Low-loss coupling between two single-mode optical fibers with different mode-field diameters using a graded-index multimode optical fiber

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We present a method for ultra-low-loss coupling between two single-mode optical fibers with different mode-field diameters using multimode interference in a graded-index multimode optical fiber. We perform a detailed analysis of the interference effects and show that the graded-index fiber can also be used as a beam expander or condenser. The results are important for devices in which optical fibers with different mode-field diameters are coupled in series, such as in ultra-short-pulse fiber ring lasers, or in optical fiber communication links.

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Multimode interference (MMI) has been successfully explored in various fiber optic applications in beam shapers, sensors, and filters [1–8]. In a multimode fiber, different guided modes have different phase velocities. Therefore, the excited modes interfere as they propagate along the fiber, creating different interference field patterns at different locations. The original field pattern is repeated after propagating a finite length, leading to a periodic self-imaging behavior.

Here, we explore the MMI effect in graded-index multimode fibers (GIMFs). GIMFs are commonly used in optical communications to equalize the group velocities of the modes. For MMI applications, GIMFs exhibit the unique property that the propagation constants of their modes are equally spaced. Consequently, their self-imaging lengths can be very short—even less than 1 mm. In this Letter, we focus on the MMI after a Gaussian beam is launched into the GIMF. The source of the Gaussian beam can be from a lens coupler or simply from a single-mode fiber (SMF), which is spliced to the GIMF. We show that the GIMF can act as a beam expander, as a beam condenser, or as a very low loss coupler between a large-core SMF and a small-core SMF. Our results can have important implications for devices in which optical fibers with different mode-field diameters are coupled in series, such as in ultra-short-pulse fiber ring lasers, or in coupling to erbium-doped and dispersion compensating fibers in optical communication links.

The refractive index profile of a GIMF is given by

$$n^2(\rho) = n_0^2\left[1 - 2\Delta\left(\frac{\rho}{R}\right)^{\alpha}\right], \tag{1}$$

where R is the core radius, n_0 is the refractive index at the center of the core, Δ is the index step, $\alpha \approx 2$ characterizes a near parabolic index profile in the core ($\rho \le R$), and $\alpha = 0$ in the cladding ($\rho > R$). The transverse electric field profile of a confined mode, with radial p ($p \ge 0$) and angular m integer numbers, can be expressed as [9]

$$E_{p,m}(\rho,\phi) = A_p^m \frac{\rho^{|m|}}{\rho_0^{|m|+1}} \exp\left(-\frac{\rho^2}{2\rho_0^2}\right) L_p^{|m|}\left(\frac{\rho^2}{\rho_0^2}\right) e^{im\phi}, \tag{2}$$

where $L_p^{|m|}$ are the generalized Laguerre polynomials and ρ_0 and A_p^m are given by

$$\rho_0 = \frac{R^{1/2}}{(k_0 n_0)^{1/2}(2\Delta)^{1/4}}, \qquad A_p^m = \sqrt{\frac{p!}{\pi(p+|m|)!}}, \tag{3}$$

where $k_0 = 2\pi/\lambda$. The coefficients A_p^m are chosen so that the modes are orthonormal. All the modes with equal group mode number $g = 2p + |m| + 1$ are almost degenerate in the value of the propagation constant given by

$$\beta_g = n_0 k_0 - \sqrt{2\Delta}\, g/R. \tag{4}$$

In order to formulate the MMI in a GIMF, consider an input optical field injected into the core of the GIMF. The injected field excites the guided modes $E_{p,m}(\rho,\phi)$ of the GIMF with different amplitudes $C_{p,m}$, which propagate with propagation constants $\beta_{p,m}$. The total electric field after propagating a distance z along the fiber is given by

$$E(\rho,\phi,z) = \sum_{p,m} C_{p,m} E_{p,m}(\rho,\phi) e^{i\beta_{p,m} z}. \tag{5}$$

In this Letter, we assume that the injected beam is in the form of a Gaussian with radius w and power P_0, centrally aligned with the GIMF, and expressed as

$$E_i(\rho) = \sqrt{\frac{2P_0}{\pi w^2}} \exp\left(-\frac{\rho^2}{w^2}\right). \tag{6}$$

The injected beam can be easily realized from the nearly Gaussian mode of an SMF, which is spliced to the GIMF. The excitation amplitudes are given by

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$$C_{p,m} = \frac{2\sqrt{P_0\eta}}{\eta+1}\Psi^p\delta_{m,0}, \qquad \eta = \frac{2\rho_0^2}{w^2}, \qquad \Psi = \frac{\eta-1}{\eta+1}. \quad (7)$$

Because of the azimuthal symmetry of the input Gaussian beam, only $m = 0$ modes can be excited. η plays a key role in determining the MMI behavior of the GIMF. In fact, $\rho_c = \sqrt{2}\rho_0$ establishes a wavelength-dependent characteristic radius for the GIMF, and η compares this characteristic radius to the radius of the input Gaussian beam. Here, we consider a commonly used GIMF, the GIF50 fiber from the Thorlabs catalog, where $R \approx 25\,\mu$m, $\Delta \approx 0.01$, and we obtain $\rho_c \approx 7.73\,\mu$m.

The total power carried by the GIMF is given by

$$\text{total power} = \sum_{p=0}^{P-1}|C_{p,0}|^2 = (1-\Phi_P)P_0, \quad (8)$$

where P is the total number of guided modes with $m = 0$ and $\Phi_P = \Psi^{2P}$ is the fractional power loss to the radiation modes. Note that Φ_P is exponentially decreasing with P, because $\Psi < 1$, and we can show that as long as $P > \eta$ for $\eta > 1$ ($P < 1/\eta$ for $\eta < 1$), $\Phi_P < 2\%$ ($P = 5$ for GIF50). Therefore, virtually lossless coupling to GIF50 can be achieved from an SMF with a mode-field diameter as small as $7\,\mu$m or as large as $35\,\mu$m.

The MMI Eq. (5) for GIMF can be written as

$$E(\rho,z) = \sum_{p=0}^{P-1} C_{p,0}E_{p,0}(\rho)e^{i\beta_{2p+1}z}$$
$$= \sqrt{\frac{8P_0}{\pi w^2}}\frac{\exp(i\beta_1 z)}{\eta+1}\exp\left(-\frac{\rho^2}{2\rho_0^2}\right)\sum_{p=0}^{P-1}\Upsilon^p(z)L_p^0\left(\frac{\rho^2}{\rho_0^2}\right). \quad (9)$$

$\Upsilon(z)$ and the beam pattern (up to an irrelevant phase) are periodic functions of z with periodicity Z

$$\Upsilon(z) = \Psi\exp(-i2\pi z/Z), \qquad Z = \pi R/\sqrt{2\Delta}. \quad (10)$$

We define Γ as the ratio of the core center intensity of the GIMF to the core center intensity of the incoupling Gaussian beam. Γ is also a periodic function of z.

$$\Gamma(z) = \frac{4}{(\eta+1)^2}\left|\frac{1-\Upsilon(z)^P}{1-\Upsilon(z)}\right|^2. \quad (11)$$

In general, if $\eta > 1$, then $\Gamma < 1$ for all values of z and P, and the GIMF acts like a *beam expander*, where the peak values of Γ occur at $z_{\text{even}} = qZ$ (q is an integer). Yet more interesting is the case of $\eta < 1$, where $\Gamma > 1$ and the GIMF acts like a *beam condenser*. The intensity of the hot spot at the center of the GIMF peaks at $z_{\text{odd}} = (2q+1)Z/2$, and it is larger than the peak intensity of the input Gaussian beam by a factor of $1/\eta^2$.

Now, suppose that the output port of the GIMF is spliced to another SMF with a mode-field radius of w'. If P_0' is the power carried by the outcoupling SMF, the power coupling efficiency P_0'/P_0 between the incoupling and outcoupling SMFs can be written as

$$\tau(L) = \frac{P_0'}{P_0} = \frac{16\eta\eta'}{(\eta+1)^2(\eta'+1)^2}\left|\frac{1-[\Psi'\Upsilon(L)]^P}{1-\Psi'\Upsilon(L)}\right|^2, \quad (12)$$

where

$$\Psi' = \frac{\eta'-1}{\eta'+1}, \qquad \eta' = \frac{2\rho_0^2}{w'^2}. \quad (13)$$

L is the length of the GIMF segment. If η and η' are both larger than unity (or are both smaller than unity), then $\tau(z)$ peaks at z_{even}. However, if $\eta > 1$ and $\eta' < 1$ (or vice versa), then $\tau(z)$ peaks at z_{odd}:

$$\tau(z_{\text{even}}) = \tau(0) = \frac{4\eta\eta'}{(\eta+\eta')^2}[1-(\Psi'\Psi)^P]^2, \quad (14)$$

$$\tau(z_{\text{odd}}) = \tau(Z/2) = \frac{4\eta\eta'}{(1+\eta\eta')^2}[1-(-\Psi'\Psi)^P]^2. \quad (15)$$

In particular, if $\eta\eta' = 1$, the GIMF acts as a very low loss coupler between a large- and a small-core SMF. This condition can be satisfied if the characteristic radius of the GIMF ρ_c is the geometrical mean of the radius of the in- and outcoupling Gaussian beams, i.e., $ww' = \rho_c^2$. The peak power coupling efficiency can be near 100%, considering that $(\Psi'\Psi)^P \approx 0$ for most practical cases. In the limit of $P \to \infty$, we can simplify Eq. (12) as

$$\frac{1}{\tau(L)} = \frac{(\eta+\eta')^2}{4\eta\eta'}\cos^2\left(\frac{\pi L}{Z}\right) + \frac{(1+\eta\eta')^2}{4\eta\eta'}\sin^2\left(\frac{\pi L}{Z}\right). \quad (16)$$

Although Eq. (12) can be considered the main result of this paper, Eq. (16) provides a very reliable approximation to be used in designing couplers.

We now report on the experimental validation of Eqs. (12) and (16). Figure 1(a) shows the setup of the experiments, where broadband light has been launched into the fiber chain under test, and the transmission spectrum has been recorded with an optical spectrum analyzer. In each plot in Figs. 1(b)–1(d), the solid curves represent the experimental measurements and the dashed curves show the theoretical results. We use standard commercial fibers from Thorlabs in our experiments. For the input and output SMFs, we use Corning SMF-28 and LMA-20 (large mode area fiber with a $20\,\mu$m core diameter), and for the GIMF, we use GIF50.

In experiment (1) of Fig. 1(a), we use identical input and output SMF-28 fibers, so $\eta = \eta' \approx 1.9$. For this experiment, we show the transmission $\tau(L)$ in decibel units as a function of the wavelength in Fig. 1(b), for a GIMF length of $L \approx 100$ cm. We note that the oscillations of the transmission are due to the weak wavelength dependence of Δ (and consequently Z), resulting from the different refractive index dispersions of the Ge-doped silica core and the undoped silica cladding of the GIMF. The gradual increase in the minimum of the oscillations with the wavelength can be explained by the wavelength dependence of the mode-field radius w of SMF-28, as well as the wavelength dependence of ρ_0 in Eq. (3). In order to obtain a good agreement between theory and experiment,

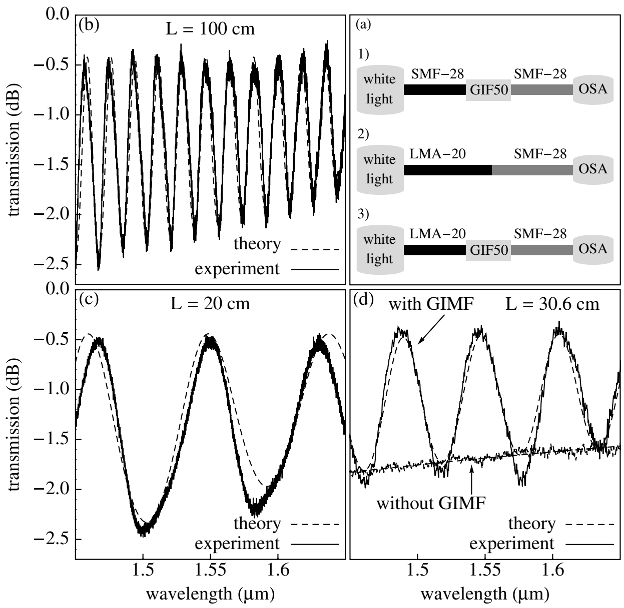

Fig. 1. Three sets of experiments are sketched in subfigure (a) and the measurement results are reported and compared with theory in subfigures (b), (c), (d). In experiment (1), a GIMF of length L is used as the intermediate coupler between two SMF-28 fibers. The transmission measurement is reported for $L \approx 100$ cm in (b), and for $L \approx 20$ cm in (c). In experiment (2), LMA-20 is directly coupled to SMF-28, and the transmission is reported in (d) as "without GIMF." In experiment (3), a GIMF of length 30.6 cm is used as the intermediate coupler between the LMA-20 and SMF-28 fibers. The transmission is reported in (d) as "with GIMF." In all cases, very good agreement between theory and experiment is observed.

we must use $\Delta = 0.01 + \delta(\lambda - \lambda_c)$, where $\lambda_c = 1.55\,\mu$m and $\delta = 0.00063/\mu$m, and $w = 5.6 + 3.6(\lambda - \lambda_c)$, where λ and w are in μm units. We must also account for an extra 0.42 dB of broadband transmission loss that might be tentatively attributed to core–cladding nonconcentricity or other geometrical imperfections and is present in all experimental results. We note that the agreement between the theory and experiment highly restricts the choice of these seemingly "free" parameters, because each parameter is directly linked to a particular feature of the spectral transmission in Fig. 1(b). We also choose not to make a global fit to the available data and use the exact same form of these parameters as stated above in all our theoretical plots in this paper. While the wavelength dependence of Δ is proprietary information of the manufacturer and cannot be verified, our suggested wavelength dependence of w is consistent with the published manufacturer specifications of the SMF-28 fiber [10]. The period of oscillations in Fig. 1(b) is given by $\Lambda = \sqrt{2\Delta}\pi R/(\delta \times L)$, where Λ is inversely proportional to L [6]. In Fig. 1(c), we reduce the length of the GIMF to $L \approx 20$ cm, repeat the experiment of Fig. 1(b) for the shorter length, and observe a consistent increase in the oscillation period. We also observe very good agreement between theory and experiment. To achieve maximum transmission at a specific wavelength, the GIMF length L must be precisely tuned. Reducing L to $L - Z$ shifts the periodic transmission pattern by one full period.

Next, in experiment (2) of Fig. 1(a), we change the input fiber to LMA-20, and we measure the loss due to direct coupling between SMFs of different mode-field diameters. The spectral transmission for this case is reported in Fig. 1(d), which is a straight line marked as "without GIMF," where the slope is dictated by the wavelength dependence of ρ_0 and the mode-field radii of the SMF-28 and LMA-20 fibers. For LMA-20, we need to consider $w = 9.97 + 3.36(\lambda - \lambda_c)$, where λ and w are in micrometer units. At $\lambda = \lambda_c$, we have $\eta \approx 0.6$ and $\eta' \approx 1.9$, so $\eta\eta' \approx 1.14$. However, because SMFs are directly coupled, we need to use $L = 0$ in Eq. (16), which results in $\tau(0) \approx -1.37$ dB at λ_c [see also Eq. (15)]. Including the broadband loss, we get ≈ -1.79 dB -1.79 dB, which agrees well with the measurements.

Finally, in experiment (3), we insert an $L \approx 30.6$ cm GIMF between the LMA-20 and SMF-28 fibers and repeat our measurement. The spectral transmission is reported in Fig. 1(d) and is marked as "with GIMF." Again, the theory agrees very well with the measurement, where we have used the wavelength dependence form of Δ, w, and w', as reported above. At $\lambda \approx \lambda_c$, the theoretical prediction of minimum stands at -1.79 dB as before. The peak of the oscillation is obtained from Eq. (15), resulting in only $\tau(z_{\mathrm{even}}) \approx -0.02$ dB. Low-loss transmission can be achieved at any desired wavelength by varying the length of the GIMF segment. After taking into account the broadband loss, we get ≈ -0.44 dB, which agrees well with the measurements.

In summary, we have introduced and experimentally verified a method that can substantially reduce the coupling losses between two dissimilar SMFs. In a proof-of-concept experiment, the coupling loss between LMA-20 and SMF-28 was reduced from -1.8 dB to -0.44 dB by adding a GIMF fiber segment of suitable length.

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