

Predicting Image Degradation from Optical Surface Metrology Data

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Abstract: A generalization of Peterson's elegant surface scatter model results in an improved capability to predict image degradation from optical surface metrology data. A new unified surface scatter theory for moderately rough surfaces provides the necessary BSDF data.

OCIS codes: Scattering (290.0290), BSDF, BRDF, and BTDF (290.1483), Scattering In-field (290.5838).

1. Introduction:

Image degradation due to scattered radiation is a serious problem in many short wavelength (X-ray/EUV) imaging systems. Most currently-available image analysis codes require the scatter behavior (BSDF data) as input in order to calculate the image quality from such systems. Predicting image degradation due to scattering effects is typically very computationally intensive. If using a conventional optical design and analysis code, each geometrically traced ray spawns hundreds of scattered rays randomly distributed and weighted according to the input BSDF. These scattered rays must then be traced through the system to the focal plane using non-sequential ray-tracing techniques. For multi-element imaging systems even the scattered rays spawn more scattered rays at each additional surface encountered in the system. In this paper we describe a generalization of Peterson's elegant analytical treatment of in-field stray light for multi-element imaging systems. In particular, we remove the smooth-surface limitation that ignores the scattered-scattered radiation which can be quite large for EUV wavelengths even for state-of-the-art optical surfaces. The resulting analytical treatment is then numerically validated by both ZEMAX and ASAP for the case of a two-mirror EUV telescope.

2. Peterson's Analytical Treatment of in-field Stray Light in Multi-element Imaging Systems:

Making use of the Lagrange invariant of 1st-order imaging theory and the brightness theorem (conservation of radiance), the scattered irradiance in the focal plane of an imaging system from the j^{th} element for an in-field point source has been shown by Peterson¹⁻² to be given by

$$E_{sj}(r) = E_{ent} \pi (na)^2 T \frac{s_{ent}^2}{s_j^2} BSDF \left((na) \frac{r}{s_j} \right) \quad (1)$$

where r is the distance from the point source image on the detector, na is the numerical aperture of the system, T is the system transmittance, s_{ent} is the radius of the entrance pupil, s_j is the radius of the beam on the j^{th} element, and E_{ent} is the irradiance in the entrance pupil of the system. *This formulation is based upon both a smooth-surface and a paraxial assumption.* For a two-mirror telescope, we can thus write

$$E_s(r) = E_{ent} \pi (na)^2 T s_{ent}^2 \left[\frac{BSDF_p((na) r / s_p)}{s_p^2} + \frac{BSDF_s((na) r / s_s)}{s_s^2} \right]. \quad (2)$$

Since $s_{ent} = s_p$, $na = 1/(2F^\#) = s_p / f'$ (f' = system focal length), and the total radiant power reaching the focal plane is given by $P_T = E_{ent} \pi s_p^2 T$, the scattered irradiance in the telescope focal plane normalized by the total radiant power is given by

$$\frac{E_s(r)}{P_T} = \left(\frac{1}{f'} \right)^2 \left[BSDF_p(r / f') + \left(\frac{s_p}{s_s} \right)^2 BSDF_s((s_p / s_s)(r / f')) \right]. \quad (3)$$

3. Generalization of Analytic Expression for Rough Surfaces:

The fraction of the total reflected power remaining in the specular beam after reflection from a single moderately rough surface, and the corresponding total integrated scatter (TIS), are given by³⁻⁹

$$A = \exp \left[- (4\pi \cos \theta_i \sigma_{rel} / \lambda)^2 \right] \quad \text{and} \quad B = TIS = 1 - A = 1 - \exp \left[- (4\pi \cos \theta_i \sigma_{rel} / \lambda)^2 \right]. \quad (4)$$

where θ_i is the angle of incidence and σ_{rel} is the rms surface roughness measured over the entire range of relevant spatial frequencies (spatial frequencies greater than $1/\lambda$ do not contribute to the scattered radiation).¹⁰ Since Eq.(4) is so important to the following discussion, we bring to the attention of the reader that a brief historical perspective of these equations is presented on page 51 of Reference 6. Our *relevant* rms surface roughness, σ_{rel} , is the same as the *effective* rms surface roughness referred to by Dittman¹¹ and by Church and Takacs.¹² The square of this relevant rms roughness is thus equal to the band-limited integral of the two-dimensional surface power spectral density (PSD) function integrated out to a spatial frequency of $1/\lambda$, whereas the square of the total, or intrinsic, rms roughness is obtained by integrating the two-dimensional surface PSD from zero to infinity. It should be noted that for two-dimensional surface PSDs exhibiting an inverse power law behavior, the total, or intrinsic, rms roughness will be infinite if the magnitude of the slope characterizing the power law behavior is less than 2. However, the effective, or relevant rms roughness will always be finite.

For a two-mirror telescope we will have a specular (direct) and a scattered component reflected from the primary mirror. After reflection from the secondary mirror there will be a diminished specular beam (direct-direct component), the scattering function from the primary mirror specularly reflected from the secondary mirror (scattered-direct component), the specularly reflected beam from the primary mirror scattered from the secondary mirror (direct-scattered component) and the scattered radiation from the primary mirror scattered again from the secondary mirror (scattered-scattered component) propagating towards the telescope focal plane as shown in Figure 1.

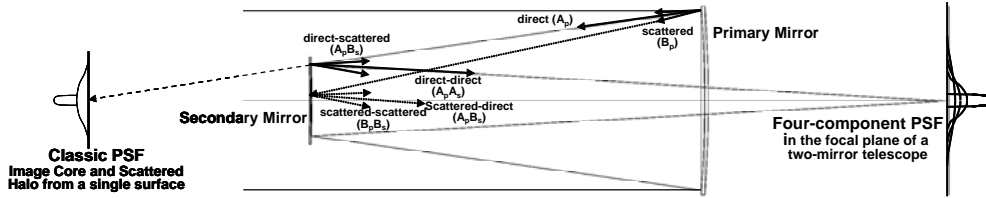


Figure 1: Illustration of scattering in a two-mirror telescope.

The point spread function (PSF) in the focal plane of the telescope will thus consist of the sum of four components whose radiant power distribution is listed below

$$\text{Direct-direct component (Specular): } P_{dd}/P_T = A_p A_s \quad (5)$$

$$\text{Scattered-direct component: } P_{sd}/P_T = B_p A_s \quad (6)$$

$$\text{Direct-scattered component: } P_{ds}/P_T = A_p B_s \quad (7)$$

$$\text{Scattered-scattered component: } P_{ss}/P_T = B_p B_s \quad (8)$$

The quantities A_p , B_p , A_s and B_s are determined from Eq.(4). Figure 2 graphically illustrates the radiant power distribution between these four components of the PSF of a two-mirror telescope as a function of the rms roughness of the mirrors expressed in wavelengths (σ/λ).

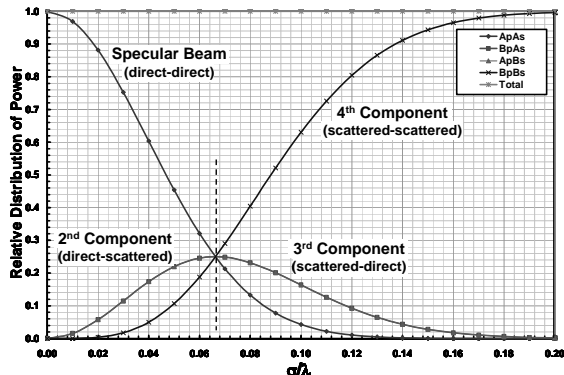


Figure 2: Energy distribution between the four PSF components.

Note that the TIS of the two-mirror telescope is equal to $1 - A_p A_s$. It is evident from Figure 2 that for $\sigma/\lambda < 0.02$ scattering effects are modest, with a TIS < 0.12 . However, as σ/λ increases, the scattered light increases rapidly. At $\sigma/\lambda = 0.066$ each of the four components contain 25% of the total power. As σ/λ continues to increase, the power in the scattered-scattered component increases and the power in all other components decreases. For $\sigma/\lambda > 0.12$ the specular beam has essentially vanished, and for $\sigma/\lambda > 0.18$ virtually all of the radiant power is in the scattered-scattered component.

For some short wavelength applications, such as solar EUV telescopes, surface scatter from state-of-the-art primary and secondary mirrors will dominate both geometrical aberrations and diffraction effects in the degradation of image quality. We will thus generalize the Peterson analytical treatment (i.e.; remove the smooth-surface limitation) by accurately calculating and adding the effects of the scattered-scattered component to the PSF in the focal plane of the telescope. Assuming isotropic roughness on both the primary and secondary mirrors, we thus construct the following expression

$$PSF(r) = PSF_{dd}(r) + PSF_{sd}(r) + PSF_{ds}(r) + PSF_{ss}(r). \quad (9)$$

The first term on the right side of Eq.(9) will be given the functional form of the image core, or specular beam, as determined by diffraction and geometrical aberrations. The two middle terms will be given the functional form provided by Peterson's analytical expression from Eq.(3). The functional form of the scattered-scattered term will be obtained by convolving the two middle terms. In general, this will be done by numerically calculating the Hankel transform of the product of the Hankel transforms of the BSDF's provided for the two mirrors. Finally, care will be taken to normalize each component of the PSF such that their respective two-dimensional integrals (fractional total reflected radiant power) will be equal to $A_p A_s$, $B_p A_s$, $A_p B_s$ and $B_p B_s$.

4. Application to a Two-mirror EUV Telescope

We assumed a 175 cm focal length Ritchey-Chretien telescope design with an aperture diameter of 19 cm and an obscuration ratio of $\varepsilon = 0.4$. There will thus be no geometrical aberrations on-axis; and the specular beam will be the well-known Fraunhofer diffraction pattern produced by the annular aperture of the telescope, normalized to have a volume of $A_p A_s$.

Figure 3 illustrates an ABC, or K-correlation, function fit to actual metrology data from a state-of-the-art EUV telescope mirror. Four separate metrology instruments were used to measure the optical fabrication errors over the entire range of relevant spatial frequencies. The BSDFs produced at eight different wavelengths from this surface PSD were calculated with the generalized Harvey-Shack (GHS) surface scatter theory that is valid for moderately rough surfaces.¹⁰ Figure 4. shows the resulting irradiance distribution in the focal plane of the above telescope as predicted from Eq.(9) when these BSDFs were sampled and substituted into Peterson's analytical model via Eq.(3). Superposed upon these predictions are the corresponding predictions from both ZEMAX and ASAP.

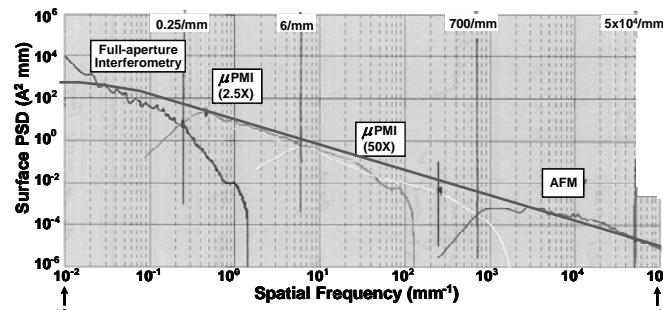


Figure 3. The surface PSD, an ABC, or K-correlation function, fit to data from four different metrology instruments characterizes the surface over the entire range of relevant spatial frequencies.

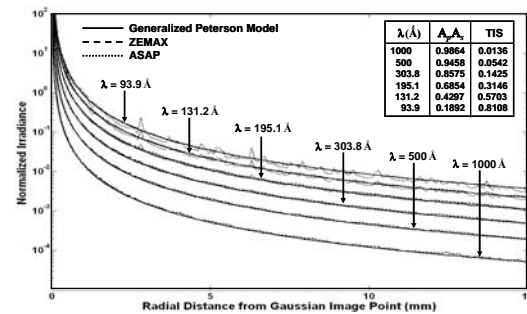


Figure 4. Comparison of the irradiance in the focal plane of the two-mirror EUV telescope as predicted by the generalized Peterson analytical model, ZEMAX and ASAP.

3. Summary and Conclusion: We have demonstrated that a generalization of Peterson's analytic approach to calculating the irradiance distribution in the focal plane of a multi-element imaging system allows one to make accurate image quality predictions even for moderately rough surfaces which do not satisfy the usual smooth-surface requirement. And we have validated that simple analytical approach to making image quality predictions with the much more computationally-intensive calculations provided by the well-known ZEMAX and ASAP codes.

6.0 References

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