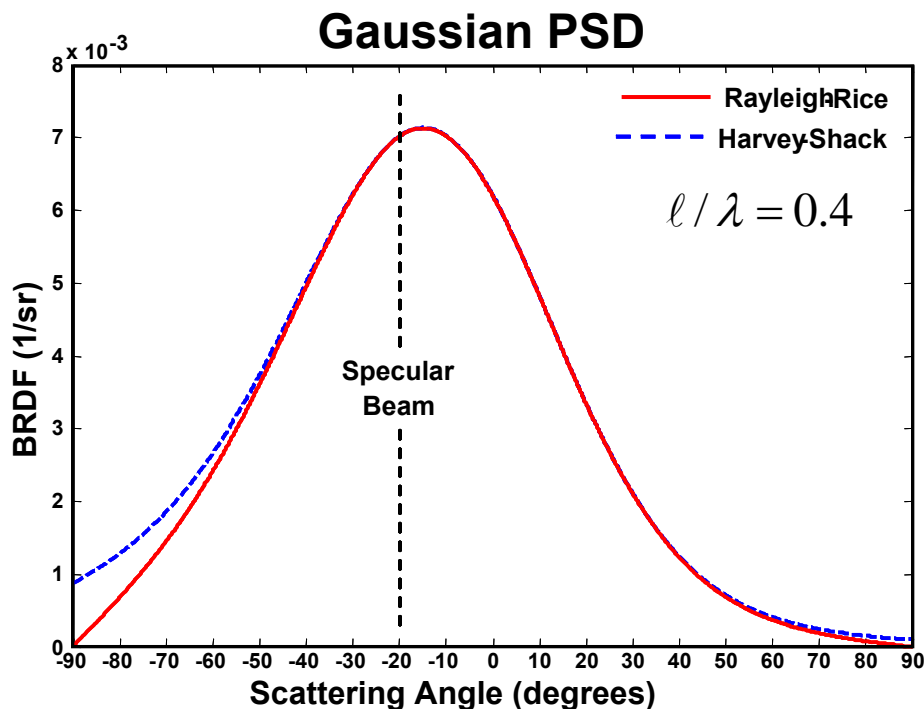## (Obliquity Factor Differs from Rayleigh-Rice Theory)

Generalized Harvey-Shack

$$BRDF = \frac{4\pi^2}{\lambda^4}(\cos\theta_o + \cos\theta_s)^2 \, Q \, PSD(f_x, f_y) \quad \textbf{(6)}$$

Rayleigh-Rice

$$BRDF = \frac{16\pi^2}{\lambda^4}\cos\theta_o \cos\theta_s \, Q \, PSD(f_x, f_y) \quad \textbf{(7)}$$

The above two equations are equivalent for small incident and scattered angles; however, the Rayleigh-Rice expression drives the BRDF to zero at ± 90 degrees regardless of the form of the surface PSD. In general, *BRDF*'s *do not go to zero* at ± 90 degrees (a Lambertian surface is an obvious counter-example). Furthermore the Rayleigh-Rice expression results in undesirable artifact in the predicted PSD when solving the inverse scattering problem (the ubiquitous "hook" at high spatial frequencies).

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    - o **Very Computationally Intense Calculations.**
    - o **Problem: Large dynamic Range of Relevant Spatial Frequencies.**
    - o **Solution: FFTLog Numerical Hankel Transform Algorithm.**

- **Example of measured metrology data from optical surface.**

- **BRDFs from Surface PSDs for increasingly short wavelengths (that violate the smooth surface approximation).**

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Total Integrated Scatter *

   The fraction of the total radiant power contained in the specular beam after reflection from a moderately rough surface is given by

$$A = \exp[-(4\pi \cos\theta_i \, \sigma_{rel} / \lambda)^2]$$
(6)

and the fraction of the total reflected radiant power that is scattered out of the specular beam, or total integrated scatter (*TIS*) is defined as

$$B = TIS = 1 - \exp[-(4\pi \cos\theta_i \, \sigma_{rel} / \lambda)^2]$$
(7)

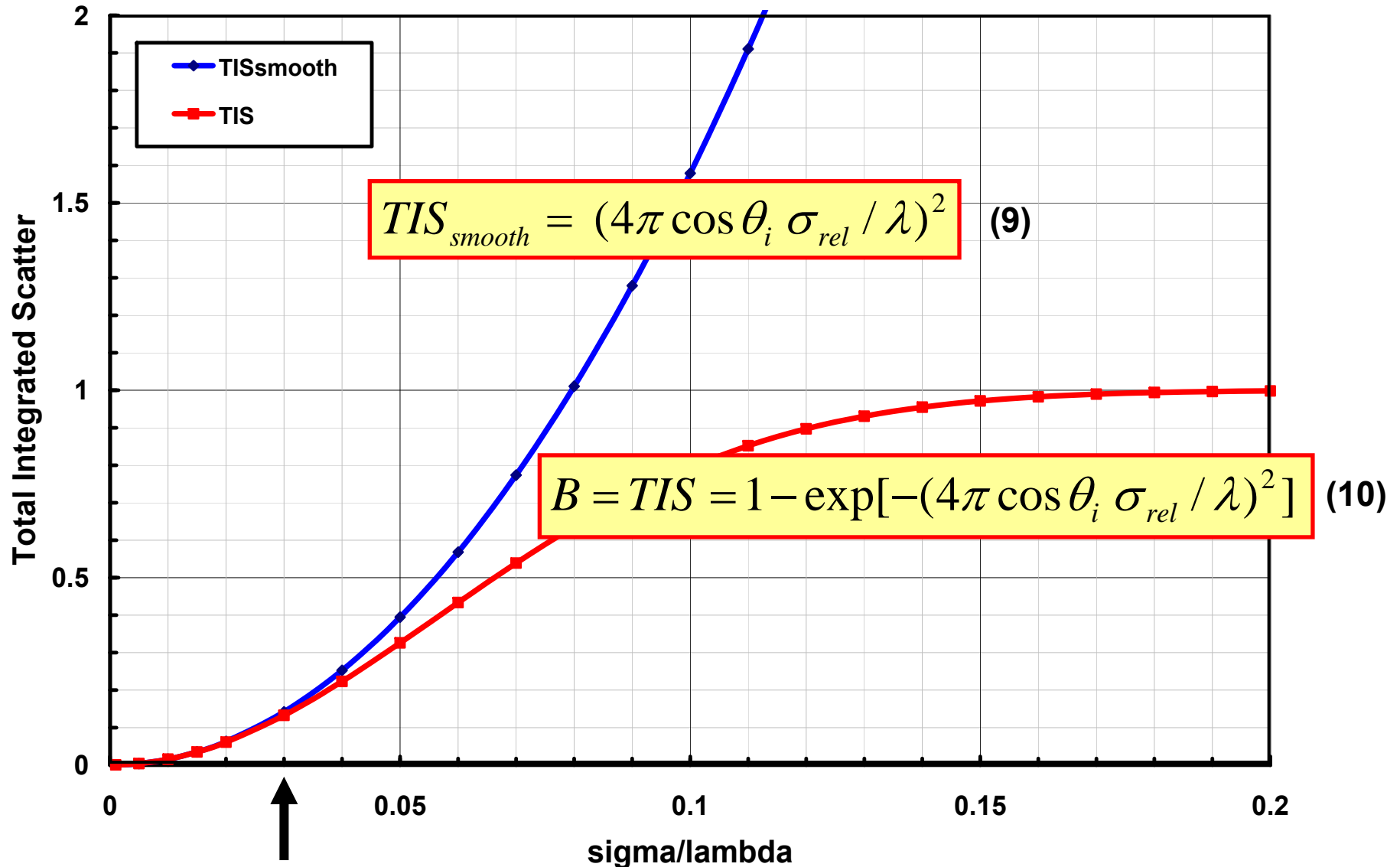where σ_{rel} is the bandlimited *relevant* roughness for $1.22/D < f < 1/\lambda$.

For smooth surfaces ($\sigma \ll \lambda$), the total integrated scatter (TIS$_{smooth}$) can thus be approximated as

$$TIS_{smooth} = (4\pi \cos\theta_i \, \sigma_{rel} / \lambda)^2$$
(8)

However, one needs to be careful in using this approximate expression as this quantity can quickly exceed unity for moderately rough surfaces.
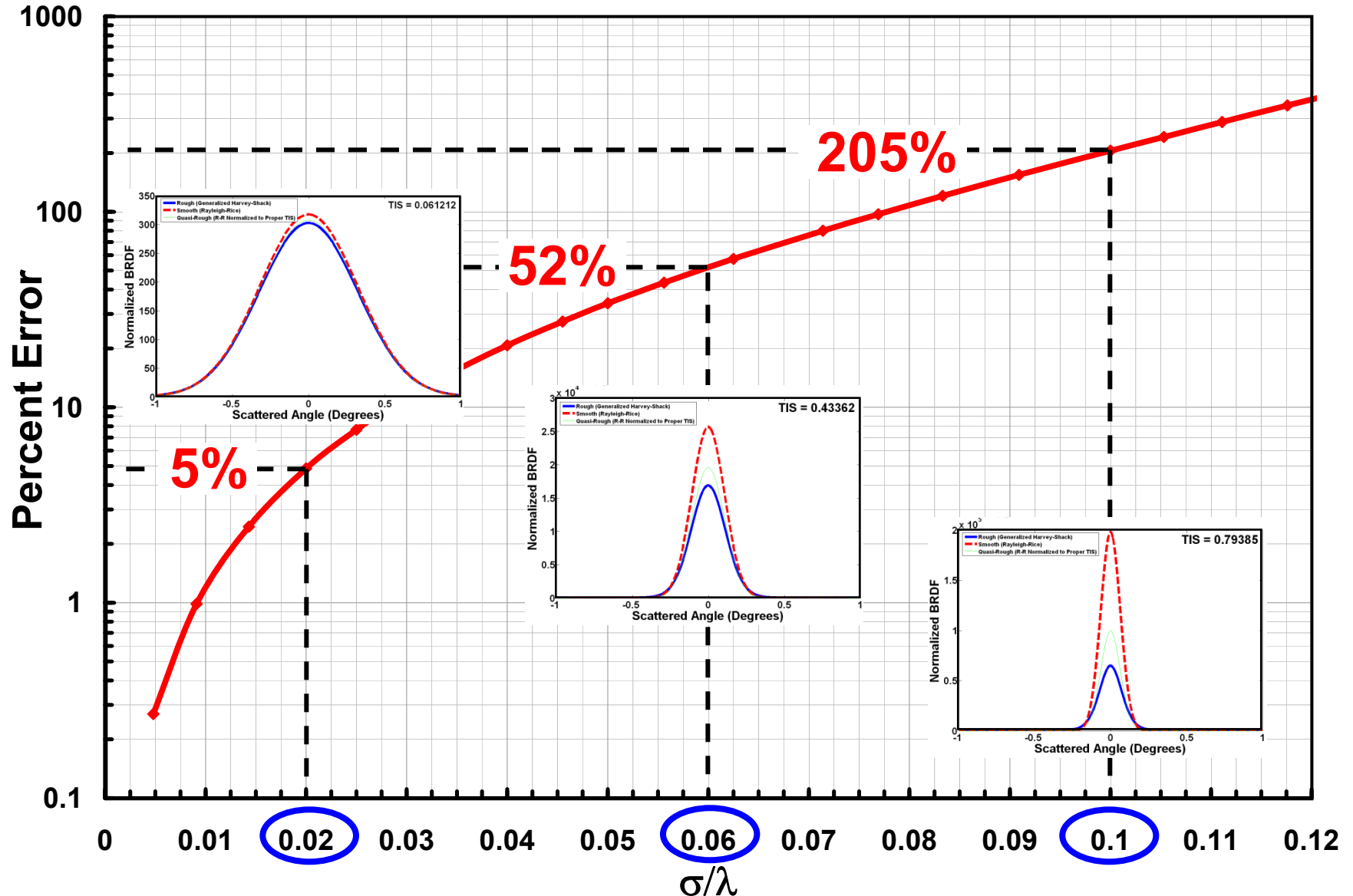
How Smooth is a Smooth Surface?

This graph shows how the smooth-surface approximation for TIS continues to grow exponentially for large σ/λ, providing an unrealistically large value for moderately rough surfaces.

$$TIS_{smooth} = (4\pi \cos\theta_i \, \sigma_{rel} \, / \, \lambda)^2 \quad \text{(9)}$$

$$B = TIS = 1 - \exp[-(4\pi \cos\theta_i \, \sigma_{rel} \, / \, \lambda)^2] \quad \text{(10)}$$
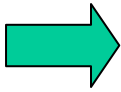
How Smooth is a Smooth Surface?

The smooth-surface approximation is a *very severe limitation* in predicting the BRDF as illustrated below for a Gaussian surface PSD. The percent error in the predicted peak value of the BRDF is illustrated below as a function of σ/λ.
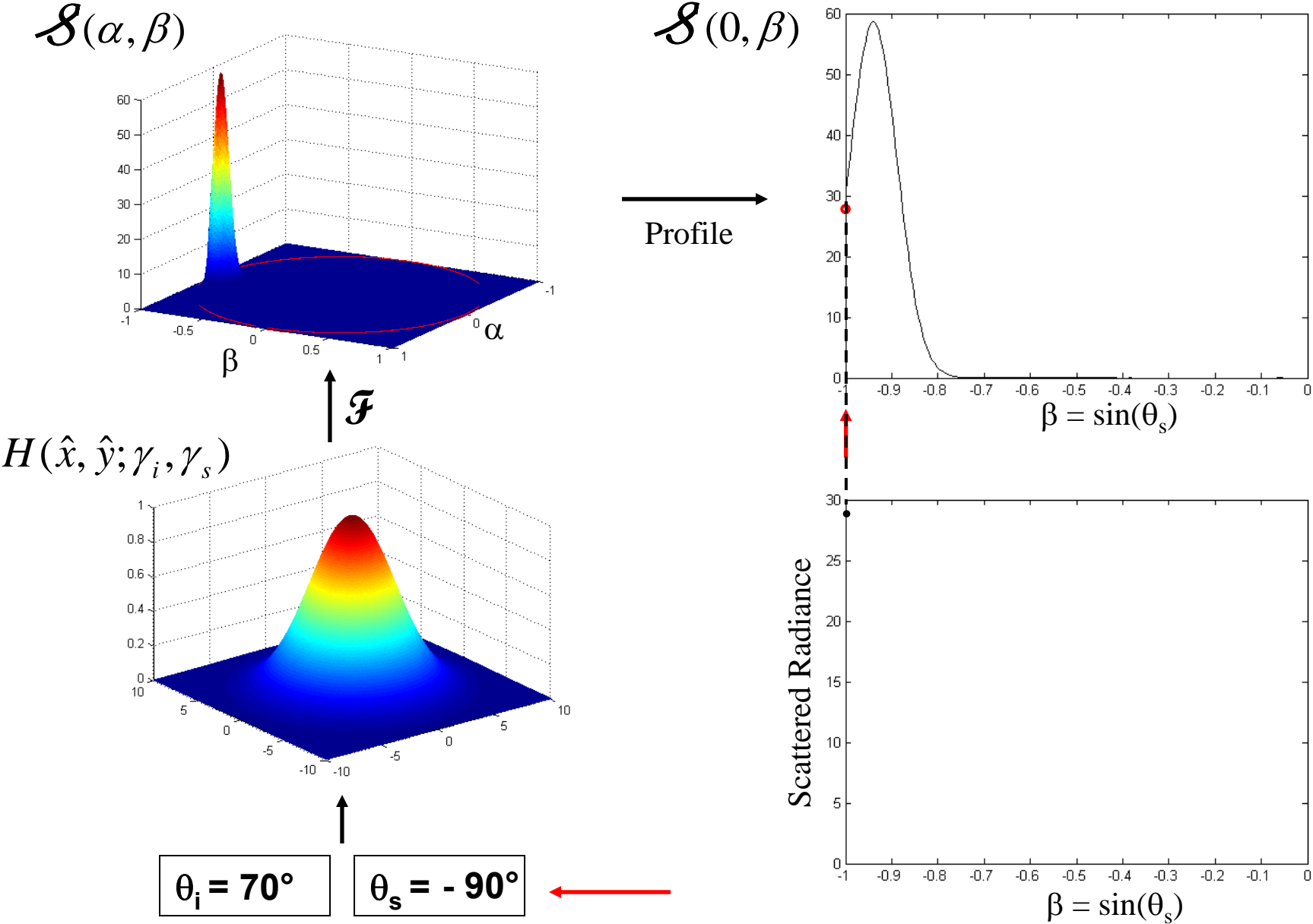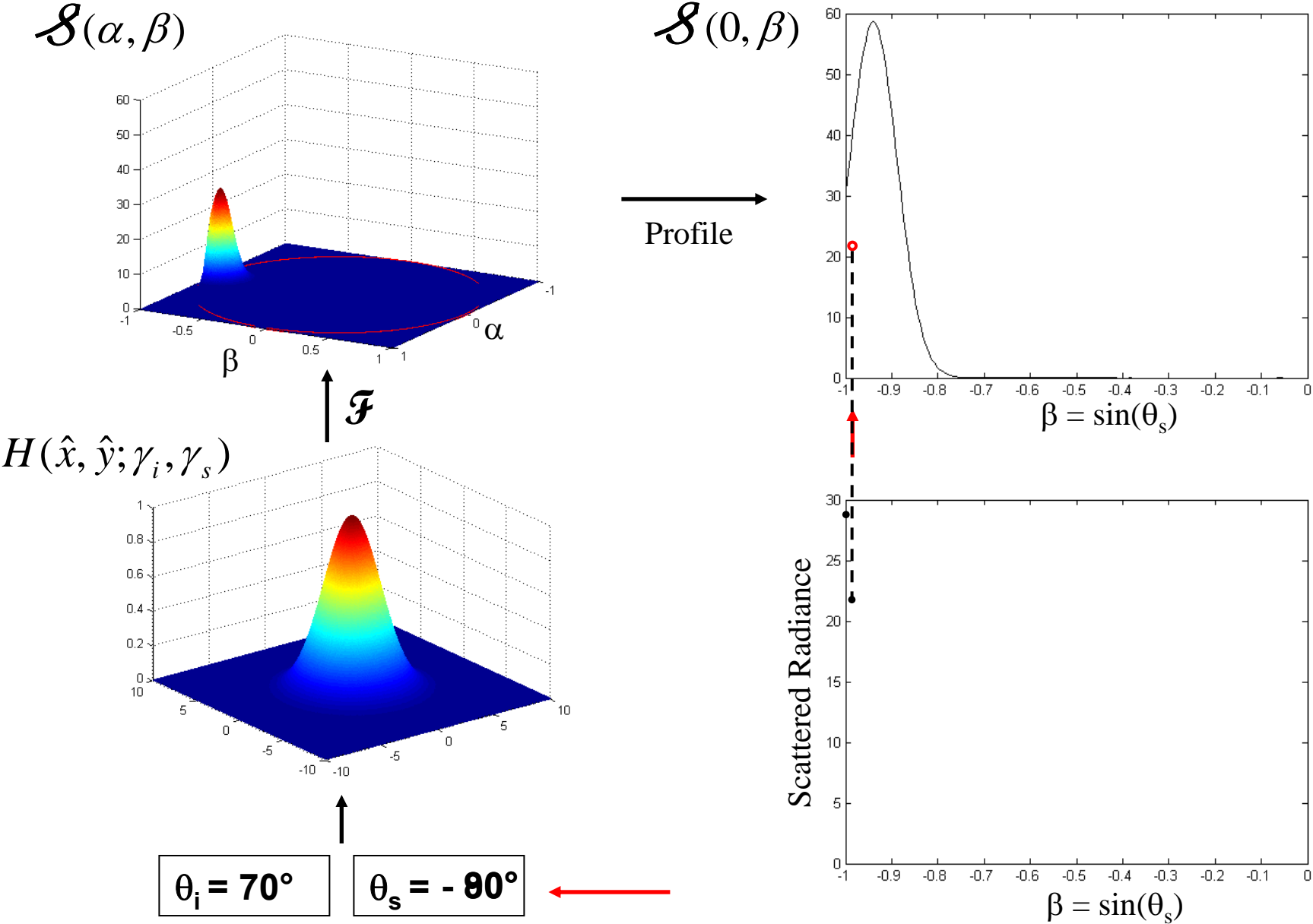
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Description of Generalized Harvey-Shack Calculations

$\mathcal{S}(\alpha, \beta)$

$\mathcal{S}(0, \beta)$

$\longrightarrow$ Profile

$\uparrow \mathcal{F}$

$H(\hat{x}, \hat{y}; \gamma_i, \gamma_s)$

$\beta = \sin(\theta_s)$

Scattered Radiance

$\theta_i = 70°$ $\theta_s = -90°$ $\longleftarrow$

$\beta = \sin(\theta_s)$

Description of Generalized Harvey-Shack Calculations

$\mathcal{S}(\alpha,\beta)$

$\mathcal{S}(0,\beta)$

$H(\hat{x},\hat{y};\gamma_i,\gamma_s)$

$\mathcal{F}$

Profile

$\beta = \sin(\theta_s)$

Scattered Radiance

$\beta = \sin(\theta_s)$

$\theta_i = 70°$ $\theta_s = -90°$

Description of Generalized Harvey-Shack Calculations

$\mathcal{S}(\alpha, \beta)$

$\mathcal{S}(0, \beta)$

Profile

$\beta = \sin(\theta_s)$

$H(\hat{x}, \hat{y}; \gamma_i, \gamma_s)$

$\mathcal{F}$

$\theta_i = 70°$ $\theta_s = 0.10°$

Scattered Radiance

$\beta = \sin(\theta_s)$

Description of Generalized Harvey-Shack Calculations

Description of Generalized Harvey-Shack Calculations

Intensity = Radiance $*$ $\cos(\theta_s)$

Scattered Intensity vs. Beta = $\sin(\theta_s)$

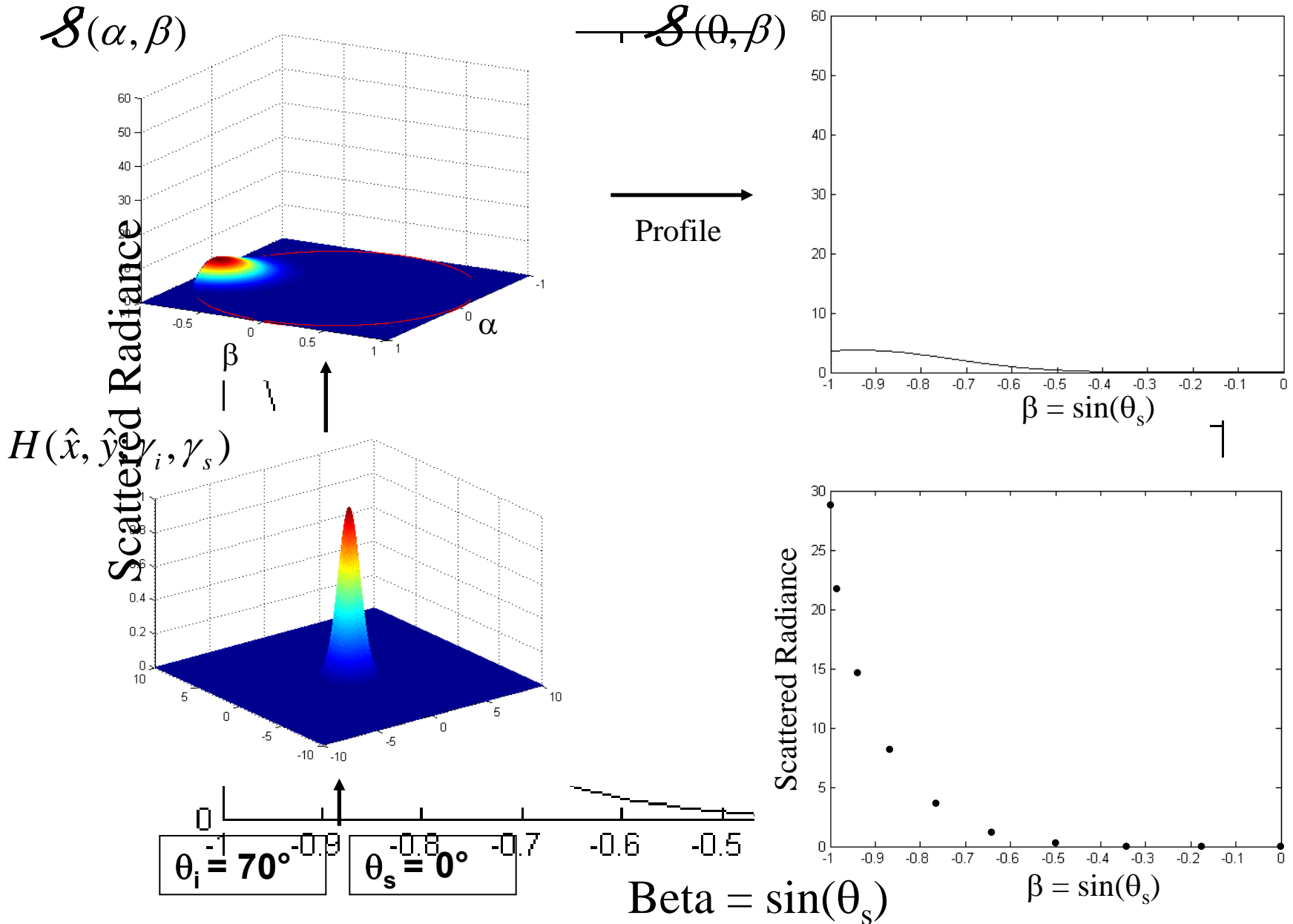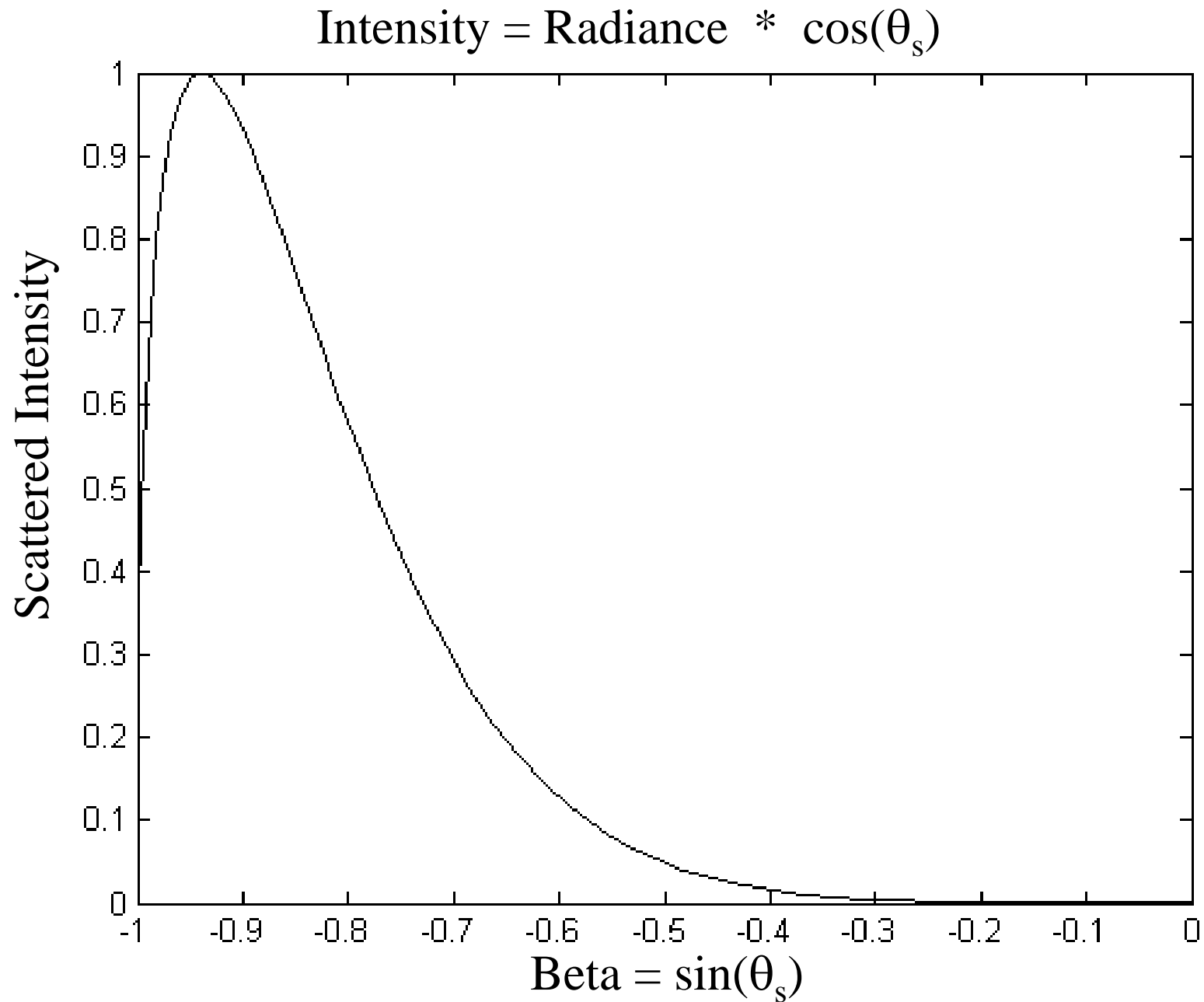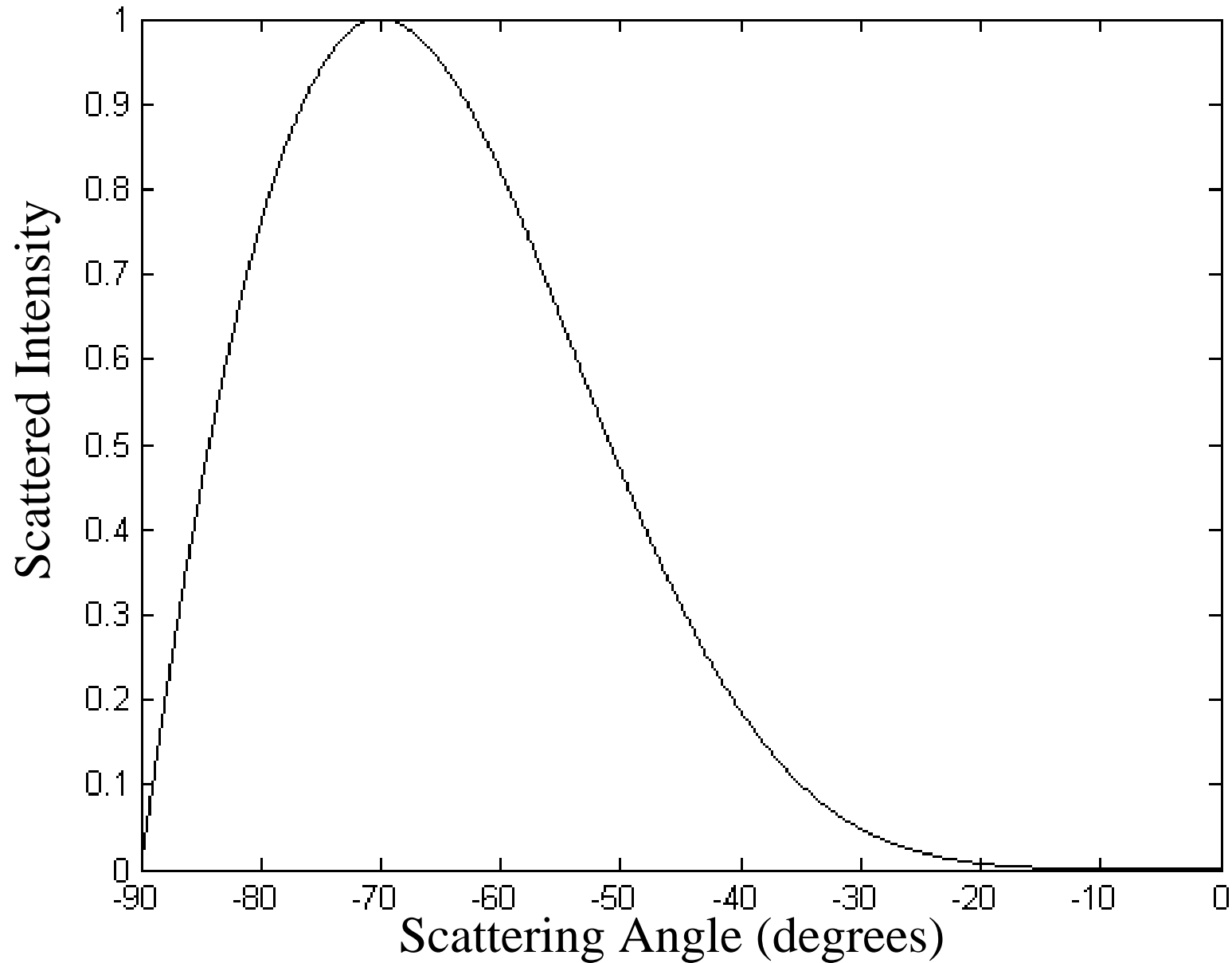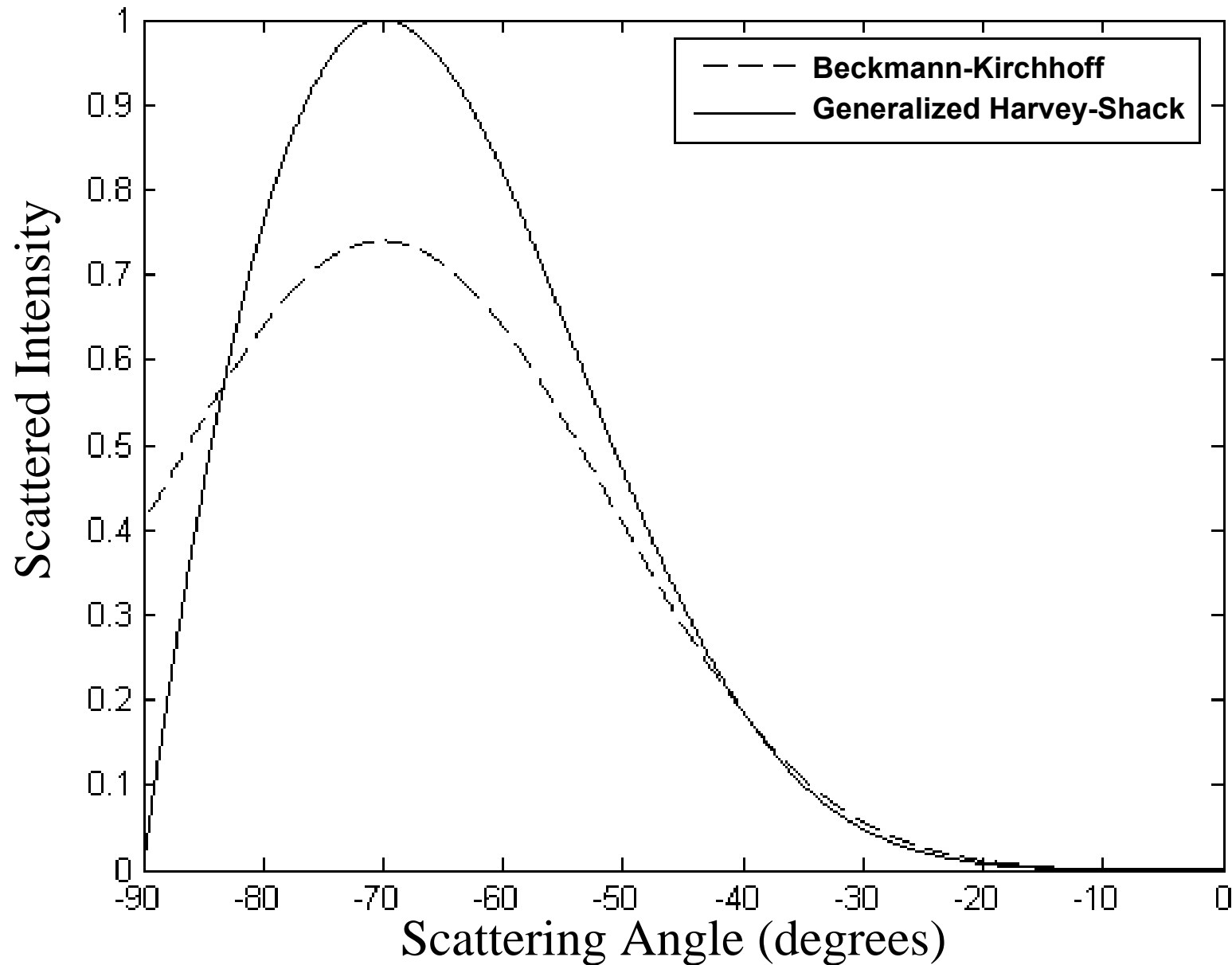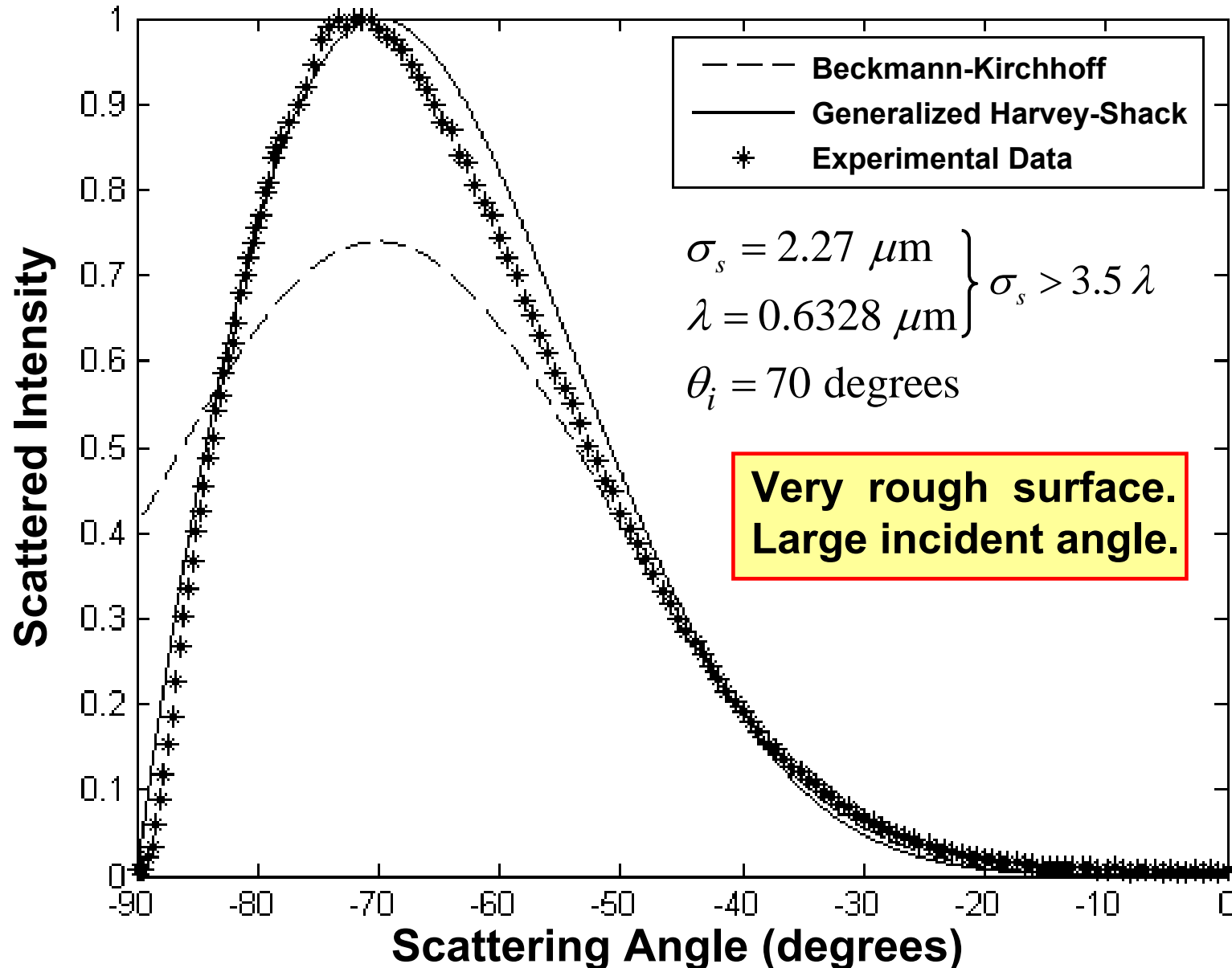# Description of Generalized Harvey-Shack Calculations

Description of Generalized Harvey-Shack Calculations

Generalized Harvey-Shack Scatter Theory
(Experimentally Validated by O'Donnell-Mendez Data)*

* K. A. O'Donnell and E. R. Mendez, "Experimental study of scattering from characterized random surfaces", J. Opt. Soc. Am. A, 4, 1194-1205 (1987).

Legend:
- - - - Beckmann-Kirchhoff
——— Generalized Harvey-Shack
✳ Experimental Data

$$\left.\begin{array}{l} \sigma_s = 2.27 \ \mu m \\ \lambda = 0.6328 \ \mu m \end{array}\right\} \sigma_s > 3.5 \lambda$$

$$\theta_i = 70 \text{ degrees}$$

Very rough surface. Large incident angle.

Axis labels: Scattered Intensity (vertical), Scattering Angle (degrees) (horizontal)

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- **Real Measured Metrology data from an EUV Telescope Mirror.**

- **BRDFs from Surface PSDs for increasingly short wavelengths (that violate the smooth surface approximation).**

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Example of Measured Metrology Data
(Including the very real "Mid" Spatial Frequencies)

It often takes three, or even four different metrology instruments to measure the surface characteristics over the entire range of relevant spatial frequencies for a given application.
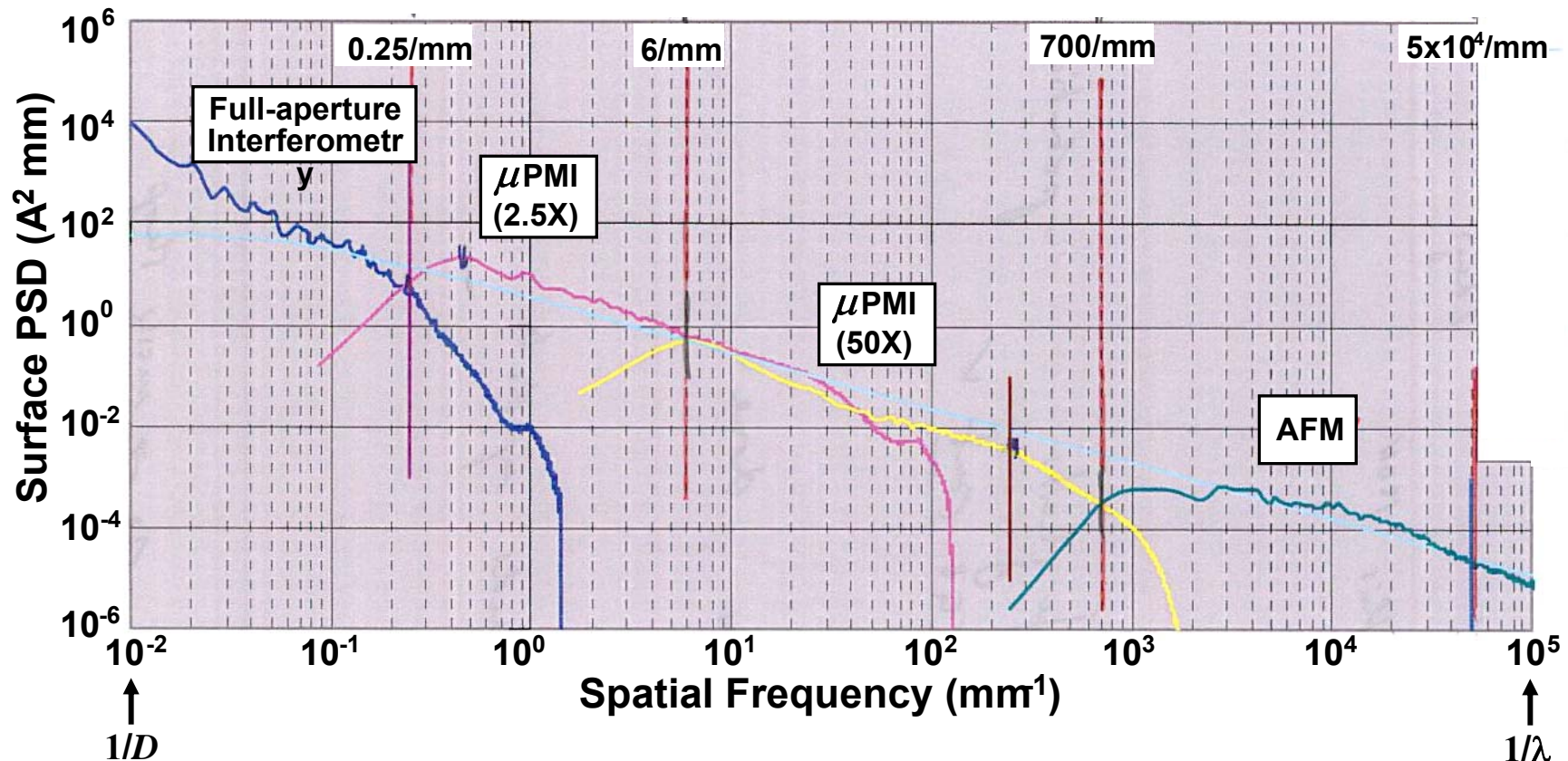

This metrology data can then be fit with an appropriate fitting function that can be used for making BRDF predictions, and then calculating image degradation. Note 7 decades of dynamic range in spatial frequency for $D = 100\,mm$ and $\lambda = 100$ A.

ABC, or *K*-Correlation Function Fit to Metrology Data*

Here we have fit the measured metrology data with an ABC or K-Correlation Function of the following form. The advantages of using a fitting function of this form is shown on the next slide.

$$PSD(f_x)_{1-D} = \frac{A}{\left[1 + (B\,f_x)^2\right]^{C/2}} \tag{11}$$
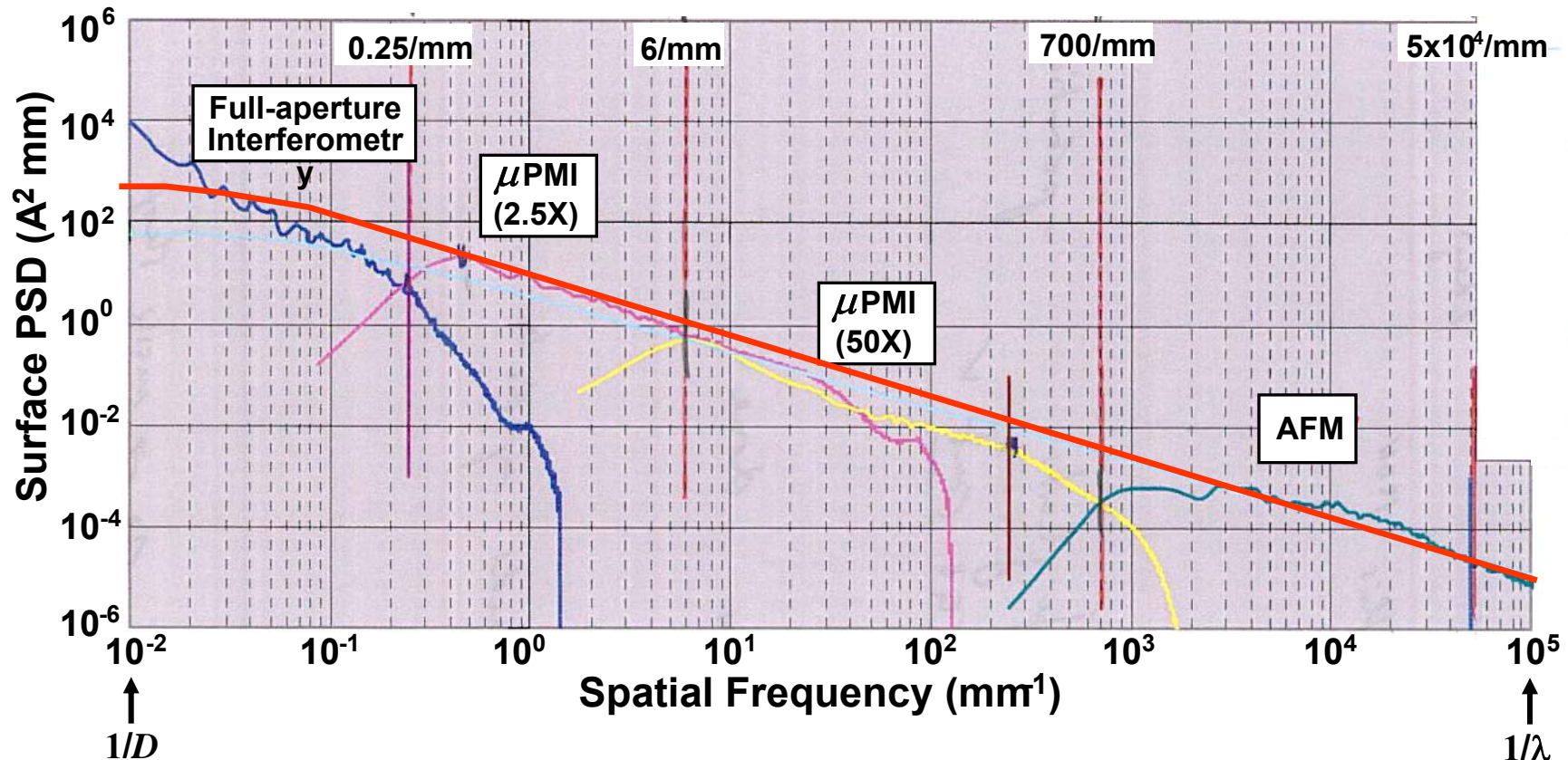
* E. L. Church and P. Z. Takacs, "The optimal estimation of finish parameters", Proc. SPIE 1530, p. 71-78 (1991).

Properties of *ABC* or *K*-Correlation Functions*

The *ABC*, or *K*-correlation function expressed by Eq.(12) has several very useful properties. The 2-D surface PSD (assuming isotropic roughness) can be obtained from the 1-D surface profile measurements by using Eq.(13). The total volume under the 2-D surface PSD is given by Eq.(14), and the Fourier transform of the 2-D K-correlation function is given by Eq.(15).

$$PSD(f_x)_{1-D} = \frac{A}{\left[1+\left(B\,f_x\right)^2\right]^{C/2}}$$
3-parameter *K*-correlation function or *ABC* function. **(12)**

$$PSD(f)_{2-D} = K\,\frac{A\,B}{\left[1+\left(B\,f\right)^2\right]^{(C+1)/2}} \;,\quad K = \frac{1}{2\sqrt{\pi}}\frac{\Gamma\big((C+1)/2\big)}{\Gamma(C/2)}$$
2-D surface PSD. **(13)**

$$f = \sqrt{f_x^2 + y_y^2}$$

$$\sigma_{Total}^2 = \frac{2\pi K A}{(C-1)B}$$
Total volume under 2-D surface PSD. **(14)**

$$ACV_s(r) = \sqrt{2\pi}\,\frac{A}{B}\frac{2^{-C/2}}{\Gamma(C/2)}\left(\frac{2\pi r}{B}\right)^{(C-1)/2}\mathcal{K}_{(C-1)/2}\left(\frac{2\pi r}{B}\right)$$
Surface Autocovariance Function. **(15)**

Where $\mathcal{K}_{(C-1)/2}$ is the modified Bessel function of the 2nd kind and $r = \sqrt{x^2 + y^2}$

E. Church and P. Takacs, "The optimal estimation of finish parameters", in *Optical Scatter: Applications, Measurement, and Theory*, J. C. Stover, Ed., Proc. SPIE **1530,** p. 71-78 (1991).
J, M. Elson, J. M. Bennett, and J. C. Stover, "Wavelength and angular dependence of light scattering from beryllium: comparison of theory and experiment", Appl. *Opt.* **32** (1993).
M. Abramowittz and I. A. Stegun, Handbook of Mathematical Functions, New York: Dover (1965).
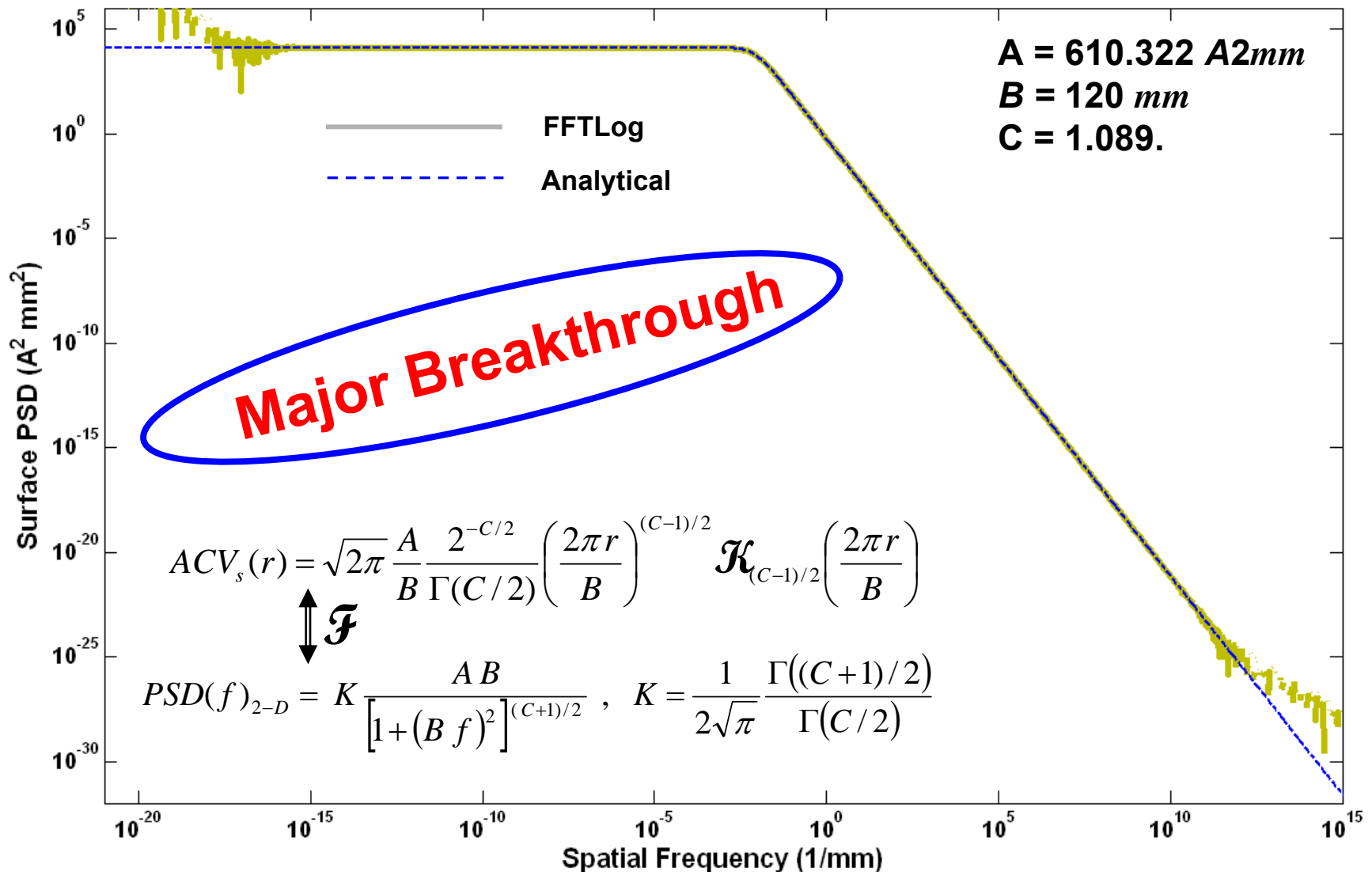
The *FFTLog* Hankel Transform Algorithm*

- *FFTLog* is a set of subroutines that compute the fast Fourier or Hankel (i.e., Fourier-Bessel) transform of a periodic sequence of logarithmically spaced data points.

- *FFTLog* can be regarded as a natural analogue to the standard Fast Fourier Transform (FFT), in the sense that, just as the normal FFT gives the exact (to machine precision) Fourier transform of a linearly spaced periodic sequence of data points, so also *FFTLog* gives the exact Fourier or Hankel transform, of arbitrary order, of a logarithmically spaced periodic sequence of data points.

- *FFTLog* shares with the normal FFT the problems of ringing (response to sudden steps) and aliasing (periodic folding of frequencies), but under appropriate circumstances *FFTLog* may approximate the results of a continuous Fourier or Hankel transform.

- The *FFTLog* algorithm is particularly useful for applications where the power spectrum extends over many orders of magnitude in wavenumber k, and varies smoothly in ln$k$.

* A. J. S. Hamilton, "Uncorrelated Modes of Nonlinear Power spectrum", Mon.Not.Roy.Astron.Soc. 312 (2000) 257-284.

Numerical Validation of the *FFTLog* Algorithm

For well-behaved ABC functions, the *FFTLog* algorithm is accurate over 25 decades of variation in spatial frequency (Note that the "ringing" and "aliasing" effects inherent to numerical Fourier transform calculations).

$$ACV_s(r) = \sqrt{2\pi}\,\frac{A}{B}\,\frac{2^{-C/2}}{\Gamma(C/2)}\left(\frac{2\pi r}{B}\right)^{(C-1)/2}\mathcal{K}_{(C-1)/2}\left(\frac{2\pi r}{B}\right)$$

$$\updownarrow \mathcal{F}$$

$$PSD(f)_{2-D} = K\,\frac{A\,B}{\left[1+\left(B\,f\right)^2\right]^{(C+1)/2}}\,,\quad K = \frac{1}{2\sqrt{\pi}}\,\frac{\Gamma\big((C+1)/2\big)}{\Gamma\big(C/2\big)}$$

A = 610.322 *A2mm*
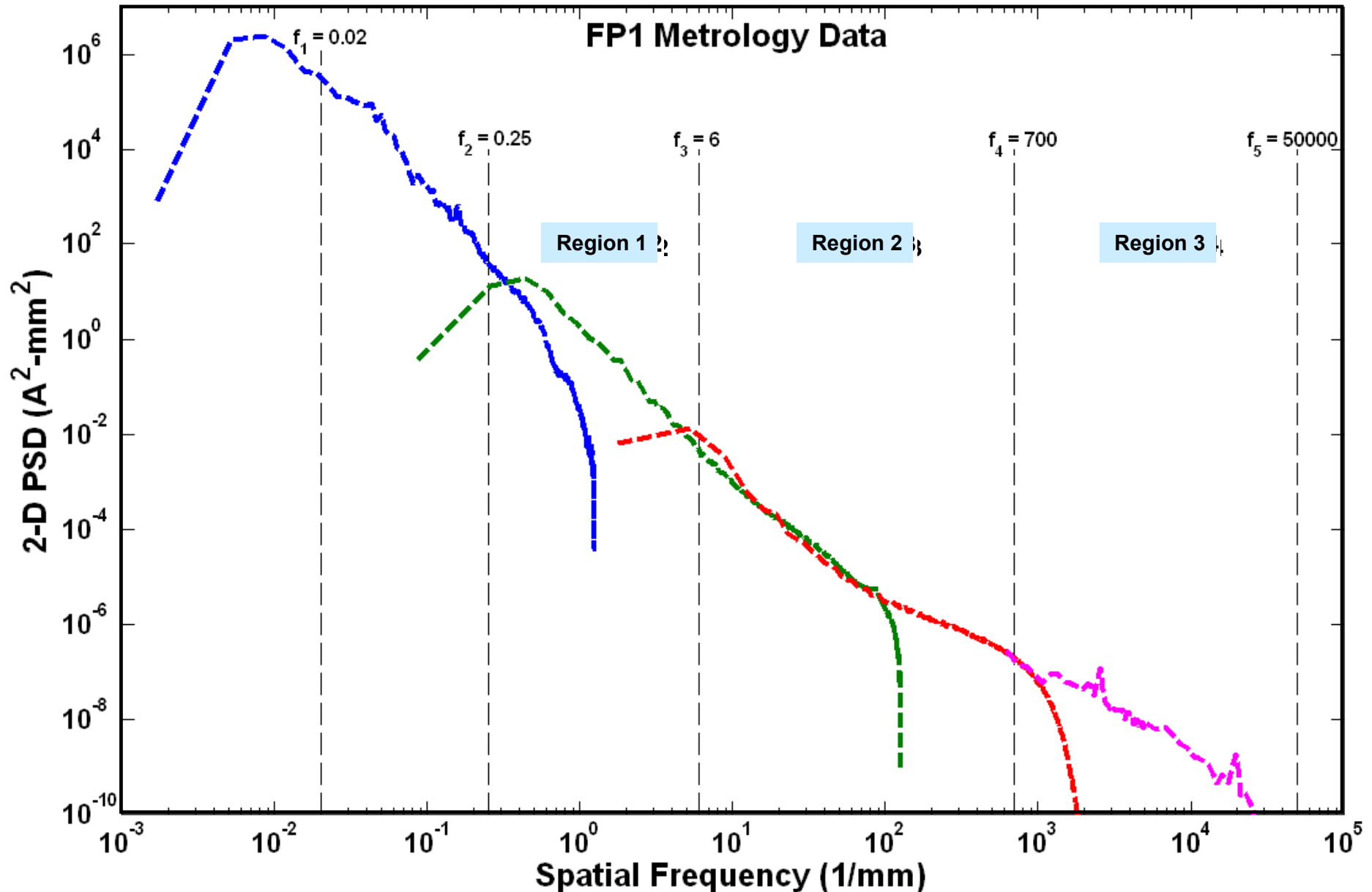B = 120 *mm*
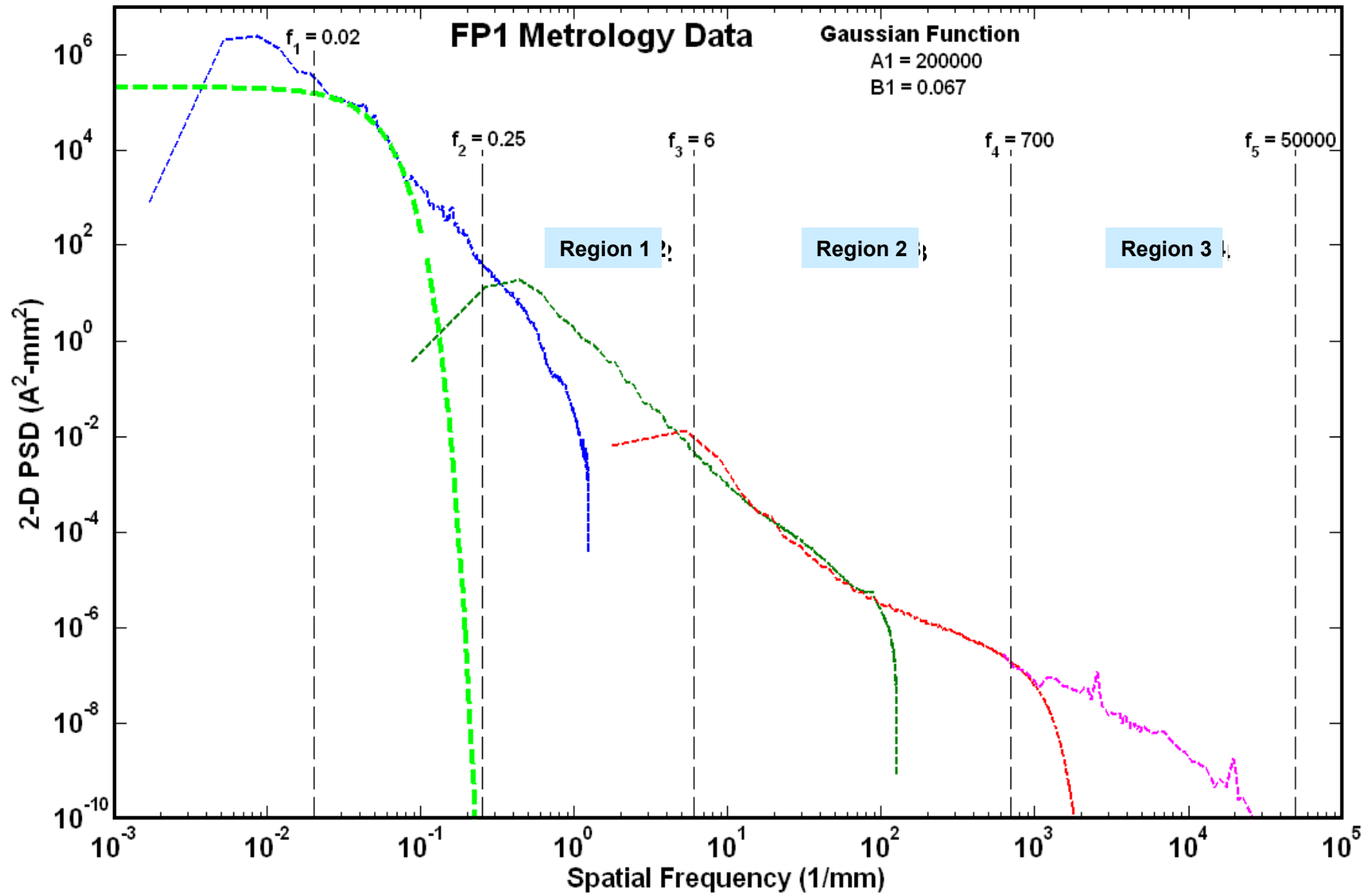C = 1.089.

Major Breakthrough

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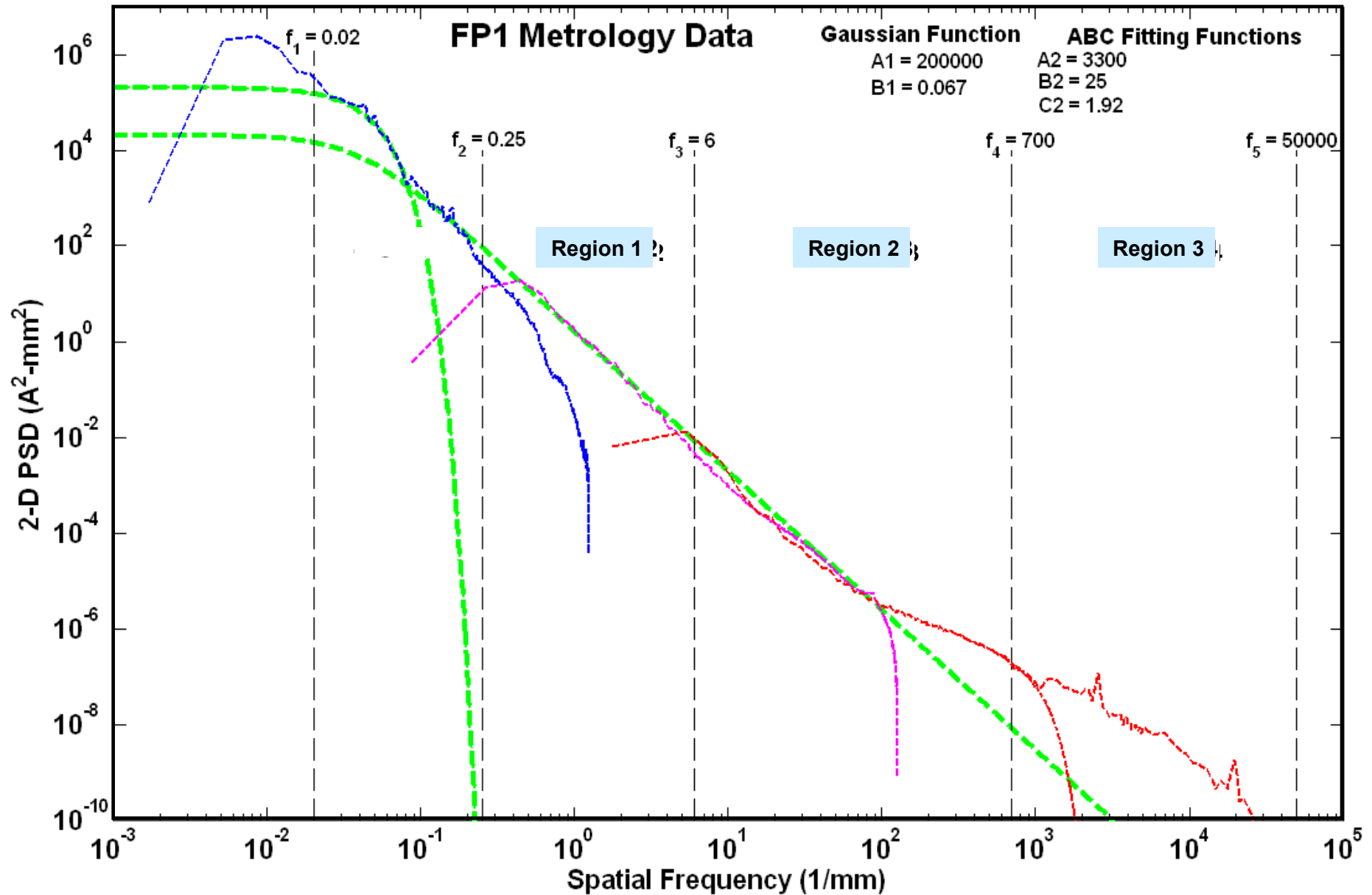
- **Results and Conclusions.**

SUVI FP1 Metrology Data
(SUVI Primary Mirror)
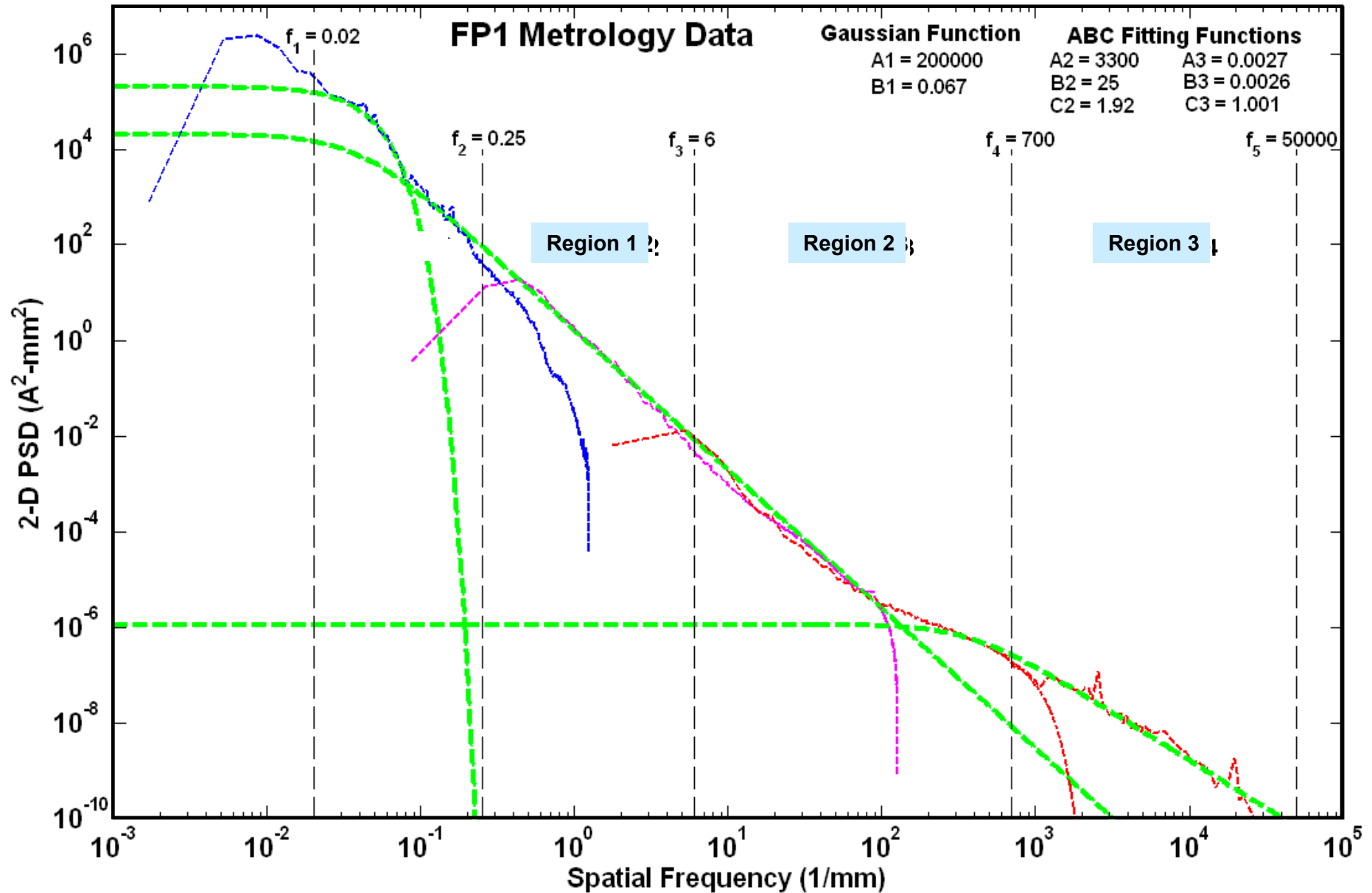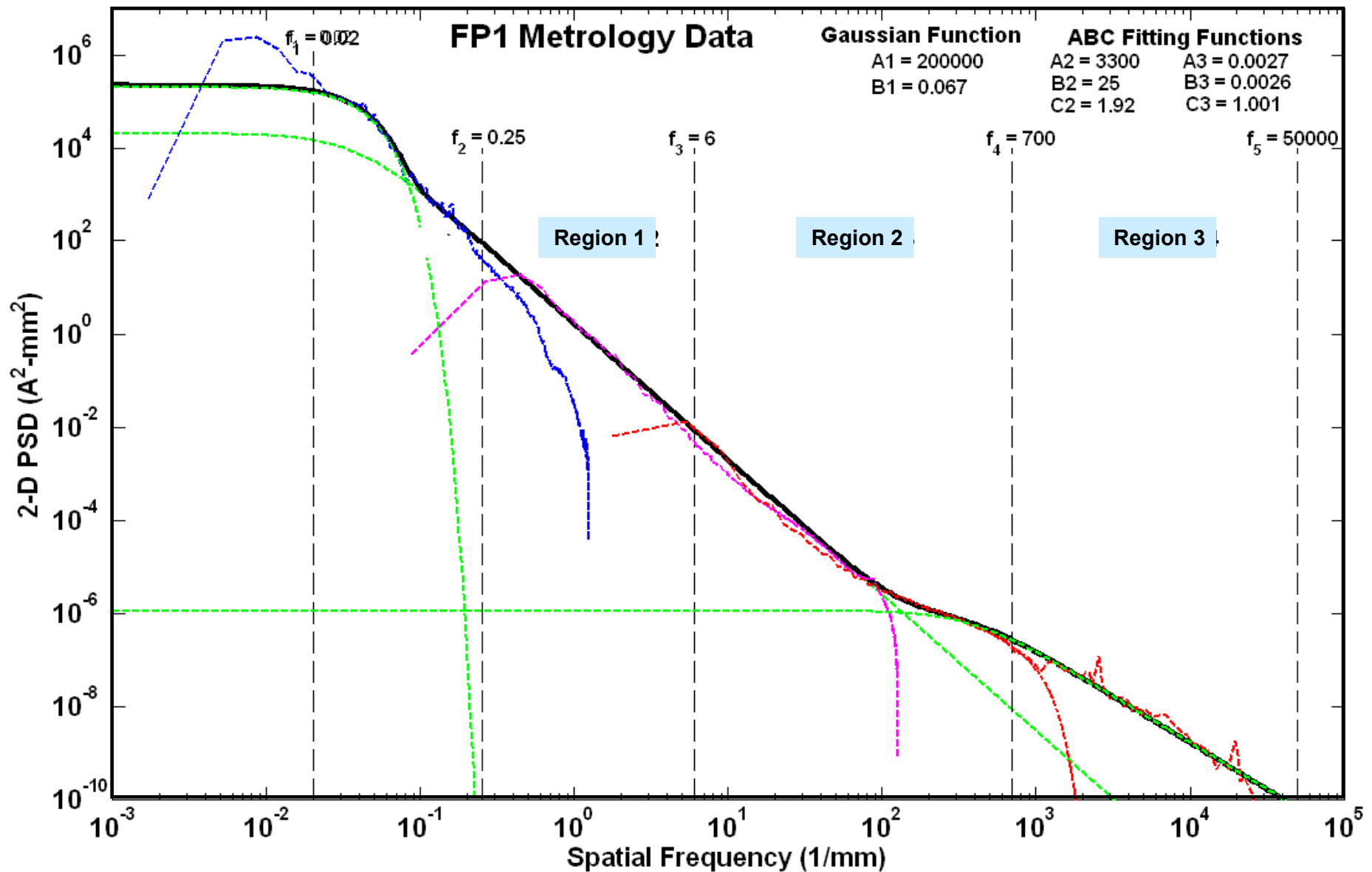
1st Fitting Function to FP1 Metrology Data

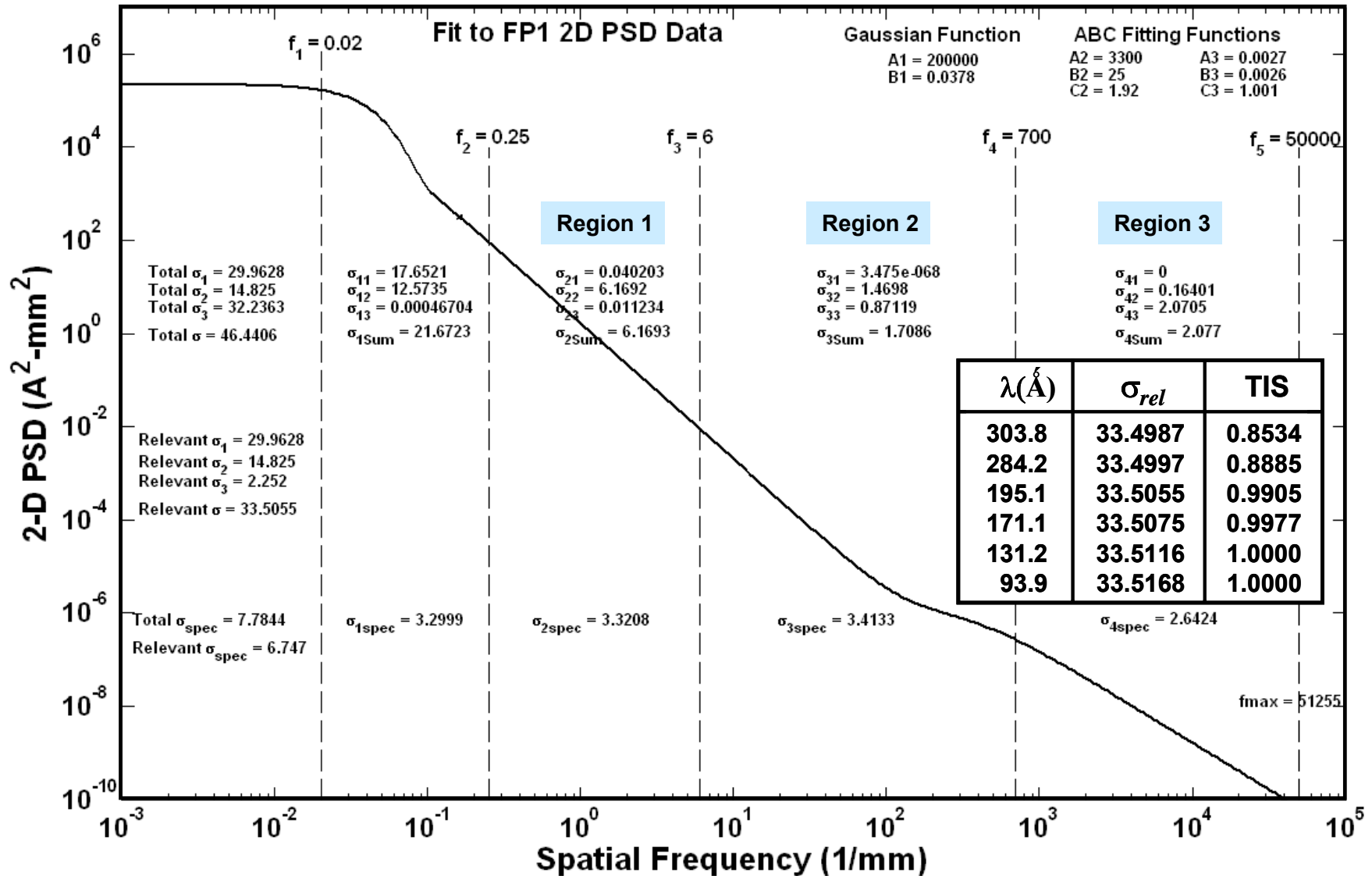# 2nd Fitting Function to FP1 Metrology Data
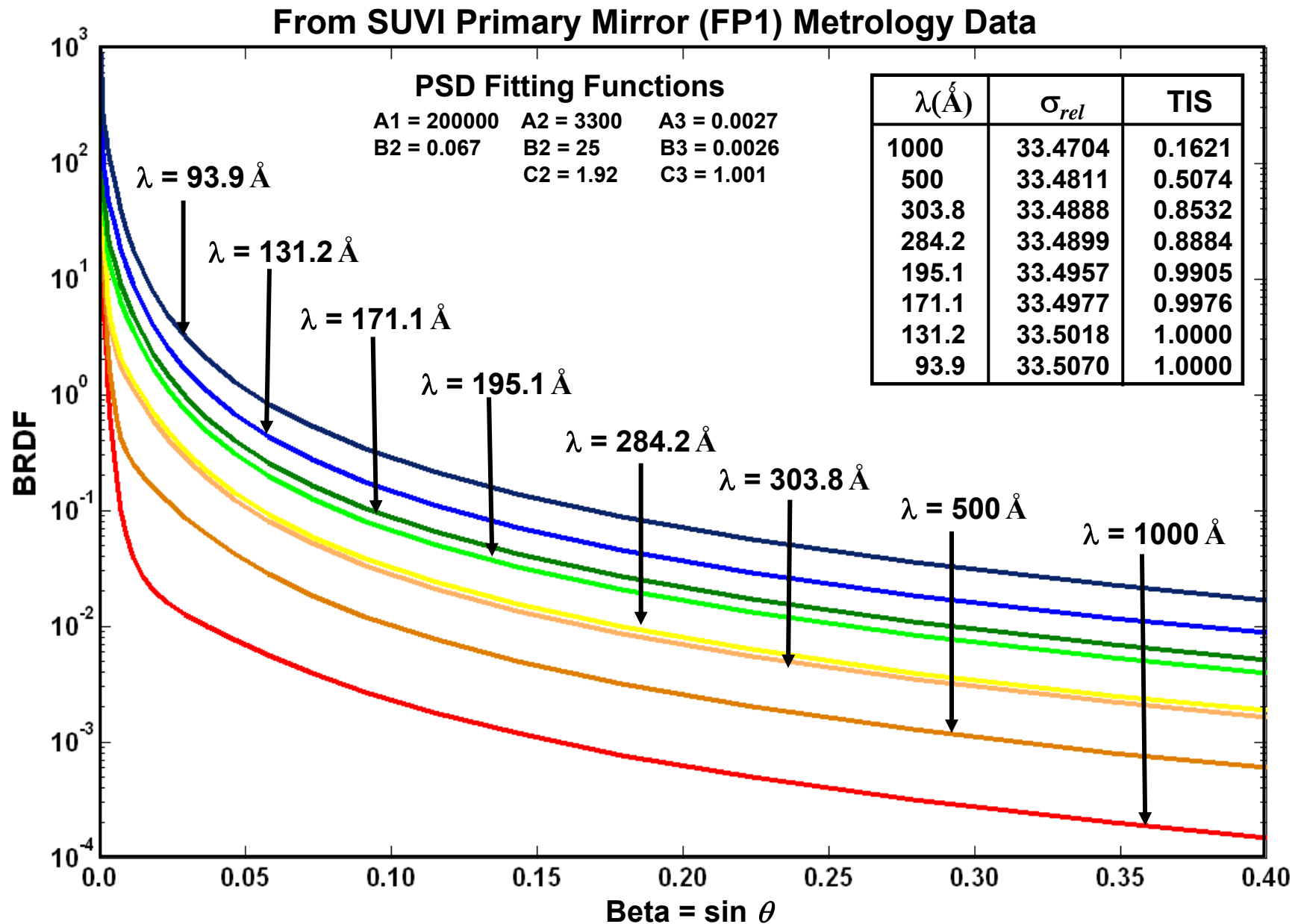
Sum of three Fitting Functions

FP1 2-D PSD Metrology Data
(With Band-limited Roughness Values)

One Gaussian + Two ABC Functions

BRDF Predictions from FP1 Metrology Data

From SUVI Primary Mirror (FP1) Metrology Data

PSD Fitting Functions

A1 = 200000   A2 = 3300   A3 = 0.0027
B2 = 0.067   B2 = 25   B3 = 0.0026
              C2 = 1.92   C3 = 1.001

$\lambda(\text{Å})$	σ_{rel}	TIS
1000	33.4704	0.1621
500	33.4811	0.5074
303.8	33.4888	0.8532
284.2	33.4899	0.8884
195.1	33.4957	0.9905
171.1	33.4977	0.9976
131.2	33.5018	1.0000
93.9	33.5070	1.0000

$\lambda = 93.9\,\text{Å}$
$\lambda = 131.2\,\text{Å}$
$\lambda = 171.1\,\text{Å}$
$\lambda = 195.1\,\text{Å}$
$\lambda = 284.2\,\text{Å}$
$\lambda = 303.8\,\text{Å}$
$\lambda = 500\,\text{Å}$
$\lambda = 1000\,\text{Å}$

BRDF

Beta = sin θ

BRDF Predictions from FP1 Metrology Data

PSD Fitting Functions

A1 = 200000   A2 = 3300   A3 = 0.0027
B2 = 0.067   B2 = 25   B3 = 0.0026
              C2 = 1.92   C3 = 1.001

$\lambda(\text{Å})$	σ_{rel}	TIS
1000	33.4704	0.1621
500	33.4811	0.5074
303.8	33.4888	0.8532
284.2	33.4899	0.8884
195.1	33.4957	0.9905
171.1	33.4977	0.9976
131.2	33.5018	1.0000
93.9	33.5070	1.0000

Outline

- **Brief Review of Surface Scatter Theory.**

- **Total Integrated Scatter (TIS) for Moderately Rough Surfaces.**

- **Demonstration of the Generalized Harvey-Shack Scatter Theory.**
  - o **Very Computationally Intense Calculations.**
  - o **Problem: Large dynamic Range of Relevant Spatial Frequencies.**
  - o **Solution: FFTLog Numerical Hankel Transform Algorithm.**

- **Real Measured Metrology data from an EUV Telescope Mirror.**

- **BRDFs from Surface PSDs for increasingly short wavelengths (that violate the smooth surface approximation).**

- **Generalized Peterson Analytical Scattering Model.**
  - o **Dealing with the "Scattered-Scattered" Light.**
  - o **Numerical Validation with ASAP and ZEMAX.**

- **Results and Conclusions.**

Analytic Expression for In-field Scattered Irradiance in Imaging Systems*

Although optical systems are complex, the distribution of scattered light from their elements is not. The halo of scattered light that surrounds a bright source image is merely the sum of the contributions from each element. Furthermore, the scattered-light irradiance distribution from any one element has the form of that element's BSDF, and its magnitude and scale depend only upon the size of the beam that passes through that element.

Most in-field scattered light distributions are obtained by very computationally-intensive calculations; i.e, by tracing millions of rays on a computer. However, the analytic formulas presented in Reference 1 makes all of this unnecessary. In-field scattered light calculations are now accessible to anyone with a pocket calculator, spreadsheet, or mathematics program. In addition, the analytic formulas provide insight and understanding that is totally absent from the conventional brute-force ray-tracing approaches. Design trades can now be performed, and limits on system performance assessed, without the need for complex computer calculations.

Gary Peterson, "Analytic Expressions for In-field Scattered Light Distribution", Proc SPIE 5178-01, 184-193 (2004).
Gary Peterson, *Analytic Expressions for In-field Stray Light Irradiance in Imaging Systems*, Master's Report, OSC/UA (2003). 35

Making use of the Lagrange invariant of 1st-order imaging systems and the brightness theorem (conservation of radiance), the scattered irradiance in the focal plane of an imaging system from the jth element for an in-field point source was derived by Peterson

$$E_{sj}(r) = E_{ent}\, \pi (na)^2\, T\, \frac{s_{ent}^2}{s_j^2}\, BRDF\left((na)\frac{r}{s_j}\right) \tag{16}$$

where r is the distance from the point source image on the detector, na is the numerical aperture of the system, T is the system transmittance, s_{ent} is the radius of the entrance pupil, s_j is the radius of the beam on the jth element, and E_{ent} is the irradiance in the entrance pupil of the system. *This formulation is based upon both a smooth-surface and a paraxial assumption.* For a two-mirror telescope, we can thus write

$$E_s(r) = E_{ent}\, \pi (na)^2\, T\, s_{ent}^2 \left[\frac{BRDF_P\left((na)\, r/s_p\right)}{s_p^2} + \frac{BRDF_s\left((na)\, r/s_s\right)}{s_s^2} \right] \tag{17}$$

Since $s_{ent} = s_p$, $na = \dfrac{1}{2F^\#} = \dfrac{s_p}{f'}$, **and** $P_T = E_{ent}\, \pi\, s_p^2\, T$ $\left(f' = \text{system focal length}\right)$

$$\frac{E_s(r)}{P_T} = \left(\frac{1}{f'}\right)^2 \left[BRDF_P\left(r/f'\right) + \left(\frac{s_p}{s_s}\right)^2 BRDF_s\left((s_p/s_s)(r/f')\right) \right] \tag{18}$$

＊ **G. Peterson, "Analytic expressions for in-field scattered light distributions", Proc SPIE 5178 (2004).**
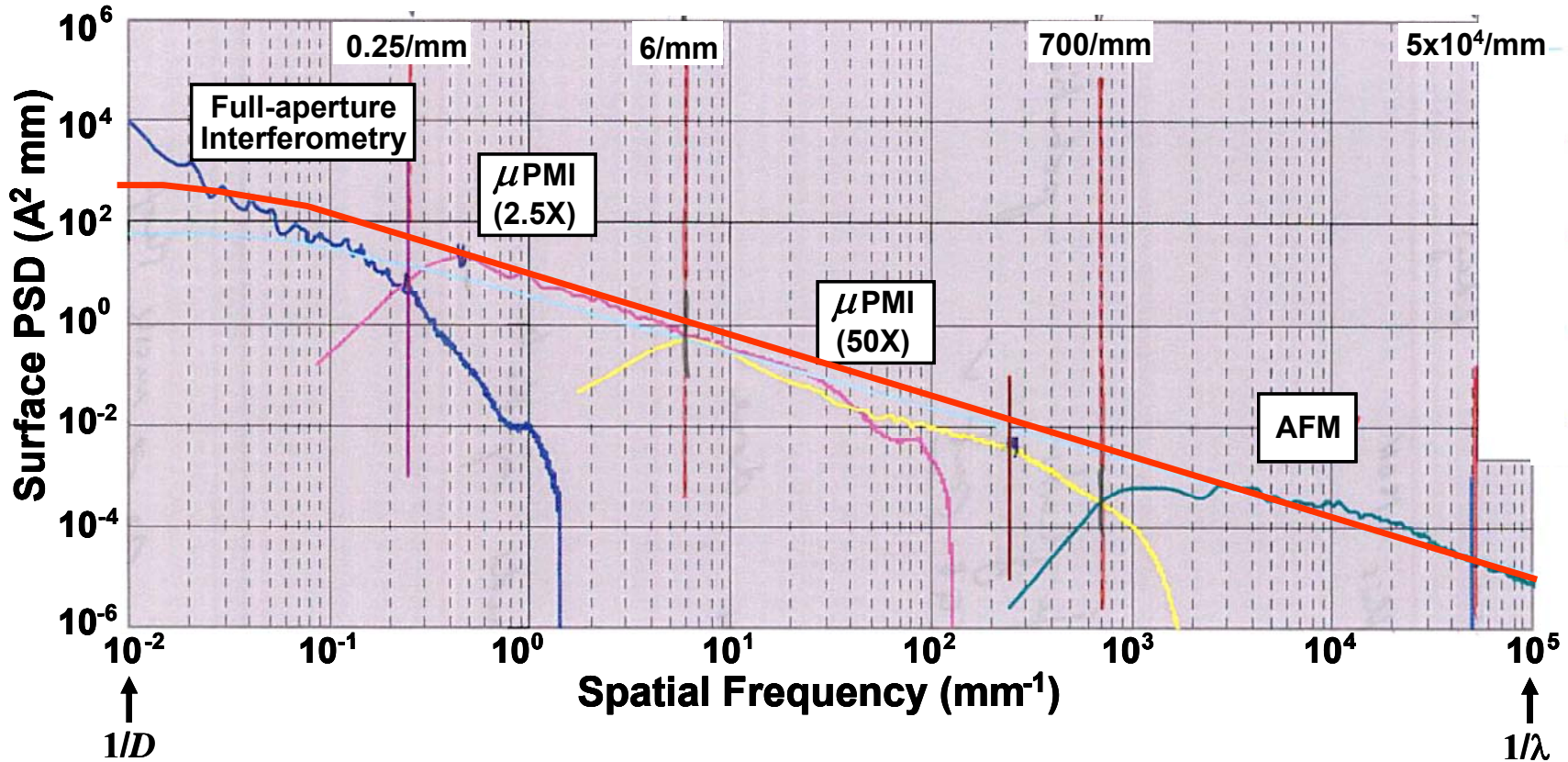
Generalized Peterson Analytical Scattering Model

Since Peterson's elegant and insightful treatment is limited by both a *paraxial* and a *smooth-surface* assumption, it must be generalized to include scattering from moderately rough surfaces before applying to the NOAA Solar UV Imager (SUVI) Program. We have thus:

- Removed the "smooth-surface" limitation by including "scattered-scattered" radiation from the two-mirror SUVI telescope.

- Verified that the SUVI application is indeed paraxial.

- The simple analytical model has then been numerically validated by comparing the results with the very computationally-intensive commercially-available ZEMAX and ASAP codes.

Gary Peterson, "Analytic Expressions for In-field Scattered Light Distribution", Proc SPIE 5178-01, 184-193 (2004).
Gary Peterson, *Analytic Expressions for In-field Stray Light Irradiance in Imaging Systems*, Master's Report, OSC/UA (2003). 37

The SUVI Spec Surface PSD
(Scattered-Scattered Light will be Very Substantial)

PSD Fitting Function

$$PSD(f_x)_{1-D} = \frac{A}{\left[1 + \left(B\, f_x\right)^2\right]^{C/2}}$$

A1 = 610.322 $Å^2 mm$

B1 = 120 mm^{-1}

C1 = 1.089

$\lambda(Å)$	σ_{rel}	TIS
1000	6.5698	0.0068
500	6.6487	0.0275
303.8	6.7020	0.0740
284.2	6.7089	0.0842
195.1	6.7470	0.1721
171.1	6.7600	0.2185
131.2	6.7857	0.3445
93.9	6.8171	0.5650

Scattering in a Two-mirror EUV Telescope

For a solar EUV telescope surface scatter from the primary and secondary mirrors sometimes dominates both geometrical aberrations and diffraction effects in the degradation of image quality.

Direct (A_p)

Scattered (B_p)

direct scattered (A_pB_s)

direct direct (A_pA_s)

scattered scattered (B_pB_s) scattered direct (B_pA_s)

Classic PSF
(Image Core and Scattered Halo)
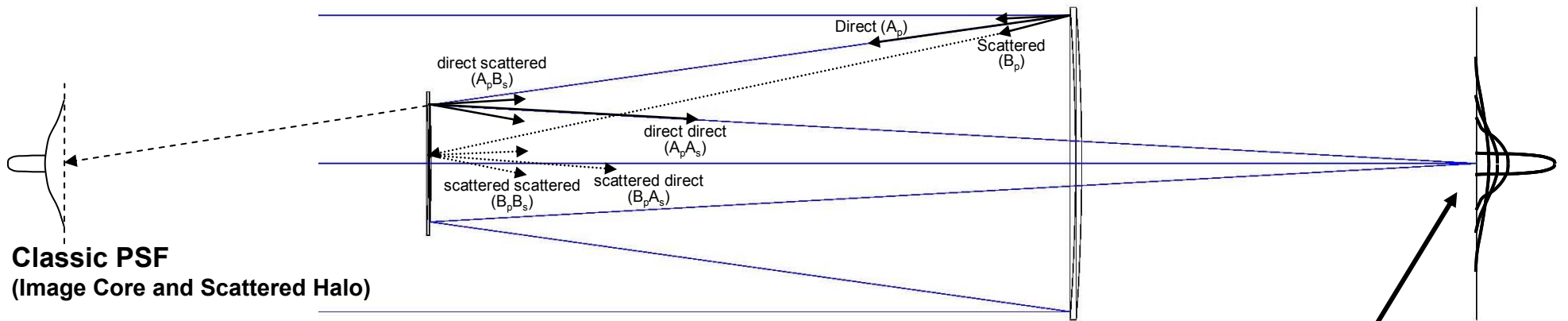
$$A = \exp\left[-\left(4\pi\sigma_{rel}/\lambda\right)^2\right] = \text{Fraction of total reflected energy in specular beam.}$$

$$B = TIS = 1 - A = \text{Fraction of total reflected energy in scattered halo.}$$

(19)

The SUVI point spread function (PSF) consists of four components with an energy distribution given by:

Direct-direct component (Specular) — $A_p\ A_s$

Scattered-direct component — $B_p\ A_s$

Direct-scattered component — $A_p\ B_s$

Scattered-scattered component — $B_p\ B_s$

Energy Distribution between PSF Components

The radiant energy distribution between the four components of the PSF is shown below as a function of σ/λ. The σ is the relevant rms roughness (PSD integrated from $f_{min} < f < 1/\lambda$). Note that for $\sigma/\lambda > 0.066$, the broad scattered-scattered component becomes dominant.
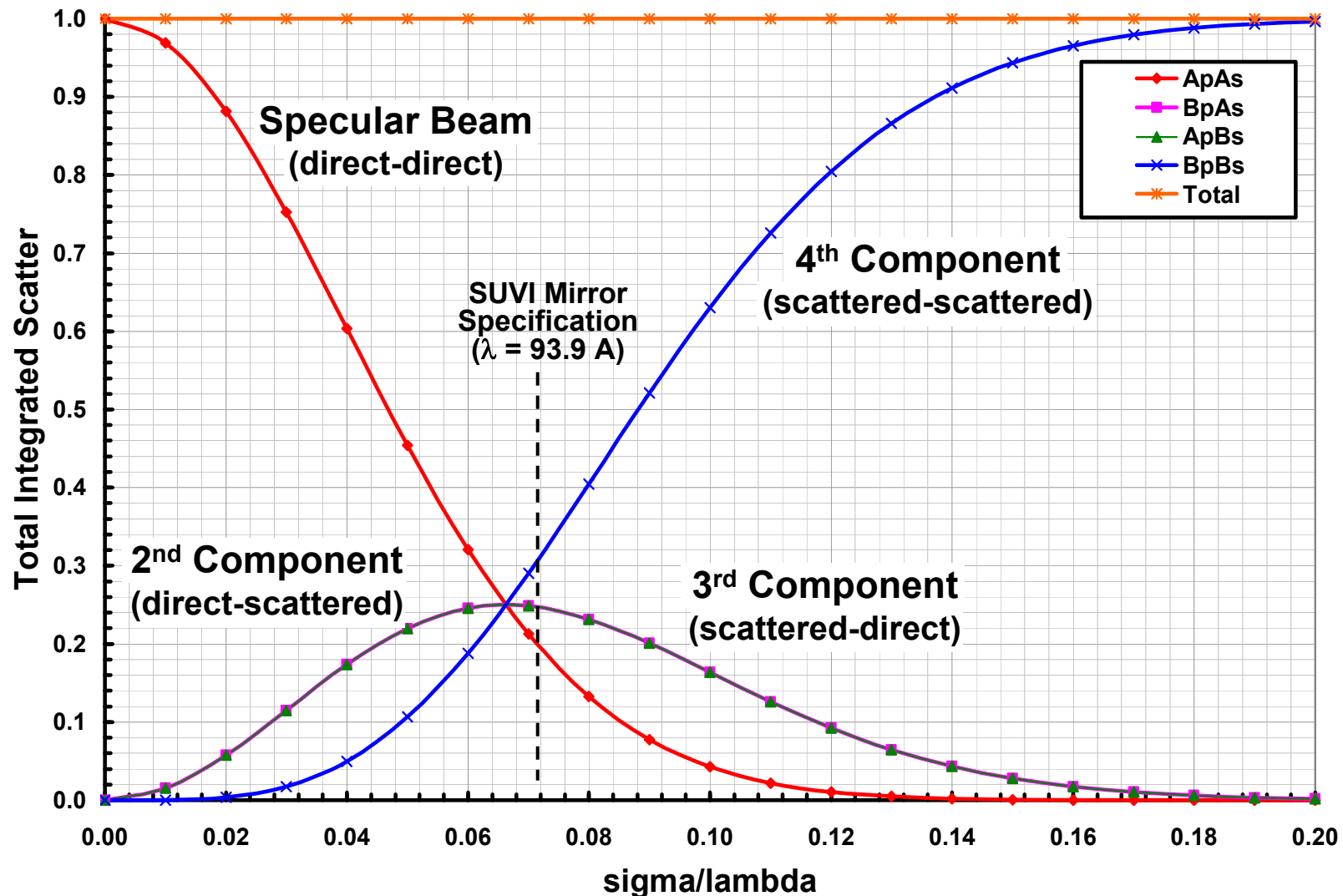
Including the Scattered-Scattered Light

Since most EUV applications clearly do not satisfy the smooth surface assumption, but are perceived to satisfy the paraxial limitation, we merely construct an expression for each of the four components making up the PSF in the focal plane of the telescope, and substitute it into Eq.(18) of Peterson's analytic treatment

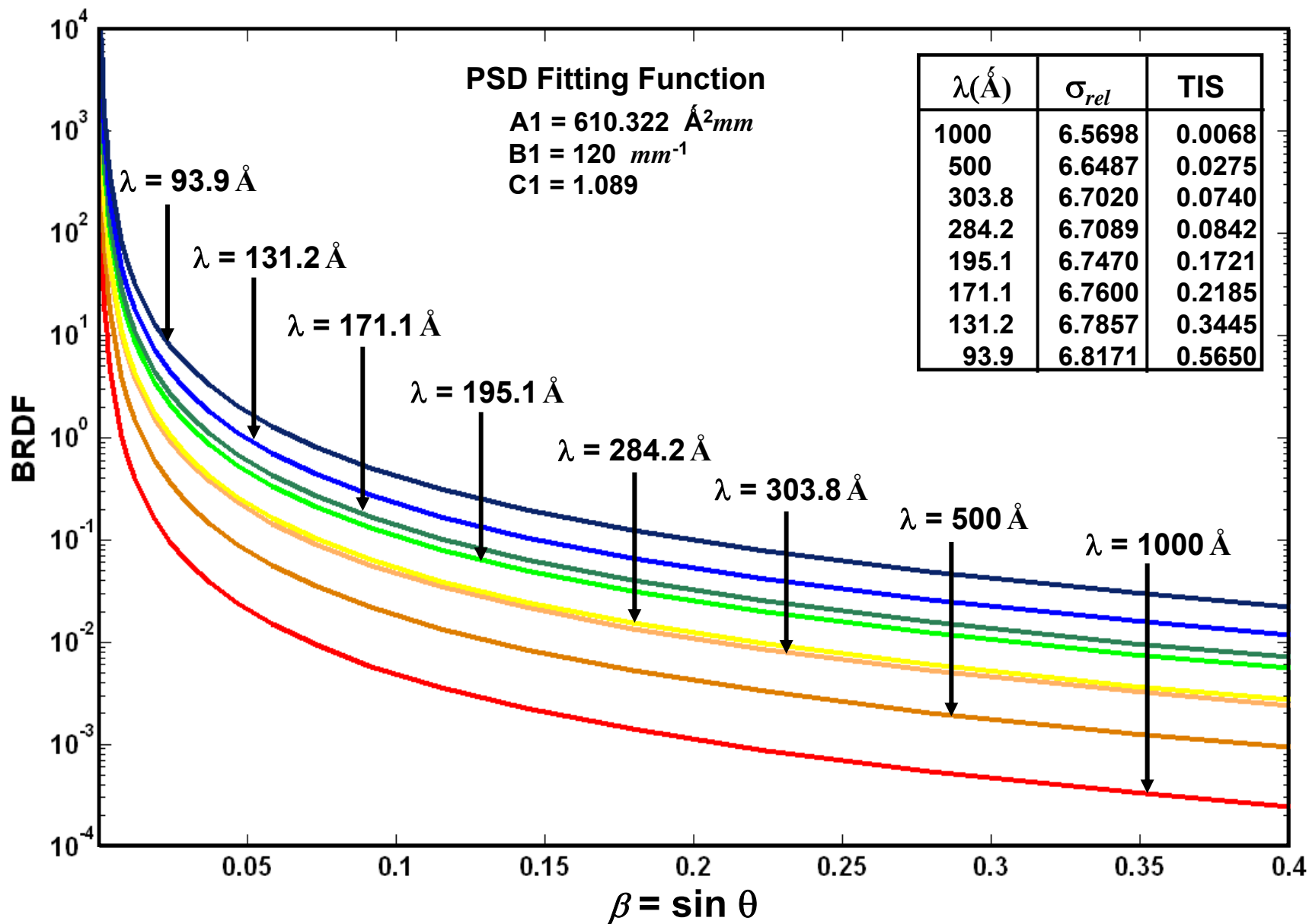$$PSF \ = \ PSF_{dd} + PSF_{sd} + PSF_{ds} + PSF_{ss} \qquad \text{(20)}$$

Care is taken to normalize each component such that their respective volumes (fractional total reflected radiant power) will be given by A_pA_s, B_pA_s, A_pB_s, and B_pB_s.

We will assume a 175 cm focal length Ritchey-Chretien telescope design with an aperture diameter of 19 cm and an obscuration ratio of $\varepsilon = 0.4$. There will thus be no geometrical aberrations on-axis; and the specular beam will be the well-known Fraunhofer diffraction pattern produced by the annular aperture of the telescope

$$PSF_{dd}(r) = \frac{1}{\left(1-\varepsilon^2\right)^2}\left[\frac{2J_1(x)}{(x)} - \varepsilon^2\frac{2J_1(\varepsilon x)}{\varepsilon x}\right]^2 \quad \textbf{where} \quad x = \frac{\pi r}{\lambda f / D}. \qquad \text{(21)}$$
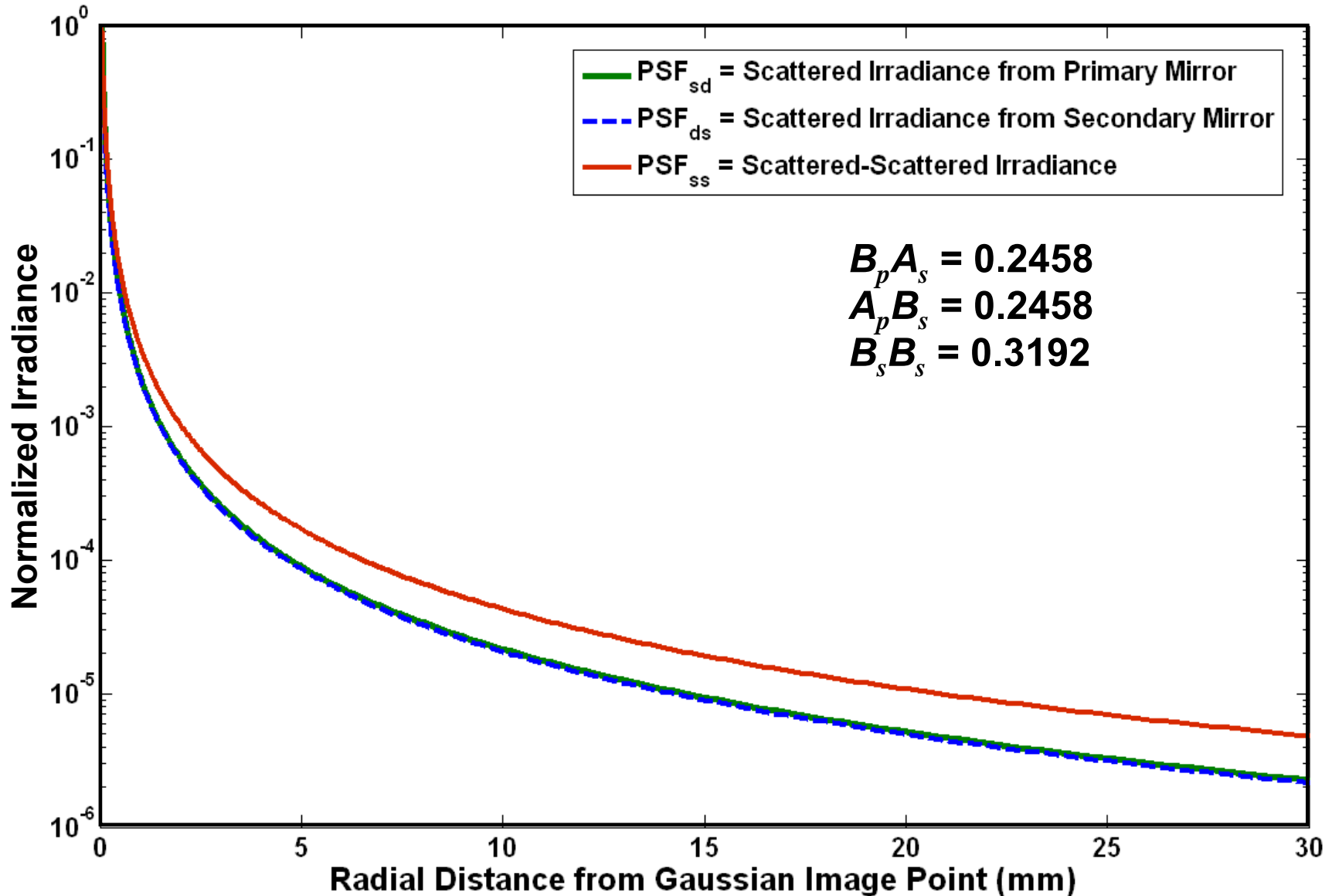
The above expression is normalized to a unit volume. It will thus need to be multiplied by the coefficient A_pA_s in the following analysis.

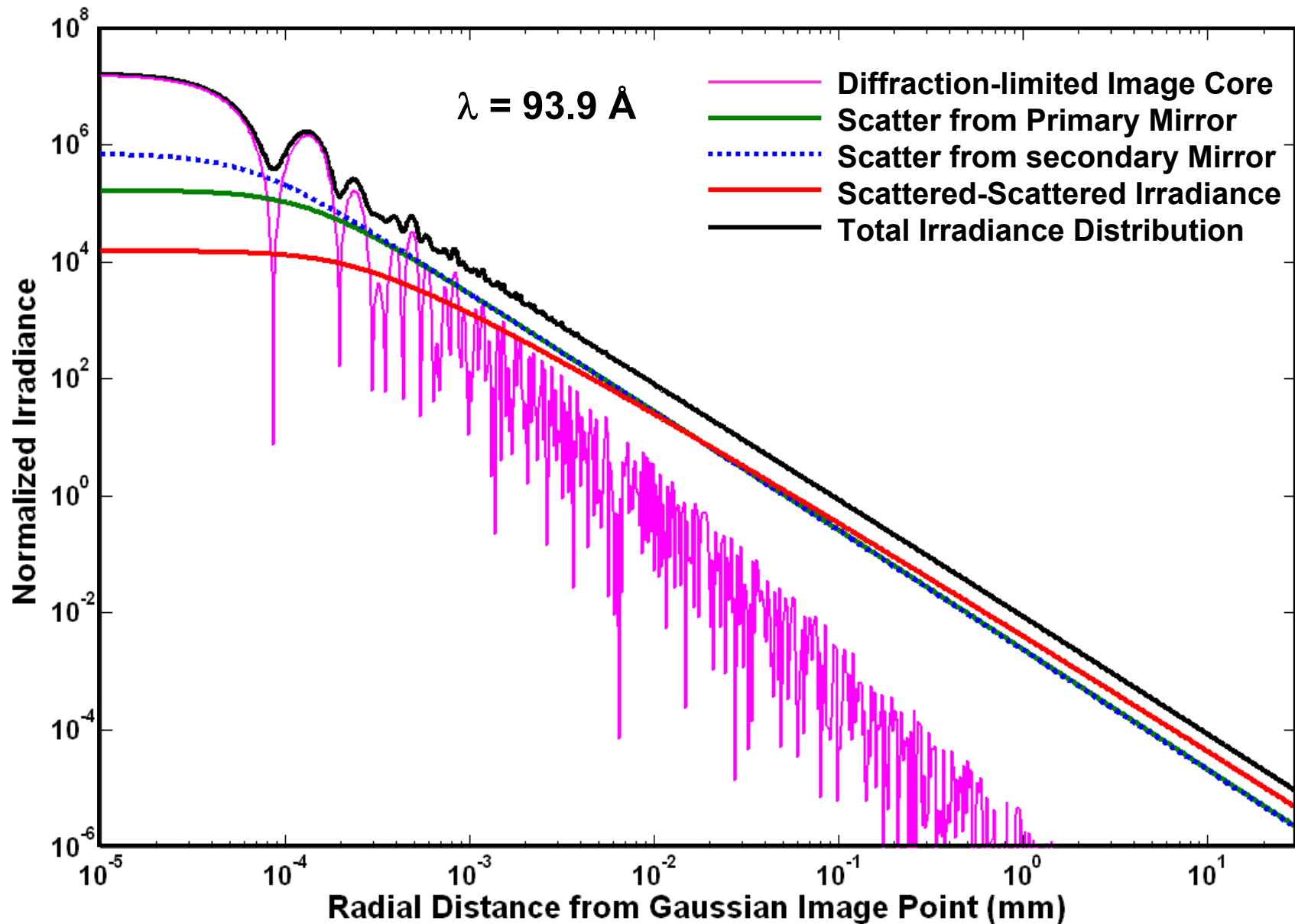# BRDF Profiles Calculated from SPEC PSD with the GHS Scattering Theory

PSD Fitting Function

A1 = 610.322 Å²mm
B1 = 120 mm⁻¹
C1 = 1.089

λ(Å)	σ_{rel}	TIS
1000	6.5698	0.0068
500	6.6487	0.0275
303.8	6.7020	0.0740
284.2	6.7089	0.0842
195.1	6.7470	0.1721
171.1	6.7600	0.2185
131.2	6.7857	0.3445
93.9	6.8171	0.5650

λ = 93.9 Å
λ = 131.2 Å
λ = 171.1 Å
λ = 195.1 Å
λ = 284.2 Å
λ = 303.8 Å
λ = 500 Å
λ = 1000 Å

BRDF

β = sin θ

Scattered-Scattered Light Dominates
(λ = 93.9 A)

PSF$_{sd}$ = Scattered Irradiance from Primary Mirror
PSF$_{ds}$ = Scattered Irradiance from Secondary Mirror
PSF$_{ss}$ = Scattered-Scattered Irradiance
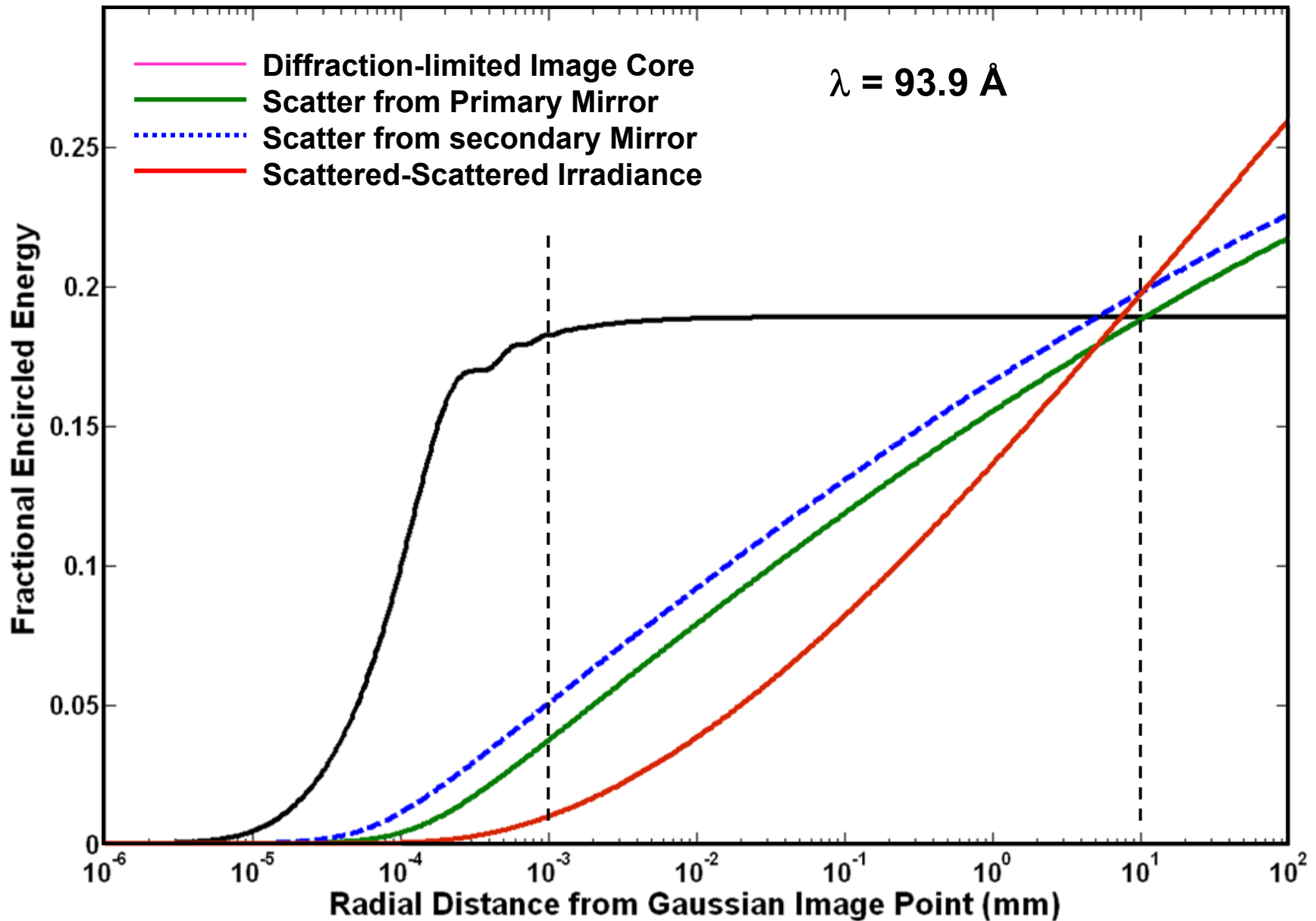
B_pA_s = 0.2458
A_pB_s = 0.2458
B_sB_s = 0.3192

Radial Profiles of Four Components
(λ = 93 A)

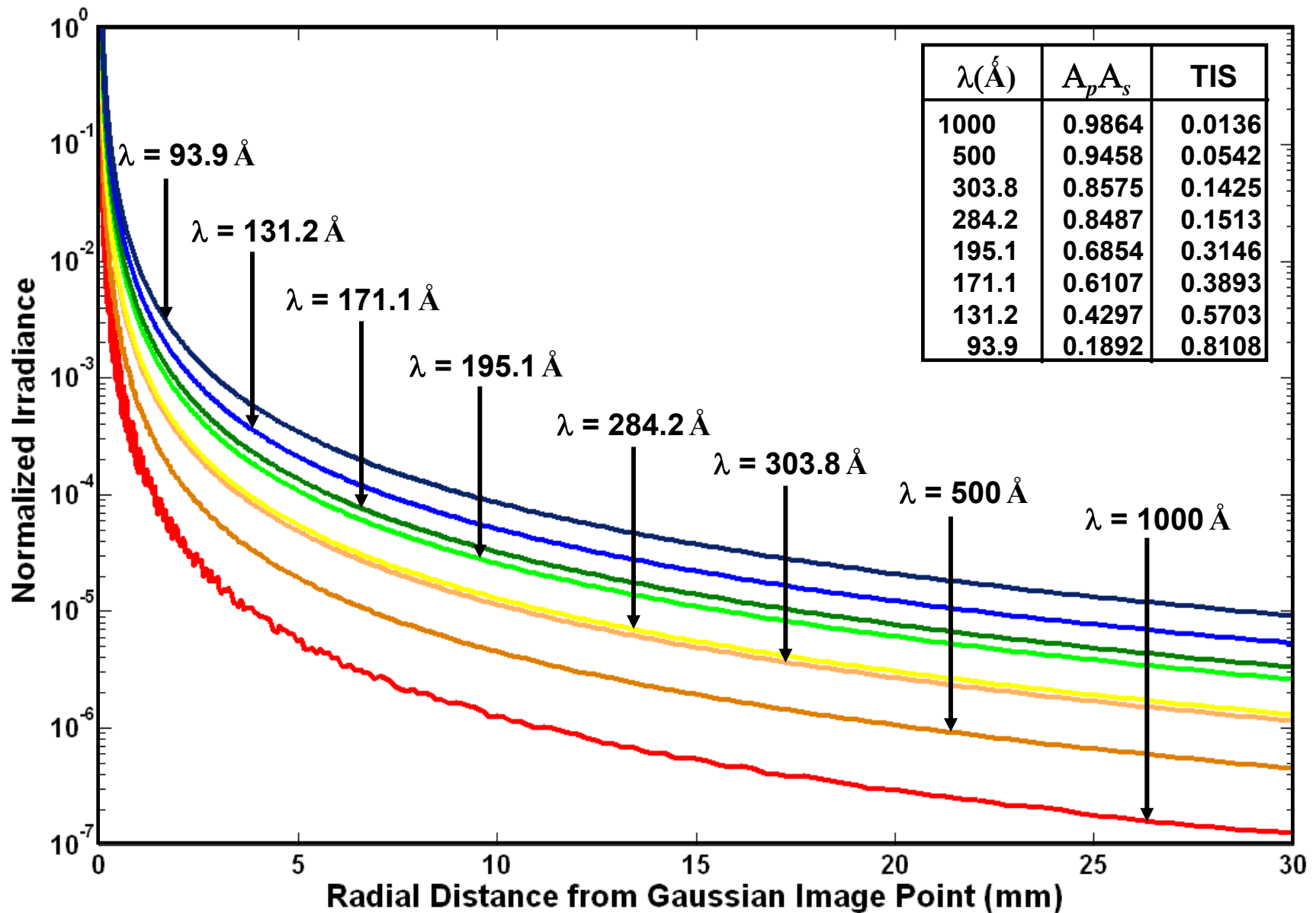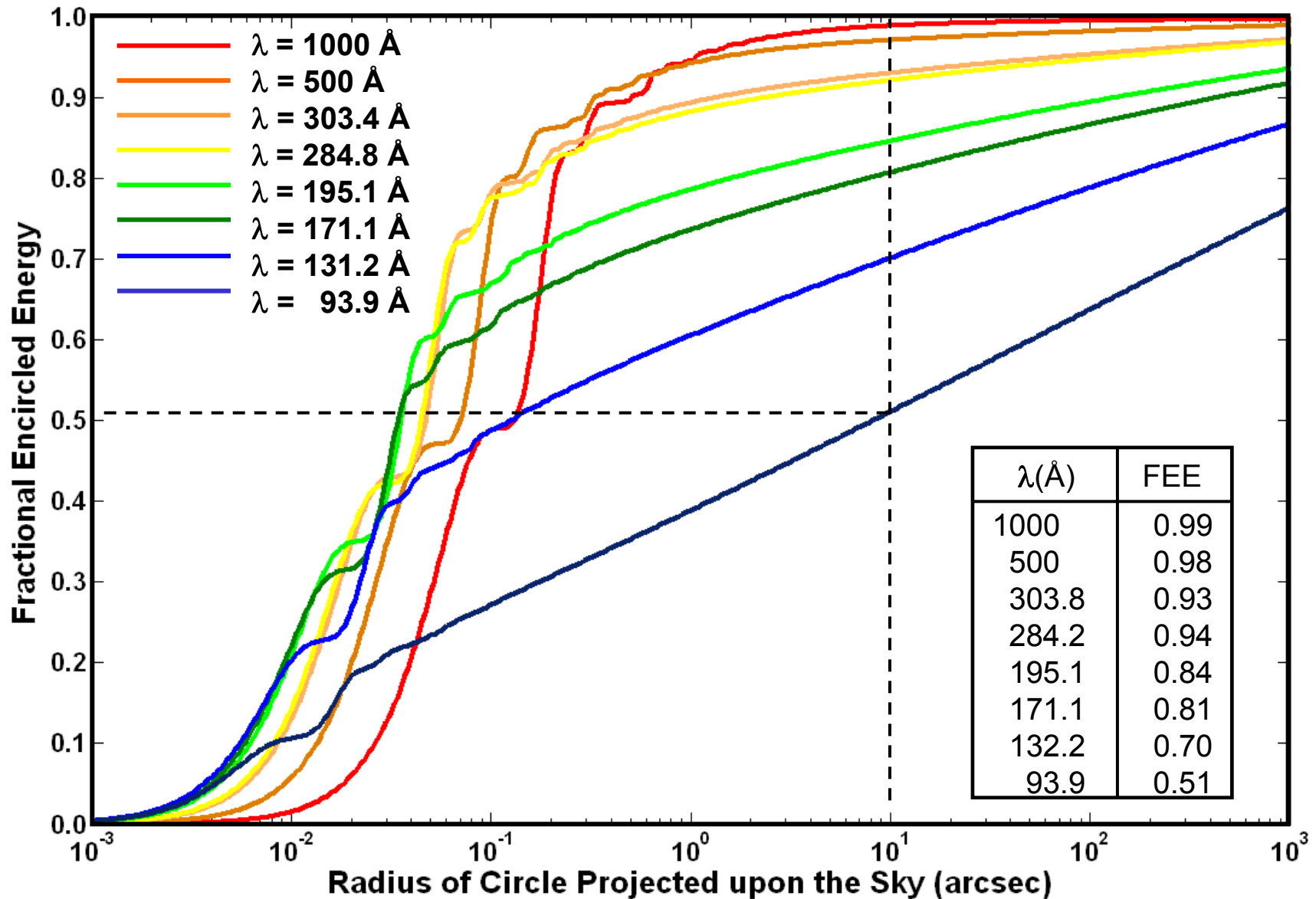# FEE Plots of the 4 Components of PSF
(λ = 93.9 A)

Irradiance Profile in Telescope Focal Plane
(Predicted by Generalized Peterson Model)
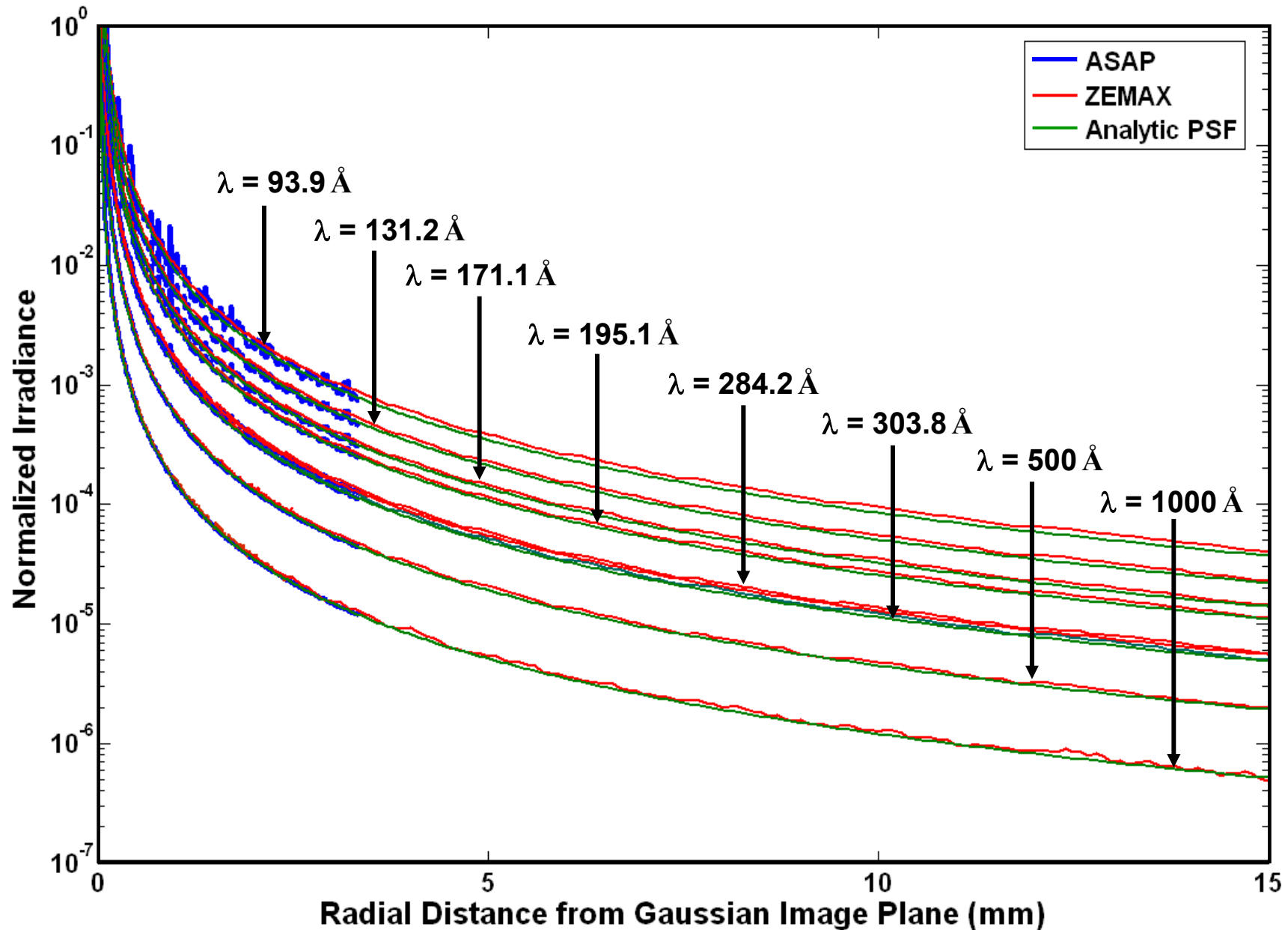
FEE Plots of the Total PSF Projected onto Sky

SUVI Image Quality Requirements
(Fractional Ensquared Energy: Expressed as %)

Square Size (arcsec)	Wavelength					
	93.9	131.2	171.1	195.1	284.2	303.8
7x7	43	50	50	50	50	50
10x10	49	53	59	60	60	60
20x20	57	61	65	65	65	65
40x40	67	69	70	70	70	70
65x65	72	75	75	75	75	75
150x150	78	82	84	85	85	85

Numerical Validation by ASAP and ZEMAX

Summary, Results and Conclusions

- **Stated a Need for Calculating Image Degradation from Measured Metrology Data.**

- **Review a Generalized Surface Scatter (GHS) Theory valid for Rough Surfaces at Large Incident and Scattered Angles.**

- **Discussed Computational Problems for Surface PSDs with Large Dynamic Range in Spatial Frequency.**

- **Introduced the FFTLog Algorithm as a Solution to the computational Problem.**

- **Demonstrated BRDFs Calculated from Surface PSDs for increasingly short wavelengths (which violate the smooth-surface approximation).**

- **Generalized the Peterson Analytical Model for Calculating Image Degradation to include surface scatter from rough surfaces.**

- **Demonstrated a variety of useful parametric performance predictions provided by the Generalized Peterson Analytical Model.**

- **Numerically validated the Generalized the Peterson Analytical Model with both ASAP and ZEMAX.**