### A passive nonresonant technique for pulse contrast enhancement and gain isolation<sup>a)</sup>

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A passive aperture-scalable device is described which provides high contrast enhancement of a laser pulse over baseline optical noise while maintaining efficient net transmission of the principal laser pulse. The technique, which is based upon the nonresonant phenomenon of self-induced ellipse rotation, is capable of providing effective gain isolation for virtually any high-power laser system. The constrast enhancement capabilities of the device are demonstrated experimentally using a single ultrashort laser pulse obtained from a cavity-dumped passively mode-locked Nd: glass laser. In addition, a discussion is given concerning the ability of the device to temporally compress or stretch the optical pulse.

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#### I. INTRODUCTION

In those applications involving a single laser pulse, it is generally essential that the ratio of the pulse intensity to any prepulse or postpulse background radiation be as large as possible. This is particularly true for high-gain laser amplifier systems where it it imperative that parasitic background light be prevented from reaching the amplifier stages. Also, it is necessary to provide stringent gain isolation between amplifier stages in order to avoid problems of amplified spontaneous emission and, in laser plasma studies, to also provide such optical isolation between the target and amplifier chain. Current approaches to contrast enhancement and gain isolation employ electro- and magneto-optic shutters or various types of intensity-dependent saturable filters. 1

This paper describes a novel approach to passive contrast enhancement which is based upon the phenomenon of self-induced ellipse rotation<sup>2</sup> (SIER). Preliminary work on this approach was given by Glenn<sup>3</sup> and, in more detail, by Thorne et al. 4 Similar but intentionally less efficient designs have also been described by Dahlström.<sup>5</sup> The device, hereafter referred to as the passive contrast enhancer (PCE), employs two retardation plates, having identical retardances, between which is situated an optical Kerr cell. Polarization selectivity is provided by then placing these elements between a pair of crossed polarizers. By choosing suitable orientation angles for the retardation plates, it is possible to arrive at an intensity-dependent transmission through the PCE which increases with SIER in the Kerr cell. The principal intense laser pulse, which experiences appreciable SIER, can be effectively transmitted by the PCE, while weaker background radiation, which experiences little or no SIER, is preferentially rejected. The degree to which the transmitted pulse is enhanced over the baseline radiation can be extremely high (>104) and is limited only by the rejection ratio of the crossed polarizers. The PCE is inherently aperture

Section II of this paper presents a general analytical study of the PCE and formulates expressions for the instantaneous intensity-dependent transmission as well as the net energy transmission of the PCE. It is shown that, by simply varying the orientations of the retarders used in the PCE, it is possible to maximize the principal pulse transmission while maintaining a crossed-polarizer-limited transmission for the baseline radiation. Effects of a nonuniform intensity distribution in the beam cross section on the transmission characteristics are dealt with briefly in the Appendix. Section III presents an experimental demonstration of the effectiveness of the PCE for optical filtration of low-level background radiation accompanying a single picosecond laser pulse from a cavity-dumped Nd: glass oscillator. With a single PCE, a pulse transmission of 45% and a contrast enhancement of  $> 10^4$  were obtained for an input pulse energy of  $\simeq 4$  mJ. Using two PCE's in series, which were capable of a contrast enhancement factor > 108, a measured enhancement of  $\simeq 10^5$  was detector limited with principal pulse transmissions of  $\simeq 20\%$ . In each case, the net transmission factor of the principal laser pulse was chiefly limited by the relatively poor transmission polarizers. Any passive device characterized by an intensity-dependent transmission must necessarily affect the temporal and spatial profile of the pulse. Section IV gives a brief analytical description of the pulse compression and stretching capabilities of the PCE.

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scalable and can be used in series or in multiple-pass mode to provide exceedingly high-contrast-enhancement factors. Moreover, since SIER is a nonresonant phenomenon, the PCE is intrinsically a nonfrequencyselective device, thus making possible its application to high-power laser systems operating at wavelengths from the ir to the uv. For instance, with potential multiamplifier excimer discharge systems<sup>6</sup> exhibiting extremely high-total small-signal gain, staged effective gain isolation should be feasible with the incorporation of one or more PCE's. Moreover, with these and other more conventional high-power shortpulse laser systems such as Nd: glass and iodine, the use of the PCE should also ensure a high degree of pulse contrast enhancement over optical baseline

<sup>&</sup>lt;sup>a)</sup>A paper describing these results was presented at the IEEE/ OSA Conference on Laser Engineering and Applications (CLEA), 1977, Washington, D.C.

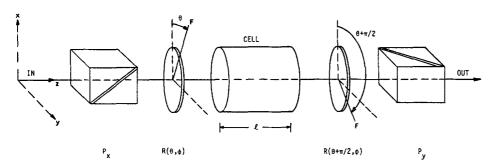


FIG. 1. The passive contrast enhancer (PCE).  $R_1$  and  $R_2$ , retardation plates;  $P_x$  and  $P_y$ , crossed polarizers.

### II. TRANSMISSION CHARACTERISTICS OF THE PCE

The PCE, shown schematically in Fig. 1, consists of a pair of retardation plates  $R_1$  and  $R_2$  having identical retardances  $\phi$  and orientation angles  $\theta$  and  $\theta + \frac{1}{2}\pi$ , respectively, which are placed between a pair of crossed polarizers  $P_x$  and  $P_y$ . Between the retardation plates is situated a cell of length l which is filled with some medium exhibiting an appreciable optical Kerr effect. The various elements of the PCE are described by their Jones matrices written here to conform with a positive phase ( $\delta$ ) convention defined as  $\exp[i(kz - \omega t + \delta)]$ . The polarizers  $P_x$  and  $P_y$  are represented by the matrices

$$P_{x} = \begin{pmatrix} p_{1} & 0 \\ 0 & p_{2} \end{pmatrix},$$

$$P_{y} = \begin{pmatrix} p_{2} & 0 \\ 0 & p_{1} \end{pmatrix},$$
(1)

where  $p_1$  and  $p_2$  are the principal amplitude transmittances  $(p_1 \gg p_2)$ . The retardation plates are described by the matrix  $R(\theta_j, \phi_j) = M(-\theta_j)R(\phi_j)M(\theta_j)$ , j = 1, 2, where  $\theta_1 = \theta$ ,  $\theta_2 = \theta + \frac{1}{2}\pi$ ,  $\phi_1 = \phi_2 = \phi$ , and where

$$R(\phi_{j}) = \begin{pmatrix} \exp\left[\frac{1}{2}(-i\phi_{j})\right] & 0\\ 0 & \exp\left[\frac{1}{2}(i\phi_{j})\right] \end{pmatrix},$$

$$M(\theta_{j}) = \begin{pmatrix} \cos\theta_{j} & \sin\theta_{j}\\ -\sin\theta_{j} & \cos\theta_{j} \end{pmatrix}$$
(2)

are the retardation and coordinate rotation matrices, respectively. As depicted in Fig. 1, the orientation angles  $\theta_j$  of the fast axes of the retarders are measured positive for rotation from the +x axis in a counterclockwise sense as seen by an observer receiving the radiation. The optical pulse(s) entering the PCE is represented by the quasimonochromatic Gaussian pulse

$$E_{in}(t) = E_0 \exp\left[-\frac{1}{2}(t/\tau)^2\right] \cos(\omega_0 t) ,$$

$$I_{in}(t) = I_0 \exp\left[-(t/\tau)^2\right] ,$$
(3)

where  $I_0 = cE_0^2/8\pi$  and  $2\tau$  is the  $e^{-1}$  full-width duration of the pulse intensity. It is assumed, for the moment, that the incident pulse is linearly polarized along the transmitting +x axis of  $P_r$ .

Before proceeding to derive exact expressions for the PCE transmission, a useful intuitive understanding of the PCE principle may be gained by first describing its characteristics qualitatively. The linearly polarized radiation emerging from P<sub>r</sub> is converted to elliptically polarized light by the retarder R<sub>1</sub>. This elliptically polarized beam then traverses the Kerrliquid-filled cell where the polarization ellipse of the radiation is rotated (without distortion) through an angle whose magnitude depends directly on both the beam intensity and its ellipticity as determined by R<sub>1</sub>. Very-low-intensity light undergoes negligible SIER and so emerges unrotated from the Kerr cell into the second retarder R2, which has a retardance identical to the first retarder but which is oriented with its fast axis at 90° to that of R<sub>1</sub>. With this configuration, the second retarder R2 thus reconverts unrotated elliptically polarized light back into light linearly polarized along the transmitting axis of P<sub>r</sub> which then, of course, is rejected completely by the second polarizer P, set orthogonal to the first. In this way, low-intensity light experiences effectively zero transmission through the PCE. High-intensity light, on the other hand, does undergo appreciable SIER in the Kerr cell, with the result that the light emerging from R<sub>2</sub> will generally be elliptically polarized. Thus, some fraction of the high-intensity beam will be transmitted through Pv. Efficient transmission of the high-intensity light is possible by, in effect, "matching" the ellipticity produced by the retarder R<sub>1</sub> (i.e., selecting the value of  $\theta$ ) to the beam intensity in such a way that the elliptically polarized light emerging from R<sub>2</sub> is very nearly linearly polarized with its major axis roughly parallel to the transmitting axis of Pv. The following calculations will now express these characteristics quantitatively.

The SIER occurring within the Kerr cell is given by the expression  $^{8}$ 

$$\epsilon(t) = (\delta n_{+} - \delta n_{-}) \omega_{0} l / 2c 
= (6\pi\omega_{0} l / n_{0} c) \chi_{3}^{1221}(\omega_{0}, \omega_{0}, \omega_{0}, -\omega_{0}) 
\times (|E_{-}|^{2} - |E_{+}|^{2}) \exp[-(t/\tau)^{2}],$$
(4)

where  $n_0 = n(\omega_0)$  and  $\chi_3^{1221}(\omega_0, \omega_0, \omega_0, -\omega_0)$  is that element of the fourth-rank nonlinear susceptibility tensor (assumed real) describing SIER. The rotation angle  $\epsilon$  is >0 for counterclockwise rotation of the beam and vice versa. The circular electric field amplitudes are  $E_{\pm} = 2^{-1/2}(E_x \pm iE_y)$ , where  $E_x$  and  $E_y$  are, respectively, the amplitudes within and at the entrance to

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the Kerr cell, given by

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = R(\theta, \phi) P_x \begin{pmatrix} n_0^{-1/2} E_0 \\ 0 \end{pmatrix} .$$
 (5)

Solving for  $E_x$  and  $E_y$  and using the identity  $(|E_+|^2 - |E_-|^2) = i(E_x^* E_y - E_x E_y^*)$ , Eq. (4) yields the result

$$\epsilon(t) = -p_1^2 q J_0 \exp[-(t/\tau)^2],$$
 (6)

where  $q = \sin 2\theta \sin \phi$  and

$$J_0 = (48\pi^2 \omega_0 l/n_0^2 c^2) \chi_3^{1221}(\omega_0, \omega_0, \omega_0, -\omega_0) I_0 \tag{7}$$

is a dimensionless generalized "intensity". Using experimentally determined<sup>9,10</sup> values of  $\chi_3^{1221}(\omega_0, \omega_0, \omega_0, \omega_0, \omega_0)$  extrapolated to  $\lambda_0 = 1.06 \ \mu^{11}$  gives the relations

$$J_0 \simeq \frac{lI_0}{800 \text{ MW/cm}}$$
 in CS<sub>2</sub>,

$$J_0 \simeq \frac{lI_0}{1200 \text{ MW/cm}}$$
 in  $C_6H_5NO_2$ .

The complete intensity-dependent rotation matrix describing the SIER is given by  $N(\epsilon) = \exp(i\Gamma)M(-\epsilon)$ , j=1,2, where

$$\Gamma = (\delta n_{+} + \delta n_{-}) \omega_{0} l / 2c$$

$$= (48\pi^{2} \omega_{0} l / n_{0}^{2} c^{2}) \chi_{3}^{1111} (\omega_{0}, \omega_{0}, \omega_{0}, -\omega_{0})$$

$$\times I_{0} \exp[-(t/\tau)^{2}]$$
(8)

is the self-phase-modulation factor. For self-induced effects in isotropic media, the relation  $\chi_3^{111}/\chi_3^{1221}=(8+R)/(6-3R)$  is valid, <sup>12</sup> where  $R=n_2^e/n_2$  is the fractional electronic contribution to the nonlinear refractive index. In CS<sub>2</sub>, for example, <sup>10</sup> where R=0.13,  $\chi_3^{111}=1.45\chi_3^{1221}$ .

With the various Jones matrices given above, the amplitude transmittance for the PCE is given by the matrix

$$\Upsilon = \left( \begin{array}{cc} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{21} & \Upsilon_{22} \end{array} \right) = P_y R(\theta + 90^\circ, \phi) N(\epsilon) R(\theta, \phi) P_x. \tag{9}$$

The matrix elements are found to be

$$\begin{split} &\Upsilon_{11} = \exp(i\Gamma)p_1p_2(\cos\epsilon + i\sin2\theta\,\sin\phi\,\sin\epsilon)\;,\\ &\Upsilon_{12} = \exp(i\Gamma)p_2^2(-\cos\phi\,\sin\epsilon - i\cos2\theta\,\sin\phi\,\sin\epsilon)\;,\\ &\Upsilon_{21}^{\cdot} = \exp(i\Gamma)p_1^2(\cos\phi\,\sin\epsilon - i\cos2\theta\,\sin\phi\,\sin\epsilon)\;,\\ &\Upsilon_{22} = \exp(i\Gamma)p_1p_2(\cos\epsilon - i\sin2\theta\,\sin\phi\,\sin\epsilon)\;. \end{split}$$

The output electric field amplitude is given by

$$\mathbf{E}_{\text{out}}(t) = \text{Re}\{(\Upsilon_{11}\hat{x} + \Upsilon_{21}\hat{y}) E_0 \exp[-\frac{1}{2}(t/\tau)^2] \times \exp(-i\omega_0 t)\}$$
(11)

and the output intensity by  $I_{\rm out}(t)=(c/4\pi)\ \langle\ |\ {\bf E}_{\rm out}(t)\ |\ ^2\rangle$ , where the brackets  $\langle\ \rangle$  denote time averaging over several optical cycles. In this way, by carrying out the time averaging with respect to the total phase term  $\exp[i(\Gamma-\omega_0t)]$ , Eqs. (8) and (11) make it possible to explicitly allow for the effects of self-phase-modulation on the transmission characteristics of the PCE (the important effects of frequency dispersion on self-phase-modulation may also be explicitly accounted for by including absolute phase factors with the

Jones matrices<sup>12,13</sup>). However, numerous studies of self-phase-modulation<sup>14</sup> have shown that the distorting effects of this phenomenon on the optical pulse envelope are generally negligible for values of  $J_0 < 5$ . The same statement is valid for the phenomenon of self-steepening. <sup>15</sup> Moreover, for a beam with a reasonably uniform intensity cross section, it is always possible<sup>3</sup> to obtain appreciable SIER in a distance much less than the self-focusing length, a condition aided by the fact that the self-focusing threshold for elliptically polarized light is substantially higher than that for linearly polarized light of the same intensity. <sup>9,16</sup>

Thus, for values of  $J_0 < 5$ , the effect of the self-phase-modulation factor  $\exp(i\Gamma)$  may be safely neglected where it follows that  $I_{\rm out}(t) = I_{\rm in}(t) (|\Upsilon_{11}|^2 + |\Upsilon_{21}|^2)$ . Hence, the instantaneous intensity transmission of the PCE,  $T_I(J_0, t) = I_{\rm out}(t)/I_{\rm in}(t)$ , is readily found to be

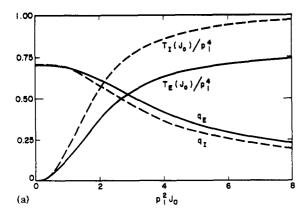
$$T_{I}(J_{0}, t) = p_{1}^{2}p_{2}^{2} + p_{1}^{4}(1 - q^{2}) \sin^{2}\{p_{1}^{2}qJ_{0} \exp[-(t/\tau)^{2}]\}.$$
(12)

The peak intensity transmission (at t=0) is thus seen to describe a simple  $\sin^2$  behavior with minima of  $p_1^2p_2^2$  at  $J_0=0$ ,  $\pi/p_1^2q$ , ..., and maxima of  $p_1^4(1-q^2)$  at  $J_0=\pi/2p_1^2q$ ,  $3\pi/2p_1^2q$ , etc. (taking q>0). It is important to note that  $T_I$  depends upon the parameters  $\theta$  and  $\phi$  only in the combined form  $\sin 2\theta \sin \phi$ . Thus, the most general choice for  $\phi$  is  $\phi=\frac{1}{2}\pi$  since only this value ensures that the range of q is not restricted to anything less than [-1,1]. Moreover, since  $T_I$  is an even function of q, only values of  $\theta$  in the range  $0-\frac{1}{4}\pi$  (for  $\phi=\frac{1}{2}\pi$ ) need be considered.

The particular design for the PCE given in Fig. 1 was arrived at by imposing the condition that the PCE transmission be a minimum at low light intensities independent of the values of  $\theta$  and  $\phi$ , i.e., that

$$\lim_{I_0 \to 0} T_I(J_0, t) = p_1^2 p_2^2 \tag{13}$$

for all  $\theta$  and  $\phi$ . This condition is indeed satisfied by the transmission function given by Eq. (12). All other alternate designs for the PCE employing two retarders and two polarizers which differ from that design given in Fig. 1 by their choice of  $\phi_1$  and  $\phi_2$ and the use of either crossed or parallel polarizers can satisfy Eq. (13) only for specific values of  $\theta_1$  and  $heta_2$  which, in general, depend upon the values selected for  $\phi_1$  and  $\phi_2$ . Clearly, such alternate designs are unnecessarily restrictive in that they allow no "fitting" of the PCE transmission. The fact that the PCE of Fig. 1 satisfies Eq. (13) for any value of q has two important consequences. First, since q is a "free" variable, its value may be chosen so as to optimize some particular transmission characteristics of the PCE. Recalling that  $T_1(J_0, t=0)$  first rises to a maximum at  $J_0 = \pi/2p_1^2q$ , i. e., at a peak intensity  $I_0$  which scales as  $(\lambda_0/l\chi_3^{1221})q^{-1}$ , it is possible to select q so as to make  $T_I$  a maximum at any arbitrary value of  $I_0$ , in effect, "stretching" the sin2 transmission curve to match exactly the principal pulse peak intensity. This is the approach adopted in the present work where q



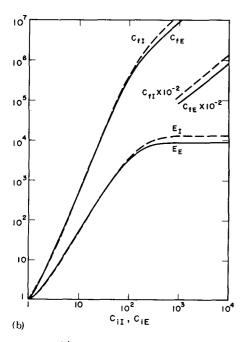


FIG. 2. (a) The optimized parameters  $q_I$  and  $q_E$  and the optimized transmissions  $T_I(J_0)$  and  $T_E(J_0)$  as functions of  $p_1^2J_0$ . (b) The contrast enhancement factors  $E_I$  and  $E_E$  and the final contrasts  $C_{fI}$  and  $C_{fE}$  as functions of the initial contrasts  $C_{iI}$  and  $C_{iE}$  for  $J_{0s}=3.5$ ,  $p_1^2=0.88$ ,  $p_2^2=5.68\times 10^{-5}$ ,  $q_I=0.45$ , and  $q_E=0.50$ .

is chosen to maximize the principal pulse transmission while maintaining crossed-polarizer-limited transmission for the weak background radiation. A second consequence of the arbitrariness of the parameter q is that, since Eq. (13) is satisfied for any value of  $\phi$ , i.e., for any wavelength  $\lambda_0$  (assuming  $\phi_1 = \phi_2$  to the same order), the elements of the PCE can, in practice, be accurately and conveniently oriented using any low-intensity laser source.

The intensity transmission of the PCE at the peak of the laser pulse at t=0 is simply

$$T_I(J_0) \equiv T_I(J_0, t=0)$$

$$= p_1^2 p_2^2 + p_1^4 (1 - q^2) \sin^2(p_1^2 q J_0).$$
(14)

An intensity contrast enhancement factor  $\boldsymbol{E}_{I}$  is defined as

$$E_{I} = C_{fI}/C_{iI} = T_{I}(J_{0s})/T_{I}(J_{0w}),$$
 (15)

where the subscripts s and w are intended to denote a "strong" principal laser pulse and a "weak" background pulse  $(J_{0w} < J_{0s})$ , respectively, and where  $C_{iI}$   $(=J_{0s}/J_{0w})$  and  $C_{fI}$  are the initial and final peak intensity contrasts, respectively, of the strong to weak pulse. For arbitrary values of  $J_{0s}$  and  $J_{0w}$ , there is, in general, no value of q which optimizes  $E_I$ . However, it is possible to maximize  $T_I(J_0)$  for the principal pulse by choosing  $q=q_I$  where  $q_I$  satisfies the transcendental equation

$$(q_I^2)^{-1} - 1 = \tan(p_1^2 q_I J_0) / (p_I^2 q_I J_0), \tag{16}$$

which follows from Eq. (14) by setting  $dT_I(J_0)/dq = 0$ . Figure 2(a) gives the value of  $q_I$  as a function of parameter  $p_1^2 J_0$  as well as the value of  $T_I(J_0)/p_1^4$  versus  $p_1^2 J_0$  calculated using  $q = q_I$ . Note that, for  $p_1^2 J_0 \gg 1$ ,  $q_I \simeq \pi/2p_1^2 J_0$  while  $T_I(J_0)$  asymptotically approaches  $p_1^4$ . The fact that  $p_1^2 q_I J_0$  is always  $< \frac{1}{2}\pi$  also ensures that the output pulse remains smooth and monomaximal; if  $p_1^2qJ_0$  were  $> \frac{1}{2}\pi$ , then "substructure" in the form of two or more intensity maxima would result. Figure 2(b) gives plots of  $E_I$  and  $C_{fI}$  versus  $C_{iI}$  using, for illustration, the values  $J_{0s} = 3.5$ ,  $q = q_I = 0.45$ , and the (experimentally measured) values  $p_1^2 = 0.88$  and  $p_2^2$ = 5.68  $\times$  10<sup>-5</sup>. Two distinct regions of contrast enhancement are evident. The first region over lower  $C_{iI}$  values is characterized by a  $C_{iI} = (C_{iI})^3$  relationship and is referred to as intensity-limited contrast enhancement in that the intensity-dependent term on the right-hand side of Eq. (14) for  $T_I(J_{0w})$  is  $\gtrsim p_1^2 p_2^2$ . The cubic relationship<sup>3,4</sup> follows by ignoring the  $p_1^2p_2^2$ term and by approximating  $\sin^2(p_1^2qJ_0) \simeq (p_1^2qJ_0)^2$  when  $T_I \propto I_0^3$ . The second region over large  $C_{iI}$  values is characterized by  $E_I$  = const and is referred to as polarizer-limited contrast enhancement since  $T_I(J_{0w})$  $=p_1^2p_2^2$  in this region. This limiting value for  $E_I$  is simply  $E_I = T_I(J_{0s})/p_1^2p_2^2$  which, for the values quoted above, is  $1.21 \times 10^4$  for a single pass through the PCE. Figure 2(b) clearly shows that the ultimate enhancement properties of the PCE are determined by the small but finite value of  $p_2$  and emphasizes the need to retain a finite  $p_2$  value in the PCE calculations if realistic conclusions are to be drawn.

In many applications, a more useful measure of the PCE efficiency is the net energy transmission derived (for a uniform beam) by integrating over all times t the product  $I_{\rm in}(t)T_I(J_0,t)$  and then dividing by the time integral of  $I_{\rm in}(t)$ . The net energy transmission of the PCE is thus found to be

$$T_E(J_0) = p_1^2 p_2^2 + p_1^4 (1 - q^2) F_1(p_1^2 q J_0), \qquad (17)$$

where

$$F_1(x) = (\pi)^{-1/2} \int_0^\infty \exp(-y^2) \sin^2[x \exp(-y^2)] dy$$
. (18)

The value of  $q = q_E$  which maximizes  $T_E(J_0)$  is given by the equation

$$\frac{1}{q_E^2} - 1 = \frac{2F_1(p_1^2 q_E J_0)}{(p_1^2 q_E J_0)F_2(p_1^2 q_E J_0)},$$
(19)

where  $F_2(x) = dF_1(x)/dx$ . The values of  $F_1$  and  $F_2$  may be generally determined numerically using a Hermite-Gaussian quadrature formula. <sup>17</sup> For values of  $p_1^2qJ_0$ 

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not too large, however, a more direct and useful approach is to expand the  $\sin^2$  term in the integrand of  $F_1$  and integrate term by term to give the result

$$T_E(J_0) = p_1^2 p_2^2 + p_1^4 (1 - q^2) \sum_{n=1}^{\infty} a_n (p_1^2 q J_0)^{2n},$$
 (20) where  $a_n = (-1)^{n+1} 2^{2n-1} [(2n)! (2n+1)^{1/2}]^{-1}$ . Figure 2(a)

where  $a_n = (-1)^{m+1}2^{2n-1}[(2n)! (2n+1)^{1/2}]^{-1}$ . Figure 2(a) shows the variation of  $q_E$  with  $p_1^2J_0$  as well as the net transmission  $T_E(J_0)/p_1^4$  calculated using  $q=q_E$ . For large  $p_1^2J_0$ ,  $q_E \simeq 1.8638/p_1^2J_0$  while  $T_E(J_0)$  approaches the limiting value of 0.7764 $p_1^4$ ; these numerical coefficients are characteristics of the Gaussian pulse model adopted. An energy contrast enhancement factor  $E_E$  is defined as

$$\begin{split} E_E &= C_{fE}/C_{iE} \\ &= T_E(J_{0s})/T_E(J_{0w}) \;, \end{split} \tag{21}$$

where  $C_{iE}$  and  $C_{fE}$  are the initial and final contrast ratios, respectively, of the energy contained in the principal pulse to that contained in the weaker background pulse. The dependence of  $E_E$  and  $C_{fE}$  on  $C_{iE}$  is plotted in Fig. 2(b) using the values  $J_{0s}=3.5$ ,  $q=q_E=0.50$ ,  $p_1^2=0.88$ , and  $p_2^2=5.68\times 10^{-5}$ . The regions of intensity- and polarizer-limited contrast enhancement are again characterized by log-log slopes for  $C_{fE}$  of 3 and 1, respectively. It is noted that the maximum value of q which appears in Fig. 2(a) is  $q=2^{-1/2}$ . Thus, in addition to the most general choice of a  $\frac{1}{4}\lambda$  retardance for the two retardation plates as noted previously, values of  $\phi$  between  $\frac{1}{4}\pi$  and  $\frac{3}{4}\pi$  may also be used without restricting the applicability of the PCE.

The calculations of E and  $C_f$  given in Fig. 2(b) are for a single transit through one PCE. In some circumstances, the use of a single PCE may fail to give a sufficiently large  $C_f$ , particularly if  $C_i$  is poor. It is a straightforward matter to employ two or more passes through the PCE or to use two or more PCE's in series having one polarizer common to each pair. This latter arrangement is preferable in that it allows the value of q for each individual PCE to be selected so as to maximize the overall principal pulse transmission. For m passes or m PCE's in series, the ultimate enhancement factor will approach  $(p_1/p_2)^{2m}$  which, for example, is  $E \simeq 2.4 \times 10^8$  for m = 2 and

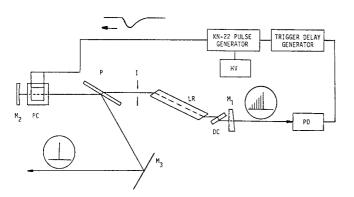
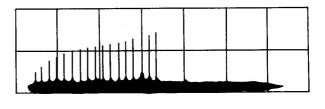


FIG. 3. The cavity-dumped passively mode-locked Nd:glass oscillator. Legend:  $M_1$ ,  $M_2$ , and  $M_3$ , mirrors; DC, saturable dye cell; LR, laser rod; I, iris; P, polarizer; PC, Pockels cell; PD, photodiode.



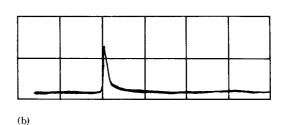


FIG. 4. (a) The output laser pulse train from the mirror  $M_1$ ; time scale is 50 nsec/div. (b) The cavity-dumped single picosecond laser pulse coupled out by the polarizer P; time scale is 2 nsec/div.

those values of  $p_1$  and  $p_2$  used the calculations for Figs. 2 and 3.

The transmission equations derived above for the PCE assumed a linearly polarized input pulse with  $I_0 = I_{0x}$  and  $I_{0y} = 0$ . It is, however, a relatively straightforward matter to show, when  $I_{0y} \neq 0$ , that the PCE transmission functions may be generalized to 12

$$T_{I}(J_{0}, t) = p_{1}^{2}p_{2}^{2} + \frac{p_{1}^{4}}{1 + r^{2}}(1 - q^{2}) \sin^{2}\left\{\frac{p_{1}^{2}qJ_{0}}{1 + r^{2}}\right\}$$

$$\times \exp\left[-\left(\frac{t}{\tau}\right)^{2}\right]$$
(22)

and

$$T_E(J_0) = p_1^2 p_2^2 + \frac{p_1^4}{1+2^2} (1-q^2) F_1 \left(\frac{p_1^2 q J_0}{1+2^2}\right),$$
 (23)

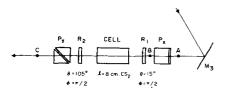
where  $r^2 = I_{0y}/I_{0x}$  and where the intensity  $I_0$  appearing in the definition of  $J_0$  is now the total intensity  $I_0 = I_{0x}$  $+I_{0v}$ . Clearly, values of  $r^2 > 0$  result in reduced transmission through the PCE although, in practice, slight ellipticities ( $r^2 = 0$ ) are quite tolerable for the principal pulse. The derived PCE transmission functions were obtained under the assumption that the pulse duration  $\tau \gg \tau_0$ , where  $\tau_0$  is the orientational relaxation time of the optical Kerr medium, so that relaxation effects could be ignored. When  $\tau \approx \tau_0$ , the amount of induced birefringence and thus the amount of SIER is reduced<sup>5,18</sup> with a corresponding reduction in the PCE transmission. In the extreme case where  $\tau\!\ll\!\tau_{\rm 0}$  , only the electronic optical Kerr effect remains.  $^{18}$  However, the transmission equations remain valid, provided that only the electronic portion of the nonlinear susceptibility element  $\chi_3^{1221}(\omega_0, \omega_0, \omega_0, -\omega_0)$  is used. As an example, the electronic portion of  $\chi_3^{1221}$  in CS<sub>2</sub> is approximately  $^{10,12}$ 

A final note concerns a second nonlinear device, here referred to as the passive contrast reducer (PCR), which is complementary to the PCE and is derived from it by simply replacing the second polarizer  $P_y$  in Fig. 1 by a polarizer  $P_x$  parallel to the first. The PCR transmission is thus

$$T_{I}^{\text{PCR}}(J_{0}, t) = p_{1}^{4} + p_{1}^{2}p_{2}^{2} - T_{I}^{\text{PCE}}(J_{0}, t)$$

$$\approx p_{1}^{4}q^{2} + p_{1}^{4}(1 - q^{2})\cos^{2}\{p_{1}^{2}qJ_{0}\exp[-(t/\tau)^{2}]\}.$$
(24)

The transmission characteristics of the PCR are exactly opposite to those of the PCE, with the PCR preferentially transmitting low-intensity light while effectively rejecting the higher-intensity radiation. The PCR may prove of value in those applications



 MEASURED
 A
 B
 C

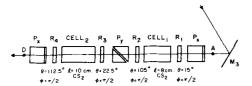
 MAIN PULSE
 4 14 mj.
 3.40 mj.
 1.86 mj.

 PEAK BACKGROUND
 0.104 mj.
 0.0171 mj.
 5.02 X10 mj.

 $C_{ie} = 39.8$   $r_s^2 = 0.072$   $r_s^2 = 4.35$ 

	MEASURED	CALCULATED
TE (JOS)	0.45	0.42
TE (JOW)	4.8 10 <sup>-5</sup>	5.4 IO <sup>-5</sup>
CfE	3.71 10 <sup>5</sup>	3.10 10 5
EE	9.3 10 3	7.8 10 <sup>3</sup>

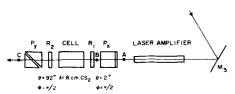
(a)



MEASURED A D
MAIN PULSE 4.14 mj. 0.768 mj
PEAK BACKGROUND 0.104 mj. <2X10 Tmj.

	MEASURED	CALCULATED
TE (Jos)	0.19	0.16
TE (Jow)	<1.9 10 <sup>-6</sup>	3.07 10 <sup>-9</sup>
CfE	>3.8 10 6	2.08 109
EE	>9.7 104	5.22 107

(b)



 MEASURED
 A
 B
 C

 MAIN PULSE
 31.2 mj.
 27.4 mj.
 17.0 mj.

 PEAK BACKGROUND
 0.470 mj.
 0.071 mj.
 2.82 X 10 mj.

 $C_{ie} = 66.4$   $r_s^2 = 0.002$   $r_w^2 = 4.83$ 

	MEASURED	CALCULATED
TE (Jos)	0.54	0.56
TE (JOW)	6.0 10-5	5.2 10 <sup>-5</sup>
CfE	6.03 10 <sup>5</sup>	7.17 105
EE	0.91 104	1.08 104

(c

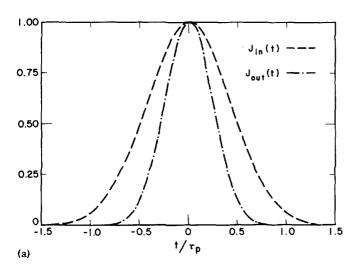
FIG. 5. (a) Passive contrast enhancement using a single PCE. (b) Passive contrast enhancement using two PCE's in series. (c) Passive contrast enhancement of the amplified laser pulses using a single PCE.

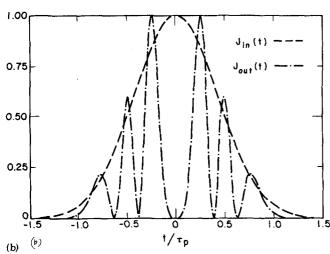
where it is necessary to have a more uniform intensity profile, either temporally or spatially. The ability of the PCR to temporally stretch an optical pulse with a Gaussian profile is described in Sec. IV.

### III. EXPERIMENTAL STUDIES WITH THE PCE

The effectiveness of the PCE as a contrast improvement device was investigated experimentally using a single isolated picosecond laser pulse obtained from a passively mode-locked and cavity-dumped Nd: glass laser oscillator, which is shown schematically in Fig. 3. The 1.2-m laser resonator, formed by a fully reflecting dielectric mirror  $\mathbf{M}_2$  of 5-m radius of curvature and a wedged plane output mirror M<sub>1</sub> having a reflectivity of 95%, utilized a 9-in.-long Brewster-angled laser rod. An iris I, which limited transverse mode development, and an antireflectioncoated thin film dielectric polarizer P were also incorporated in the resonator. The laser was passively mode locked using a Brewster-angled 3-mm-thick cell DC of Kodak 9740 dye. At a point during the buildup of the mode-locked pulse, a fast rising  $\frac{1}{4}\lambda$ voltage step, derived from a (KN22) krytron pulse generator triggered by a photodiode PD through a variable delay generator, was applied to an unterminated Pockels cell PC situated in the laser resonator. Thus, the ultrashort pulse on its next transit experienced a polarization rotation of 90° for the double pass through the PC and was coupled out of the resonator via polarizer P with, ideally, an efficiency of 100%. Figure 4(a) is a sample of the output from the mirror M<sub>1</sub> and clearly shows the sudden and complete extinction of the laser pulse train. The single picosecond laser pulse coupled out of the resonator by the polarizer P is shown in Fig. 4(b). The exact time of switch-out of the single pulse could be controlled by varying the attenuation in front of photodiode PD or the time delay of the delay generator, and was found to remain relatively stable from shot to shot. This resulted in a good reproducibility for producing a single dumped pulse with an amplitude jitter ( $\pm 20\%$ ) determined chiefly by the variation in the Q-switched pulse envelope from shot to shot and a time jitter between the appearance of the highvoltage pulse at PC and the selection of the laser pulse of approximately 2 nsec. Because the Pockelscell-polarizer combination could never be made perfectly nonbirefringent in its passive state, the single pulse was always accompanied by prepulses of much lesser intensity. By very carefully aligning the Pockels cell and polarizer, the intensity contrast between the principal pulse and these prepulses was typically a factor of  $\approx 1500$ . For the purposes of investigating the PCE, however, the PC was deliberately misaligned slightly so as to provide relatively poor initial contrasts (<100). The contrast enhancement properties of the PCE were then measured by comparing the single laser pulse to the largest of the background prepulses.

The principal measurements of the contrast enhancement of the single pulse over the background prepulse using the PCE in three different configurations are summarized in Fig. 5. The retardation plates used were all zero-order, quartz  $\frac{1}{4}\lambda$  retarders, antireflection coated (r < 0.2%) on both sides, and suitably mounted in continuously rotatable mounts having a setting accuracy for  $\theta$  of 0.1°. The CS<sub>2</sub> Kerr





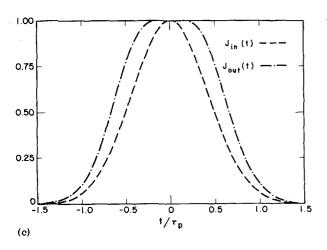


FIG. 6. Optical pulse compression by the PCE with (a)  $p_1^2qJ_0=0.20$  and (b)  $p_1^2qJ_0=3\pi$ . (c) Optical pulse stretching by the PCR with q=0.618 and  $p_1^2J_0=1.0$  The FWHM duration of the input pulse is  $\tau_p$ .

cells were made of Teflon cylinders of 1-in. inside diameter to which were attached quartz windows with outside faces antireflection coated. Glan-Taylor calcite polarizers were employed having measured principal transmittances of  $p_1^2 = 0.88$  and  $p_2^2 = 5.68 \times 10^{-5}$ giving an extinction ratio  $p_1^2 p_2^2$  of  $\approx 5.0 \times 10^{-5}$ . The orientation angle  $\theta$  for each PCE was chosen to make  $q = q_E$  for the principal laser pulse, where  $q_E$  is given in Fig. 2(a). The amplifier used in the configuration in Fig. 5(c) was a  $60 \times 2.5$ -cm. Nd: glass rod with wedged faces having a single-pass small gain of  $\approx 30$ . The estimated error in the measured energies is ± 20% and is primarily due to the shot-to-shot variation of the pulse amplitudes. Agreement between measured and calculated values was generally within 30%. The most difficult quantity to measure accurately was  $T_E(J_{0w})$  since its value was greatly affected by even slight misalignments in either the retarder or polarizer orientations. This difficulty was overcome by aligning the PCE with the Kerr cell temporarily removed and by using the intense principal laser pulse. A number of shots were taken while adjusting first P, (with R2 removed) and then the reinserted R2 until a minimum transmission  $(p_1^2p_2^2)$  was measured, after which the Kerr cell was replaced. In this way, it was possible to consistently obtain polarizerlimited transmissions for the very weak background pulses.

As Fig. 5(a) reveals, the use of a single PCE after the laser oscillator resulted in an enhancement which was effectively crossed-polarizer limited, the weak background pulse experiencing a transmission of only 0.005%, while the principal pulse, with an incident energy of  $\simeq 4$  mJ, experienced a transmission of 45%. Nevertheless, the background radiation after the PCE was still detectable. To further increase the pulseto-background contrast, a second PCE in series with the first with a common polarizer P, was added [Fig. 5(b)]. The principal pulse transmission was reduced by a factor of approximately 2-19%, while the background pulse transmission was reduced by a factor of at least 25 to a value < 0.0002% which was an upper-limit measurement limited by the sensitivity of the detection system. The calculated values indicated a background pulse transmission of  $\simeq 3 \times 10^{-9}$ yielding a background pulse energy after the two PCE's of <1 pJ. The measurements given in Fig. 5(c) using the amplified pulses show that it is possible to obtain principal pulse net transmission > 50% while effectively eliminating the background radiation.

The results exemplified in Fig. 5 represent a significant improvement over the use of saturable absorbers for passive contrast enhancement in that relatively high main pulse transmissions are maintained while very strongly attenuating weaker background radiation. The use of two PCE's in series as in Fig. 5(b) should prove able to satisfy even the most stringent contrast enhancement requirements. However, a major area for improvement on the PCE would be the use of more efficient polarizers having  $p_1 \simeq 1$ , since the principal pulse transmission is  $\propto p_1^4$  as Fig. 2(a) shows. To illustrate this point, if a

value  $p_1 = 1$  is assumed, then the principal pulse net transmissions for the pulse energies given in Figs. 5(a)-5(c) would become 63, 30, and 78%, respectively, representing a marked improvement over the actual measured values of 45, 19, and 54% where  $p_1$ was = 0.938. Improving the transmission efficiency of the PCE may prove possible with the aid of thinfilm multilayer dielectric reflection polarizers for which values of  $p_1 = 0.99$  are readily obtainable. However, if high contrast enhancement factors are not to be sacrificed for the sake of efficiency, then it is mandatory that the crossed polarizer extinction coefficient  $(p_1^2p_2^2)$  be as small as possible. Thus, since multilayer dielectric reflection polarizers generally have relatively large  $p_2$  values (typically  $6 \times 10^{-2}$ ), it may prove necessary in practice to use such polarizers in groups of two or more to form the individual polarizers  $P_x$  and  $P_y$  used in the PCE.

## IV. PULSE COMPRESSION AND PULSE STRETCHING WITH THE PCE

For an intense input pulse, the output pulse from the PCE is, from Eq. (12),

$$J_{\text{out}}(t) = p_1^4 J_0(1 - q^2) \exp[-(t/\tau)^2] \sin^2\{p_1^2 q J_0 \times \exp[-(t/\tau)^2]\}.$$
 (25)

This pulse will be compressed in time in comparison to the input pulse  $J_0 \exp[-(t/\tau)^2]$ . However, as noted previously, if  $J_{\rm out}(t)$  is to remain smooth and have only one intensity maximum (at t=0), then q must satisfy  $q \le 1.8638/p_1^2J_0$ . If this condition is not satisfied, then  $J_{\rm out}(t)$  becomes modulated with two or more intensity maxima. In the limit where  $p_1^2qJ_0$  is small, Eq. (25) simplifies to

$$J_{\text{out}}(t) = p_1^8 q^2 (1 - q^2) [J_{\text{in}}(t)]^3$$
  
=  $p_1^8 q^2 (1 - q^2) J_0^3 \exp[-3(t/\tau)^2],$  (26)

revealing a pulse compression of  $3^{-1/2} = 0.577$  in this limit, which in fact is the maximum possible for a single pass through the PCE. Thus, in order to obtain maximum compression of the input pulse, q must be chosen to make  $p_1^2qJ_0$  small, say,  $p_1^2qJ_0 \le 0.20$ . This is contrary to the behavior of  $q = q_E$  in Fig. 2(a), so that, in general, the requirement of compression results in lower net transmissions than are actually possible (if  $q = q_E$  were chosen, then the compression factor would approach 0.881 for large  $J_{\rm 0}$  values). Figure 6(a) shows  $J_{in}(t)$  and  $J_{out}(t)$ , both normalized to unity, versus t for the parameter value  $p_1^2qJ_0$ = 0.20. Figure 6(b) shows the result for  $p_1^2qJ_0=3\pi$ = 9.425, and illustrates the pulse "breakup" when  $p_1^2qJ_0$  is > 1.8638. The individual "substructure" pulses of  $J_{out}(t)$  in Fig. 6(b) represent compression factors of the order of 10.

While the PCE results in compression of the input pulse, the PCR, which is derived from the PCE by replacing the polarizer  $P_y$  in Fig. 1 by  $P_x$  as discussed at the end of Sec. II, generally results in a temporal stretching of the input pulse. The output pulse from the PCR is simply  $J_{in}(t)T_I^{PCR}(J_0, t)$ , where  $T_I^{PCR}(J_0, t)$  is given in Eq. (24). Figure 6(c) shows a

stretched output pulse profile (by a factor of 1.32) against the input pulse for the values q = 0.618 and  $p_1^2qJ_0=1.0$ . As in the case for the compressed pulse, it is possible to establish conditions on the value of q which maximize the degree of stretching while maintaining a smooth monomaximum output pulse profile. However, the equations defining such conditions are much more complex than those for the pulse compression case since the shape of the stretched pulse depends upon the parameters q and  $p_1^2 J_0$  independently while, as Eq. (25) reveals, the shape of the compressed pulse depends only upon the single combined parameter  $p_1^2qJ_0$ . The values used in plotting Fig. 6(c) were obtained by requiring that the second time derivative at t=0 of the stretched pulse vanish exactly.

### V. SUMMARY

The PCE has been shown to be an efficient and versatile device for optical pulse isolation. Using the output from a mode-locked cavity-dumped Nd: glass laser, single-pass contrast enhancement factors of  $>10^4$  were measured with principal pulse net transmissions of  $\approx 50\%$ . The use of two PCE's in series led to a detector-limited contrast enhancement calculated to be  $>10^7$  while maintaining an overall transmission for the principal pulse of  $\approx 20\%$ . The adaptation of more efficient polarizers should lead to much improved pulse transmissions without adversely affecting the enhancement capabilities of the device.

Since the PCE is founded on a nonresonant phenomenon and thus may be translated to any wavelength, it can be employed with a wide variety of lasers. In particular, it may find application in potentially large short-pulse gas or excimer-laser systems. The small-signal gain of such systems is generally very high and of short duration. 6 With the incorporation of such discharges into multiamplifier systems to yield an extremely high total small-signal gain, a major problem will be that of suppressing parasitic amplified spontaneous emission building up with and traveling in the same direction as the propagating pulse. In such a system, the use of several PCE's staged throughout the amplifier system should effectively isolate the gain of individual amplifier stages as well as maintain or improve the contrast factor of the principal propagating pulse.

### **ACKNOWLEDGMENTS**

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# APPENDIX: PCE TRANSMISSION FOR A BEAM WITH NONUNIFORM INTENSITY CROSS SECTION

The Gaussian input pulse, Eq. (3), used in the analysis in the text was assumed to be uniformly constant in intensity over some finite cross-sectional area. The transmission of the PCE was thus found to

depend only upon the time t insofar as  $I_{\rm in}=I_{\rm in}(t)$ . For a beam with a nonuniform spatial profile, however, the PCE transmission will also depend upon the transverse coordinates of the beam. To explicitly calculate the effects of a nonuniform cross section on the transmission characteristics, the input pulse is generalized to a Gaussian both in time t and transverse coordinate r, in the form  $I_{\rm in}(t)=I_0\exp[-(t/\tau)^2-(r/R)^2]$ . The total power in the pulse is  $P_{\rm in}(t)=\pi R^2 I_0\exp[-(t/\tau)^2]$ , while the input energy is  $E_{\rm in}=\pi^{3/2}\tau R^2 I_0$ . Referring to Eq. (12), the instantaneous intensity transmission through the PCE will now be

$$T'_{I}(J_{0}, r, t) = p_{1}^{2}p_{2}^{2} + p_{1}^{4}(1 - q^{2})\sin^{2}\{p_{1}^{2}qJ_{0} \times \exp[-(t/\tau)^{2} - (r/R)^{2}]\}.$$
(27)

The instantaneous power transmission is thus

$$T_{P}'(J_{0}, t) = p_{1}^{2}p_{2}^{2} + \frac{1}{2}p_{1}^{4}(1 - q^{2})$$

$$\times \left(1 - \frac{\sin\{2p_{1}^{2}qJ_{0}\exp[-(t/\tau)^{2}]\}}{2p_{1}^{2}qJ_{0}\exp[-(t/\tau)^{2}]}\right)$$
(28)

and the net energy transmission is

$$T_E'(J_0) = p_1^2 p_2^2 + p_1^4 (1 - q^2) F_3(p_1^2 q J_0), \qquad (29)$$

where

$$F_3(x) = \frac{1}{2} - (\pi^{-1/2}/4x) \int_{-\infty}^{\infty} \sin[2x \exp(-y^2)] dy$$
.

Expanding the sin term in a power series and integrating term by term gives

$$T_E'(J_0) = p_1^2 p_2^2 + p_1^4 (1 - q^2) \sum_{n=1}^{\infty} b_n (p_1^2 q J_0)^{2n}, \qquad (30)$$

where  $b_n = (-1)^{n+1}2^{2n-1}[(2n+1)!(2n+1)^{1/2}]^{-1}$ . As expected, the net transmission  $T_E'(J_0)$  for the pulse Gaussian in both t and r is, for  $p_1^2qJ_0<1$ . 8638, always less than that for the uniform intensity cross-section pulse, reflecting the fact that the outer low-intensity edges of the pulse are effectively rejected by the PCE. Parameters  $q_p'$  and  $q_E'$  may be defined which maximize the transmissions  $T_p'(J_0)$  and  $T_E'(J_0)$ , respectively; note, however, that  $q_I' = q_I$ . The description given in Sec. IV of temporal pulse compression and stretching by the PCE applies equally well to spatial compression and "smoothing" since the r and t dependence of  $I_{in}(r,t)$  are independent but identical in their effect on  $T_I'(J_0,r,t)$ . Thus, the PCE will generally produce a compression or narrowing of the transverse beam distribution. The PCR, on the other hand, will

enhance the lower-intensity areas by discriminating against the higher-intensity regions of the beam profile. This property could make the PCR a potentially useful device<sup>4</sup> for spatial filtering of small localized "hot spots" which often occur in the transverse intensity distribution of a laser beam.

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