Resonant delocalization and Bloch oscillations in modulated lattices

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We study the propagation of light in Bloch waveguide arrays exhibiting periodic coupling interactions. Intriguing wave packet revival patterns as well as beating Bloch oscillations are demonstrated. A new resonant delocalization phase transition is also predicted. © 2011 Optical Society of America

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Optical waveguide arrays have been a subject of intense study in the past few years. Such structures are known to provide a versatile means to control the behavior of light in both the linear and nonlinear regimes. For example, in the linear domain, beams propagating in uniform optical arrays experience discrete diffraction [1,2]—a process with altogether different characteristics from its free space counterpart. At high enough powers, however, optical discrete solitons can also form [1,2]. The propagation dynamics and stability properties of these nonlinear states have been thoroughly studied [2] in several settings. One fascinating aspect of optical lattices is their ability to mimic idealized solid-state systems [3–7]. For example, the action of a constant bias field in a periodic quantum potential forces an electron to execute oscillations around its mean position, better known as Bloch oscillations. In optical array configurations, this can be readily accomplished by linearly ramping the onsite effective refractive index or optical potential. Optical Bloch oscillations were first suggested in the late 1990s and were subsequently observed in AlGaAs and polymer waveguide lattices [3,4]. Nonaxially uniform configurations have also been used to excite optical Bloch oscillations [8,9]. Another intriguing phenomenon associated with Bloch lattices is dynamic localization—first predicted within the context of solid state physics [10]. This effect arises from the interaction between a periodic potential and a time harmonic bias force. In this regime, when the bias strength and modulation frequency are related through the zeros of Bessel functions, the coupling between sites is suppressed, and hence dynamic localization occurs. Quite recently, curved periodic structures were investigated and used to unequivocally demonstrate dynamic localization in optics along with many other interesting phenomena [11,12].

In this Letter we study the propagation dynamics of optical beams in 1D waveguide Bloch lattices when both the coupling constants and the strength of the ramping potential vary as a function of propagation distance. One possible realization of such an array configuration is depicted in Fig. 1, where the linear transverse index profile can be achieved through a temperature gradient as in the case of [4]. Another possibility in implementing such structures is to change the width of each waveguide in an ascending fashion [3].

The periodic modulation of the width of each element makes the coupling among successive elements also vary periodically along the propagation direction. We show that the analytical solution of this problem predicts a number of interesting phenomena. Under constant bias conditions, localization (with or without revivals) can occur. Under a certain condition however, the dynamics change entirely and the input experiences diffraction or delocalization. Depending on the design parameters, Bloch oscillations exhibiting two main different frequencies (beating oscillations) can exist.

We begin our analysis by considering the evolution equation that describes light transport in a periodically modulated Bloch array, similar to that shown in Fig. 1. In the tight binding approximation, this equation reads

$$i\frac{d\varphi_n}{dz}+\kappa(z)[\varphi_{n+1}+\varphi_{n-1}]+f(z)n\varphi_n=0. \qquad (1)$$

In Eq. (1), φ_n represents the optical field modal amplitude at the waveguide site n, and $\kappa(z)$ is the coupling constant between any two adjacent channels, and in general it can vary along the propagation direction z. On the other hand, $f(z)$ is the ramping strength of this Bloch array, which can also be a function of z. By introducing a new coordinate via the transformation $\eta(z)=\int_0^z \kappa(z')\mathrm{d}z'$, we find that $\frac{\partial}{\partial z}=\frac{\partial \eta}{\partial z}\frac{\partial}{\partial \eta}=\kappa(z)\frac{\partial}{\partial \eta}$, and hence Eq. (1) takes the form

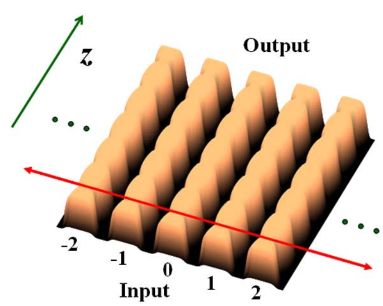

Fig. 1. (Color online) Possible realization of a periodically modulated Bloch lattice.

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$$i\frac{d\varphi_n}{d\eta} + (\varphi_{n+1} + \varphi_{n-1}) + ng(z)\varphi_n = 0, \quad (2)$$

where $g(z) = f(z)/\kappa(z)$. Note that Eq. (2) still contains η and z. However, because η varies with z, it follows that one can invert this dependence and express z as a function of η. If we write $z = h(\eta)$, we find that $g(z) = g(h(\eta)) = s(\eta)$, and Eq. (2) finally becomes

$$i\frac{d\varphi_n}{d\eta} + (\varphi_{n+1} + \varphi_{n-1}) + ns(\eta)\varphi_n = 0. \quad (3)$$

By comparing Eqs. (1) and (3), we find that geometries with modulated coupling can be mathematically mapped on other structures with varying propagation constants. It must be noted however that physical realizations of both systems is completely different. The modulus of the field amplitude $|\varphi_n|$ in Eq. (3) can then be obtained from the Dunlap and Kenkre solution [10,13]:

$$|\varphi_n|^2 = J_n^2\left(2\sqrt{v^2(\eta) + u^2(\eta)}\right). \quad (4)$$

In Eq. (4) $u(\eta) = \int_0^\eta \cos[\vartheta(\eta')]d\eta'$, $v(\eta) = \int_0^\eta \sin[\vartheta(\eta')]d\eta'$ and $\vartheta(\eta) = \int_0^\eta s(\eta')d\eta'$. Note that $\vartheta(\eta) = \vartheta(\eta(z)) = \vartheta(z) = \int_0^z g(z')(\partial\eta'/\partial z')dz'$. By using the relationships $g(z') = f(z')/\kappa(z')$ and $(\partial\eta'/\partial z') = \kappa(z')$, we arrive at the result of $\vartheta(z) = \int_0^z f(z')dz'$. Following similar arguments, u and v in Eq. (4) can now be expressed as

$$u(z) = \int_0^z \cos[\vartheta(z')]\kappa(z')dz', \quad (5a)$$

$$v(z) = \int_0^z \sin[\vartheta(z')]\kappa(z')dz'. \quad (5b)$$

This solution applies in general, irrespective of the nature of the modulating functions. We now focus our attention on the special case under consideration where the Bloch slope $f(z) = f_0$ is constant, and $\kappa(z) = \kappa_0 + \varepsilon\sin(\Gamma z)$ where κ_0 is the average coupling constant between waveguides, ε is the coupling modulation amplitude, and Γ is the modulation frequency along the propagation direction. Under these conditions,

$$u(z) = \int_0^z \cos(f_0 z')[\kappa_0 + \varepsilon\sin(\Gamma z')]dz' = \frac{\kappa_0}{f_0}\sin(f_0 z) + \varepsilon\int_0^z \cos(f_0 z')\sin(\Gamma z')dz', \quad (6a)$$

$$v(z) = \int_0^z \sin(f_0 z')[\kappa_0 + \varepsilon\sin(\Gamma z')]dz' = \frac{\kappa_0}{f_0}[1 - \cos(f_0 z)] + \varepsilon\int_0^z \sin(f_0 z')\sin(\Gamma z')dz'. \quad (6b)$$

Depending on the relative values of f_0 and Γ, Eq. (6) predicts different scenarios for the system's dynamics. The first terms in both Eqs. (6a) and (6b) are periodic and hence bounded. The second terms in these equations, however, can lead to different behavior depending on the two oscillatory functions under the integral. More specifically, if $f_0 \neq \Gamma$, the kernel will be bounded, and hence u and v themselves are bounded and the general solution exhibits localization. We note that in this case, the dynamic localization is different from that considered in earlier studies. Figures 2(a) and 2(b) depict such a situation for two different cases, e.g., when $\Gamma/f_0 = 0.5$ and $\Gamma/f_0 = 1.1$, respectively.

In these simulations, the optical beam was assumed to excite only the center channel. It is evident that the impulse response is indeed localized and periodic. On the other hand, when $\Gamma/f_0 = 1$, the situation is completely altered because the integral kernel in Eq. (6b) has a nonzero dc component and v grows monotonically, i.e., it ceases to be bounded. Consequently, the input beam undergoes diffraction as a result of this resonant delocalization (RD) process, as shown in Fig. 2(c).

Another interesting property of this system is the occurrence of localization without periodicity. As explained in Fig. 2(a) and 2(b), the input beam is not only localized but also experiences revivals. The revival period and the diffraction pattern between any two consecutive revivals depend on the ratio Γ/f_0. However, when this ratio is an irrational number, we find that the solution is still bounded but the periodicity disappears completely. Figure 3 illustrates this effect when $\Gamma/f_0 = 1/\sqrt{2}$. The top view of the beam evolution in the array is shown in Fig. 3(a), where one can observe localization effects, while Fig. 3(b) singles out the field amplitude in the middle channel, clearly indicating the aperiodicity of the field

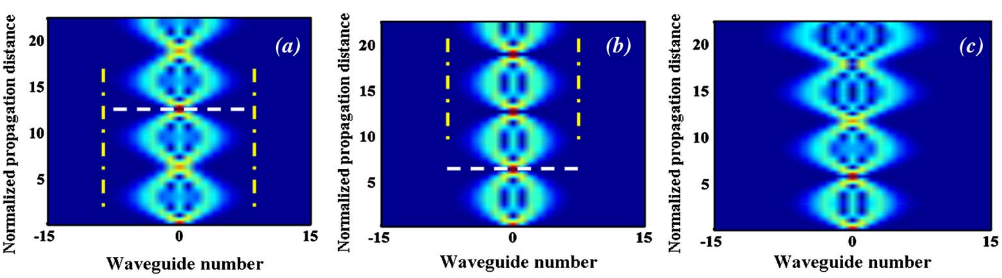

Fig. 2. (Color online) Numerical results for a single input excitation for the lattice parameters $\kappa_0 = 1$, $f_0 = 1$, and $\epsilon = 0.2$ when (a) the modulation frequency $\Gamma = 0.5f_0$, (b) $\Gamma = 1.1f_0$, and (c) the resonance condition $\Gamma = f_0$ is satisfied. In (a) and (b), the white dashed line marks the cross section at which a complete revival occurs, while yellow lines indicate that the pattern is always localized in some waveguide range. Note that in (c), the input beam is no longer confined; instead, it experiences oscillatory diffraction.

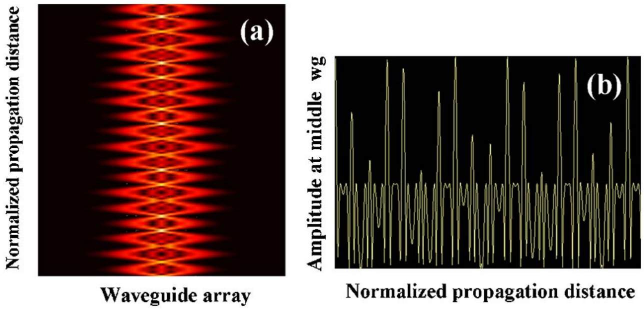

Fig. 3. (Color online) Localization without revivals when $\kappa_0 = 1, f_0 = 1, \epsilon = 0.2$, and $\Gamma = f_0/\sqrt{2}$: (a) top view and (b) evolution of field amplitude in the excitation channel.

amplitude. It is important to note that light localization can also arise in some other geometries, such as with the uniform optical lattice with an alternate sign of the coupling coefficients [14,15], where the process involves the coherent destruction of tunneling.

Next we consider how the system behaves under Gaussian beam excitation. Figure 4 shows the Gaussian beam evolution corresponding to two different scenarios, namely $\Gamma/f_0 = 0.9$ and $\Gamma/f_0 = 1.1$, respectively. Evidently, the beam undergoes Bloch oscillations. However, in contrast to regular Bloch oscillations, in the present case we observe two main different oscillation frequencies. An important remark here is the existence of a phase difference between the slow and fast periods. As shown in Fig. 4, this phase difference depends on the ratio Γ/f_0 and manifests itself in the initial direction in which the input beam bends.

We would like to emphasize that the behavior described above is by no means a result of the specific forms of the functions $\kappa(z)$ and $f(z)$. To further elucidate this point, consider the case when $\kappa(z) = \kappa_0 + \varepsilon\cos(\Gamma z + \theta)$ and $f(z) = f_0\cos(\omega z)$. In this latter arrangement, the propagation constant is now modulated along the propagation direction instead of being constant, as in the previous case. Under these conditions, the solution assumes the same form as before with the difference that

$$\vartheta(z) = \int_0^z f_0\cos(\omega z')\mathrm{d}z' = (f_0/\omega)\sin(\omega z),$$

and hence Eq. (5) reads

$$u(z) = \int_0^z \cos[(f_0/\omega)\sin(\omega z')](\kappa_0 + \varepsilon\cos(\Gamma z' + \theta))\mathrm{d}z',$$

$$v(z) = \int_0^z \sin[(f_0/\omega)\sin(\omega z')](\kappa_0 + \varepsilon\cos(\Gamma z' + \theta))\mathrm{d}z'.$$

Resonant delocalization will take place if, in the above equations, the kernel of either $u(z)$ or $v(z)$ contains a dc component. By using the relations $\cos[(f_0/\omega)\sin(\omega z)] = J_0(f_0/\omega) + 2\sum_{m=1}J_{2m}(f_0/\omega)\cos(2m\omega z)$ and $\sin[(f_0/\omega)\sin(\omega z)] = 2\sum_{m=0}J_{2m+1}(f_0/\omega)\sin[(2m+1)\omega z]$, we find that delocalization occurs whenever $J_0(f_0/\omega) \neq 0$. This last condition was also obtained within the context of dynamic localization [10]. When $J_0(f_0/\omega) = 0$, RD can still be observed if $\Gamma = (2m+1)\omega$ and $\theta \neq l\pi$ or when $\Gamma = 2m\omega$ and $\theta \neq \pi/2 + l\pi$, where m and l are integers. We would like to point out that these last two scenarios are absent in the dynamic localization case and are only inherent to systems in which coupling constants among waveguides are modulated.

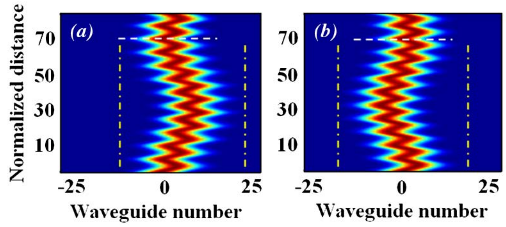

Fig. 4. (Color online) (a), (b) Beating Bloch oscillations (for a Gaussian input beam) corresponding to $\Gamma = 0.9f_0$ and $\Gamma = 1.1f_0$, respectively. White dashed lines mark a complete period, while yellow lines indicate the localization domains. Optical array design parameters in this case are $\kappa_0 = 1$, $f_0 = 1$, and $\epsilon = 0.5$.

In conclusion, we studied the propagation of optical beams in dynamic 1D Bloch waveguide lattices having periodic coupling coefficients. In this regime, several interesting phenomena are expected. Under specific conditions, the evolution dynamics can undergo an RD. A new type of beating Bloch oscillation was also predicted.

References

1. D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13**, 794 (1988).
2. D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature **424**, 817 (2003).
3. U. Peschel, T. Pertsch, and F. Lederer, Opt. Lett. **23**, 1701 (1998).
4. T. Pertsch, P. Dannberg, W. Elflein, and A. Bräuer, Phys. Rev. Lett. **83**, 4752 (1999).
5. H. Trompeter, W. Krolikowski, D. N. Neshev, A. S. Desyatnikov, A. A. Sukhorukov, Yu. S. Kivshar, T. Pertsch, U. Peschel, and F. Lederer, Phys. Rev. Lett. **96**, 053903 (2006).
6. T. Schwartz, G. Bartal, S. Fishman, and M. Segev, Nature **446**, 52 (2007).
7. Y. Lahini, A. Avidan, F. Pozzi, M. Sorel, R. Morandotti, D. N. Christodoulides, and Y. Silberberg, Phys. Rev. Lett. **100**, 013906 (2008).
8. G. Lenz, R. Parker, M. Wanke, and C. M. de Sterke, Opt. Commun. **218**, 87 (2003).
9. G. Lenz, I. Talanina, and C. M. de Sterke, Phys. Rev. Lett. **83**, 963 (1999).
10. D. H. Dunlap and V. M. Kenkre, Phys. Rev. B **34**, 3625 (1986).
11. S. Longhi, M. Marangoni, M. Lobino, R. Ramponi, P. Laporta, E. Cianci, and V. Foglietti, Phys. Rev. Lett. **96**, 243901 (2006).
12. A. Szameit, I. L. Garanovich, M. Heinrich, A. A. Sukhorukov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann, and Y. S. Kivshar, Nat. Phys. **5**, 271 (2009).
13. V. M. Kenkre and S. Raghavan, J. Opt. B: Quantum Semiclass. Opt. **2**, 686 (2000).
14. N. K. Efremidis, P. Zhang, Z. Chen, D. N. Christodoulides, C. E. Rüter, and D. Kip, Phys. Rev. A. **81**, 053817 (2010).
15. Z. Peng, N. K. Efremidis, A. Miller, Y. Hu, and Z. Chen, Opt. Lett. **35**, 3252 (2010).