# Oblique Airy wave packets in bidispersive optical media

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Using hyperbolic rotations, we show that a new class of skewed, nonspreading, accelerating Airy wave packets is possible in optical bidispersive systems. Their obliquity factor is found to have a profound effect on their spatiotemporal acceleration dynamics. Pertinent examples are provided. © 2010 Optical Society of America
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Diffraction and dispersion occur ubiquitously in nature. As is well known in optics, these processes lead to wave packet broadening in space and time, respectively [1]. In fact, over the years, strategies have been developed to contain and manage these effects that, from a practical perspective, are typically considered undesirable [2–10]. One such promising avenue involves the use of nonspreading spatiotemporal solutions like O-waves [7] and X-waves [3,7,8], to mention a few. Also of great interest is the recently demonstrated class of Airy wave packets [11,12]. Apart from the fact that they are the only nondispersing, localized solutions in one dimension, Airy waves can also exhibit several intriguing characteristics [12–17]. Perhaps the most prominent of them is their very ability to freely accelerate in both space and time. While spatial diffraction effects in standard media act the same way in both directions, dispersion on the other hand may affect wave dynamics in different ways, depending on whether it is normal or anomalous. Under normal dispersive conditions, the space–time operators exhibit opposite signs and, hence, the system acts in a bidispersive fashion [8]. Bidispersion is an intriguing process that occurs frequently in many common physical systems, and it leaves its mark in both the linear and nonlinear domain.

In this Letter, we show that a new class of nonspreading Airy wave packets is possible under bidispersive conditions. What distinguishes these waves from other families considered in previous studies is the fact that they owe their existence to hyperbolic rotations (a special class of Lorentz transformations) allowed only in the presence of bidispersion. The effect of the obliquity factor on their acceleration dynamics is studied in detail. Pertinent examples are provided in standard material systems to illustrate these effects.

Bidispersive systems can be realized in space–time configurations. For example, the propagation of wave packets in bulk dispersive media can be described by the $(3+1)D$ paraxial wave equation $i\psi_z + (1/2k)\nabla_\perp^2\psi - (k''/2)\psi_{\tau\tau} = 0$ [8]. Here, the process of diffraction is being described by the transverse Laplacian operator, dispersive effects are associated with the term involving the moving time coordinate $\tau = t - (z/v_g)$, $k = 2\pi n/\lambda$ represents the wavenumber, and $k'' = \partial^2 k/\partial\omega^2$ is the dispersion coefficient at the carrier frequency $\omega_0$ [4]. The system will be bidispersive if the material exhibits normal dispersion, i.e., $k'' > 0$.

To demonstrate this new class of oblique Airy solutions, let us first consider a $(2+1)D$ bidispersive optical paraxial system. This situation can arise in normally dispersive planar waveguides where diffraction is one dimensional. By equalizing dispersion and diffraction effects $(\tau_0^2/|k''| = kw_0^2)$, the spatiotemporal evolution of the wave packet will be described by [8]

$$i\frac{\partial\psi}{\partial Z} + \frac{\partial^2\psi}{\partial X^2} - \frac{\partial^2\psi}{\partial T^2} = 0, \qquad (1)$$

where the normalized coordinates are related to the actual ones via $Z = z/(2kw_0^2)$, $X = x/w_0$, and $T = \tau/\tau_0$, respectively. A direct calculation reveals that Eq. (1) allows the following exact Airy wave solution:

$$\psi(X,T,Z) = \mathrm{Ai}\left[\frac{\xi(X,T)}{\sqrt{2}} - \frac{Z^2}{4}\right]\mathrm{Ai}\left[\frac{\vartheta(X,T)}{\sqrt{2}} - \frac{Z^2}{4}\right] \times \exp\left[\frac{i}{2\sqrt{2}}Z[\xi(X,T) - \vartheta(X,T)]\right]. \qquad (2)$$

In Eq. (2), $\mathrm{Ai}(\cdot)$ represents the Airy function while the new oblique axes $\xi, \vartheta$ are defined through the Lorentz transformation $H(\phi)$: $\xi(X,T) = X\cosh\phi + T\sinh\phi$, $\vartheta(X,T) = X\sinh\phi + T\cosh\phi$, where $\phi$ denotes an obliquity factor. It is well known that the choice of orientation of a coordinate system in media not exhibiting bidispersion is rather arbitrary, while this is not true for bidispersive systems. In these media, one deals with two distinct axes, which lead to a preferential orientation of the frame of reference and, hence, do not allow arbitrary coordinate rotations. One can show that, for every existing solution $\psi(X,T,Z)$, the hyperbolically rotated version $H(\phi)\psi(X,T,Z)$ is a also solution to Eq. (1). Note that the new coordinates $(\xi,\vartheta)$ are no longer mutually orthogonal, but instead they intersect at an angle $\theta$, which is related to the obliquity factor $\phi$ through the relation $\phi = -(1/2)\tanh^{-1}(\cos\theta)$. The solution presented in Eq. (2) and the associated transformation from $(X,T)$ to $(\xi,\vartheta)$ lead to a new family of nonspreading oblique Airy wave packets whose propagation dynamics depend critically on the angle $\theta$. Intensity profiles of such skewed,

 

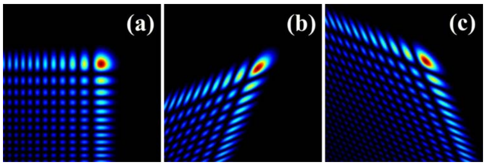


Fig. 1. (Color online) Airy beam intensity profiles for (a) $\theta = 90°$, (b) $\theta = 45°$, and (c) $\theta = 135°$.

nondiffracting Airy waves are shown in Fig. 1 for various obliquity factors or angles $\theta$. Equation (2) indicates that these solutions accelerate along the 45° axis in the $(X, T)$ frame (or the angle-bisector of the oblique $(\xi, \vartheta)$ system) and that the acceleration of the main lobe is described by $X_d = T_d = (e^{-\phi}\sqrt{2}/4)Z^2$. This latter result suggests that the acceleration of this wave can be directly controlled by using the obliquity factor $\phi$ as a free parameter. In other words, for $\phi < 0$ ($\theta$ being an acute angle), the wave will experience enhanced self bending, whereas for $\phi > 0$ ($\theta$ being obtuse) the acceleration will strongly decrease. When the obliquity factor $\phi$ is equal to zero, i.e., when the $\xi$ and $\vartheta$ axes are perpendicular to each other ($\theta = 90°$), the beam profile becomes identical to that of [11,12].

So far, we have only considered infinite-energy oblique Airy wave packets in bidispersive systems. As previously discussed in [12], experimental realization of such beams would demand amplitude truncation. Perhaps the most convenient way to do so is the use of an exponential field containment at $Z = 0$, viz.,

$$\psi(X, T, Z=0) = \mathrm{Ai}\left[\frac{\xi(X,T)}{\sqrt{2}}\right]\mathrm{Ai}\left[\frac{\vartheta(X,T)}{\sqrt{2}}\right] \times \exp\left[(\alpha_1 + i\beta_1)\frac{\xi(X,T)}{\sqrt{2}}\right] \times \exp\left[(\alpha_2 + i\beta_2)\frac{\vartheta(X,T)}{\sqrt{2}}\right]. \quad (3)$$

Here, $\alpha_{1,2} > 0$ are typically small positive constants that confine the field envelope and lead to finite-energy power spectra, while $\beta_{1,2}$ are real parameters related to the initial angle tilt of the optical wavefronts and can lead to a wide range of ballistic trajectories. The propagation dynamics of these finite-energy waves are analytically given by

$$\psi(X, T, Z) = \prod_{j=1}^{2} \mathrm{Ai}\left[\frac{\chi_j(X,T)}{\sqrt{2}} + ic_j\zeta_j - \frac{\zeta_j{}^2}{4}\right] \times \exp\left[-\frac{1}{12}(2c_j + i\zeta_j)(2c_j^2 - 3\sqrt{2}\chi_j(X,T) - 4ic_j\zeta_j + \zeta_j{}^2) + \frac{1}{3}c_j^3\right], \quad (4)$$

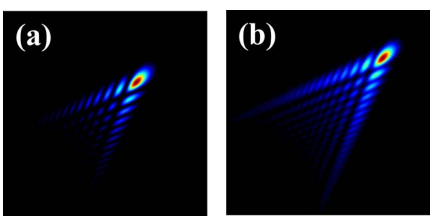


Fig. 2. (Color online) Propagation dynamics of a finite-energy Airy beam when $\theta = 45°$ at (a) $Z = 0$ and (b) $Z = 3$.

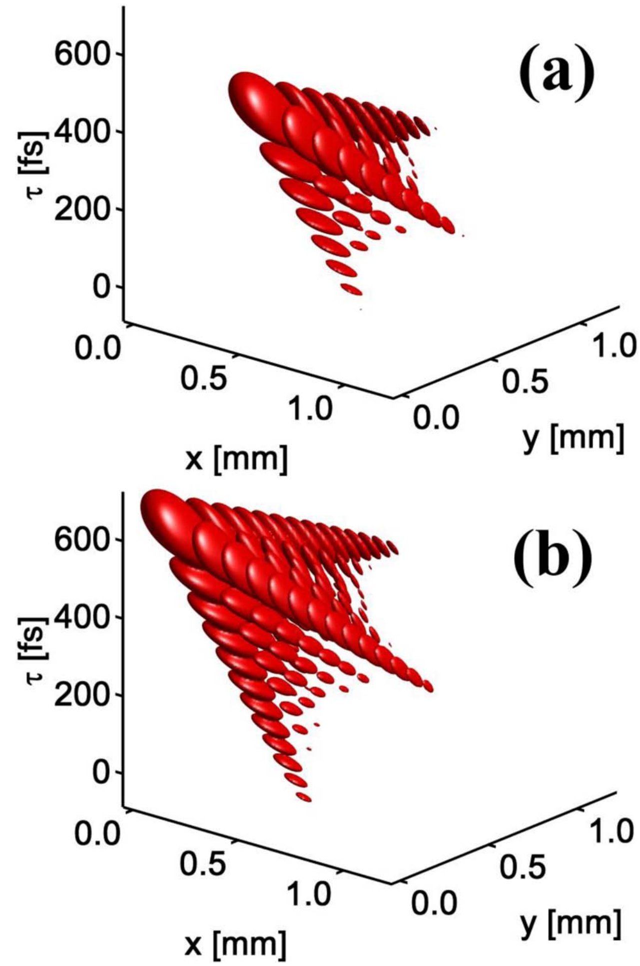


Fig. 3. (Color online) Isointensity plots of an ultrafast oblique spatiotemporal Airy wave packet in silica glass at (a) $z = 0$ cm and (b) $z = 17$ cm. For this example $\alpha_{1,2} = 0.08$, $\alpha_3 = 0.1$, and $\lambda = 800$ nm.

where $c_j = \alpha_j + i\beta_j$, $\chi_1 = \xi$, and $\chi_2 = \vartheta$, respectively; also, $\zeta_1 = Z$ and $\zeta_2 = -Z$. As is illustrated in Figs. 2(a) and 2(b) (for $\theta = 45°$, $\alpha_{1,2} = 0.1$), an oblique finite-energy Airy beam can propagate quasi-dispersion-free over a relative long distance, while for the same distance the transverse deflection is still approximately equal to that of the ideal wave packets of Eq. (2). Their experimental realization can be achieved by using Fourier synthesis techniques [12,18]. For this purpose, it can be shown that the Fourier transform of the oblique wavefront of Eq. (3) is given by

$$FT[\psi(X, T, Z=0)] = 2\exp\left[\frac{i}{3}\left(\sqrt{2}k_\xi + ic_1\right)^3\right] \times \exp\left[\frac{i}{3}\left(\sqrt{2}\Omega_\vartheta + ic_2\right)^3\right]. \quad (5)$$

Here, $k_\xi = k_X\cosh(\phi) - \Omega_T\sinh(\phi)$ and $\Omega_\vartheta = -k_X\sinh(\phi) + \Omega_T\cosh(\phi)$.

These results can be extended to three-dimensional configurations. In this case, the evolution equation takes the form [8]

$$i\psi_Z + \psi_{XX} + \psi_{YY} - \psi_{TT} = 0. \quad (6)$$

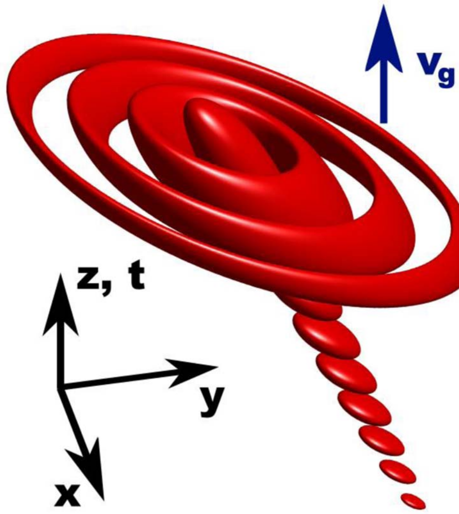


Fig. 4. (Color online) Isointensity plot of an oblique spatiotemporal Bessel–Airy wave packet.

In fact, in $(3+1)D$, a more general class of solutions can be obtained using polar rotations $R(\omega)$: $X' = X\cos\omega - Y\sin\omega$, $Y' = X\sin\omega + Y\cos\omega$, and by subsequently applying a hyperbolic transformation $H(\phi)$ involving the $T$ coordinate. For example, the intensity profile depicted in Fig. 3 was obtained via a similarity transformation $R(-\omega)H(\phi)R(\omega)$, starting from a product of three Airy solutions. The results depicted in Fig. 3 were obtained for silica glass, where at $\lambda = 800$ nm the refractive index is $n = 1.46$ and $k'' = 3.611\times10^{-26}$ $s^2$ $m^{-1}$ [19]. For these parameters, the first lobe in this example has a spatial FWHM of 100 $\mu$m, and its pulse width is 70 fs. The structure of these wave packets could be advantageous in filamentation studies where spatially separated targets can now be illuminated by one pulse in one position and by a series of subpulses in another [20,21].

We would like to emphasize that these same transformations are not limited to Airy wave packets but can be extended to all solutions of Eq. (6), such as Bessel-X-waves [8] as well as the recently observed Airy–Bessel bullets [18]. This latter case (Fig. 4) is of special interest because it could be readily realized experimentally [18].

In conclusion, we have shown that a new class of oblique, nonspreading Airy wave packets is possible in spatiotemporal optical bidispersive systems. Their obliquity factor is found to have a profound effect on their acceleration dynamics. Versatile optical bullets in normally dispersive media have also been discussed.

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