Reverse optical forces in negative index dielectric waveguide arrays

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Nonconservative optical forces acting on dipolar particles are considered in longitudinally invariant optical fields. We demonstrate that the orientation of these forces is strictly dictated by the propagation vector associated with such field configurations. As a direct consequence of this, it is impossible to achieve a reversal of optical forces in homogeneous media. We show instead that translation invariant optical tractor fields can in fact be generated in the negative index environment produced in a special class of fully dielectric waveguide arrays. © 2011 Optical Society of America

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One of the most intriguing properties of light–matter interaction is the ability of an electromagnetic field to exert mechanical forces upon polarizable objects. Such a phenomenon is a direct consequence of the process of light scattering and is dictated by a principle as fundamental as momentum conservation itself.

The first experimental observations of optomechanical interactions date back to the beginning of the past century with the seminal experiments conducted by Lebedev [1] and by Nichols and Hull [2]. The idea of harnessing optical forces culminated in the celebrated work of Ashkin [3] on optical tweezers, which endowed the most diverse fields of science with a new and powerful tool for the remote manipulation of microscopic objects.

The amount of momentum transferred to a scattering and/or absorbing body immersed in electromagnetic radiation can, at least in principle, be exactly calculated by evaluating the flux of the Maxwell's stress tensor through any arbitrary surface enclosing the object. This in general requires the exact knowledge, not only of the incident field, but also of the field scattered by the object itself. The formulation becomes substantially simpler when light interacts with particles of dimensions much smaller than the wavelength of the incident radiation. Under such circumstances the scattered field is essentially dipolar. The expression of the total force acting on the particle is then amenable to the following simpler expression [4] in terms of the particle complex polarizability $\alpha = \alpha_r + i\,\alpha_i$ and the electromagnetic field at the interaction site:

$$\langle \mathbf{F} \rangle = \frac{1}{4}\alpha_r \nabla |\mathbf{E}|^2 + \frac{k_0 \alpha_i}{\varepsilon_0}\left[\frac{\langle \mathbf{S} \rangle}{c} + c\nabla \times \langle \mathbf{L}_S \rangle\right]. \qquad (1)$$

Equation (1) represents the average force exerted by a monochromatic field of frequency $\omega = k_0 c$ on a dipolar scatterer. Notice that the imaginary part of the particle polarizability is simply related to the total cross section σ through $\sigma = k_0 \alpha_i / \varepsilon_0$. The first term in the force of Eq. (1) is a conservative field contribution derived from the scalar potential $|\mathbf{E}|^2$ and is commonly referred to as the *gradient force*. The second term, proportional to the average Poynting vector $\mathbf{S}$, represents the so-called *radiation pressure*. Finally, the last contribution is directly related to a nonuniform spin angular momentum density of the electromagnetic field $\langle \mathbf{L}_S \rangle = (\varepsilon_0 / 4\omega i)\mathbf{E} \times \mathbf{E}^*$. We will refer to this contribution as the *spin force* throughout this Letter. The second and the third components of the optical force are often designated in the literature as nonconservative.

The gradient force offers a high degree of versatility in trapping or manipulating particles, based on the simple fact that it will in general push or pull a polarizable object toward the regions of highest modulus of $|\mathbf{E}|^2$ [3,5]. More subtle is harnessing the behavior of the other two components in Eq. (1). The very fact that radiation pressure is proportional to the Poynting vector intuitively suggests that the effect on a dipolar particle would be to push it away from the radiation source. Yet, this is not always the case, due to the complex interplay between radiation pressure and spin force, as recently shown by Sukhov and Dogariu [6] using a combination of Bessel beams. Other possibilities include the use of active particles [7].

In this Letter we would like to address the following question: Is it possible to devise a *translation invariant* field distribution capable of *uniformly* attracting a dipolar object toward the source of radiation? Requiring a translation invariant field, such as along z, has important conceptual and practical implications, most notably the fact that such a *tractor field* would exert the same force on a particle regardless of its axial z position.

From an electromagnetic point of view, any electric or magnetic component of a z translation invariant field is constrained to have a general functional dependence of the form $f(x,y)\exp(i\beta z)$. Such a requirement is in general satisfied by waveguide modes, but also by free space diffraction-free beams like, for example, those of the Bessel family [8]. Based on these assumptions, it is straightforward to show that the Maxwell set decouples into TE(z) and TM(z) modes, expressible solely in terms of the longitudinal electric and magnetic fields $E_z(x,y)e^{i\beta z}$ and $H_z(x,y)e^{i\beta z}$ as follows:

$$\bar{\mathbf{E}} = \left[i\frac{\beta \nabla_T E_z - \omega\mu_0 \hat{\mathbf{z}} \times \nabla_T H_z}{k_0^2 \varepsilon - \beta^2} + \hat{\mathbf{z}} E_z\right] e^{i\beta z},$$
$$\bar{\mathbf{H}} = \left[i\frac{\omega\varepsilon_0 \varepsilon \hat{\mathbf{z}} \times \nabla_T E_z + \beta \nabla_T H_z}{k_0^2 \varepsilon - \beta^2} + \hat{\mathbf{z}} H_z\right] e^{i\beta z}, \qquad (2)$$

where ∇_T stands for the transverse gradient operator and ε is the dielectric constant of the host medium. Using Eq. (2) in the general expression in Eq. (1) leads to the first conclusion of this work regarding longitudinal forces in translation invariant fields:

$$\langle F_z \rangle = \frac{\alpha_i [(k_0^2\varepsilon - \beta^2)^2 |E_z|^2 + |\beta \nabla_T E_z + k_0 \varepsilon \eta \nabla_T H_z \times \hat{\mathbf{z}}|^2]}{2(k_0^2\varepsilon - \beta^2)^2} \beta, \tag{3}$$

where η is the vacuum characteristic impedance. For passive particles $\alpha_i > 0$, and therefore the longitudinal force is the product of a definite positive prefactor and the field propagation constant β. Based on the result [Eq. (3)], the possibility of translation invariant tractor beams in homogeneous dielectric media is ruled out, since $\beta > 0$. The present formulation applies to particles with isotropic polarizability, such as nanospheres. The extension to other geometries such as ellipsoids [9] or more complex composite nanostructures [10] is straightforward and leads to similar conclusions.

At this point a question naturally arises as to whether and under what conditions a translation invariant tractor field could be obtained. The easiest answer in principle, but most elusive in practice, would be considering a background material with a negative index of refraction. In that case power could flow away from the source while $\beta < 0$, therefore reversing the direction of F_z. Unfortunately, though, the current realizations of bulk optical negative [11] index metamaterials involve complex and densely packed inclusions that would, in fact, impede the motion of a test particle. A more careful consideration of this problem indicates that the requirement of a negative index medium is far too stringent and that, indeed, it can be relaxed by instead considering negative index *modes*.

Backward-wave propagation is generally achieved in waveguide configurations by periodic perturbations of the guiding structure [12]. In these instances the mode propagation constant β is defined, and is to be understood, as the "crystal momentum" is intended in solid-state physics [13]. The "modes" of a periodic waveguide are hence invariant only under discrete translations, as opposed to the case of longitudinally uniform structures where the modes display continuous translational symmetry. This also marks a departure from negative index bands in photonic crystals [14]. Based on this, longitudinally modulated structures cannot provide the translation invariance that we are seeking.

Radically different is the case of longitudinally uniform backward-wave waveguides. To the best of our knowledge, the only configuration falling in this category (not relying on surface plasmons) is given by the Clarricoats–Waldron waveguides [15], sketched in Fig. 1. The structure consists of a hollow metallic waveguide coaxially loaded with a high permittivity rod.

Introduced for the first time in 1960 by Clarricoats and Waldron in the microwave domain, these waveguides have the unique property of supporting modes with negative phase velocity without resorting to any longitudinal perturbation or negative index material loading. As an example, in Fig. 2 we show the dispersion curve of the backward mode in a square Clarricoats–Waldron waveguide loaded with a germanium rod and operating around 2 μm.

A field analysis of such configurations [16] reveals extended regions of reverse Poynting vector in every transverse cross section. The origin of such a phenomenon lies in the vorticity of the magnetic lines of force induced by the strong dielectric discontinuity in the waveguide interior.

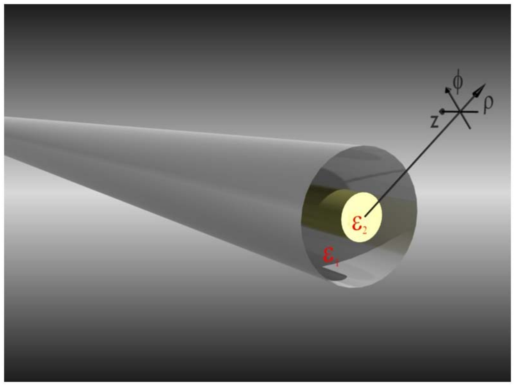

Fig. 1. (Color online) Layout of a circular Clarricoats–Waldron waveguide. Different cross sections are also possible.

The extension of such interesting configurations to the optical domain is made difficult by the need for low loss metallic walls and high permittivity materials for the waveguide core. Moreover, the fact that such structures are fully enclosed in metal renders access to the internal region virtually impossible from outside.

Of interest will be to devise a new class of structures that could support backward-wave propagation as in a Clarricoats–Waldron waveguide, without the need for metallic walls or any other reflective enclosures that could prevent access to the internal regions. In other words, is there any way to remove the metallic boundaries without affecting the field distribution of a Clarricoats–Waldron waveguide? With this idea in mind, the dispersion relation and the field distribution shown in Fig. 2 were obtained for a structure enclosed in ideal lossless metal. The single Clarricoats–Waldron element is, in fact, for our purposes, just an intermediate theoretical step to the realization of a new structure where perfect electric conductor boundary conditions are created by the very symmetry of the mode propagating in the structure.

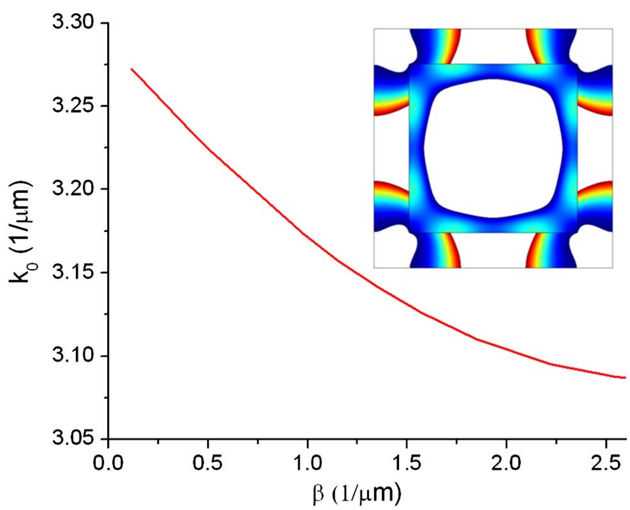

Fig. 2. (Color online) Dispersion of a Ge loaded square Clarricoats–Waldron waveguide. Inset: regions of negative Poynting vector.

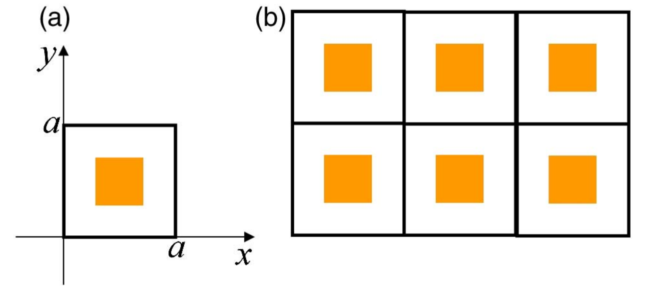

Fig. 3. (Color online) (a) Single square element, (b) two-dimensional array of isolated elements.

As a consequence of the uniqueness theorem in electrodynamics, a necessary and sufficient condition in order to remove the metallic walls without affecting the internal field is that the tangential components of both the electric and the magnetic fields at the waveguide boundary remain unaltered. To this end we consider an array of Clarricoats–Waldron waveguides with square cross sections, as shown in Fig. 3(b). In this case there is no interaction between adjacent elements, since each waveguide is fully shielded by a metallic boundary and independently operates in the backward-wave regime. We further make the assumption that neighboring waveguides are excited π out of phase. These field configurations correspond to Floquet–Bloch modes at the Brillouin zone boundaries.

Given the fact that the tangential electric field vanishes at each of the metallic boundaries, it is easy to understand that removing them would not affect the continuity of the transverse field. Concerning the magnetic field, eliminating the waveguide walls suppresses the surface currents that ordinarily extinguish the external field. With reference to Fig. 3(a), a necessary and sufficient condition to ensure the continuity of the tangential magnetic fields in the absence of the metallic boundary is that the mode under consideration is endowed with the following symmetry:

$$H_x|_{y=0} = H_x|_{y=a}; \qquad H_y|_{x=0} = H_y|_{x=a}. \tag{4}$$

If the condition in Eq. (4) holds, all the walls separating adjacent elements can be removed without affecting the modal fields. Notice that in principle the external wall enclosing the whole array is still necessary, except for infinite arrays. Nevertheless, if one considers the elements far from the periphery of a large array, they will be affected only marginally by the absence of the external enclosure. The full-wave finite element simulation presented in Fig. 4 confirms this expected behavior in the dielectric array obtained by removing the internal walls from the structure of Fig. 2. In this specific example we considered an array of germanium ($\varepsilon = 16$) square rods in air, of side length 600 nm operating at $\lambda = 2\,\mu$m. The center-to-center distance between elements is 850 nm. The propagation constant associated with such a mode is $\beta = -85.6 \cdot 10^4\,\mathrm{m}^{-1}$, corresponding to an effective mode index of $n_e = -0.27$. Similar results can be obtained at visible frequencies by using gallium phosphide rods ($n = 3.45$ at $\lambda = 550$ nm).

As a consequence of Eq. (3), a particle placed in the empty regions between the dielectric rods would be propelled backward, against the power flow and the radiation pressure. As previously pointed out, such optical force is uniform along the longitudinal direction. This is, of course, due to the translation invariance of the modal field. To the best of our knowledge, this is currently the only configuration allowing a translation invariant reversal of the nonconservative optical forces.

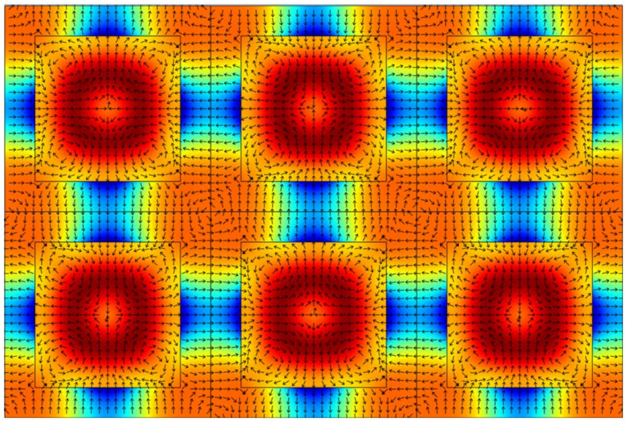

Fig. 4. (Color online) Electric field and intensity distribution in a six-element array of square dielectric rods of permittivity 16.

In conclusion, we have shown that a translation invariant tractor beam cannot be generated in a homogeneous medium such as free space. Instead we showed that a negative index environment can be created in a properly designed dielectric array. A particle immersed in the modal field of such optical array would be propelled upstream, against the power flow and the radiation pressure of the incident mode. A tractor force uniform with respect to the longitudinal coordinate is ensured by the longitudinal translation invariance of such an electromagnetic field distribution.

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