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Classical analogues to quantum nonlinear coherent states in photonic lattices

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ABSTRACT

We show that light evolution occurring in waveguide arrays with a particular n-functional square root dependence of coupling coefficients can be used to produce classical analogues of nonlinear quantum coherent states. Using operator algebras we obtain closed-form expressions describing the optical field dynamics in such structures. In addition, by numerically monitoring the Mandel's parameter, we obtain the conditions necessary to generate sub-Poissonian and super-Poissonian classical intensity distributions in the proposed photonic lattices.

1. Introduction

Optical discrete systems have been recently employed to observe a host of optical processes in both the linear and nonlinear regime [1,2]. On many occasions, these effects have direct analogues in optics and solid state physics. In particular, optical waveguide arrays provide a versatile environment where discrete diffraction can be highly controlled and tailored at will to match specific applications [3]. Along these lines, effects ranging from Rabi oscillations and Talbot revivals, to Anderson localization and resonant localization have been successfully observed.

On the other hand, considerable experimental and theoretical work has been carried out on optical systems exhibiting quantum/classical correspondences [4–8]. One topic of great interest in quantum computing is that associated with the dynamics of trapped ions. The description of these configurations is typically carried out through the so-called nonlinear coherent states (NLCS) [9]. In principle such NLCS can be generated by applying a deformed displacement operator on vacuum states. Particular versions of nonlinear coherent states have been frequently encountered in several nonlinear quantum optics arrangements, as for example in Kerr nonlinear media [10]. Of interest will be to devise optical systems where classical analogues of NLCS can be directly observed and studied.

In this communication we show that it is possible to classically emulate NLCS by using photonic lattices with a particular n-functional dependence in their coupling coefficients, namely $\kappa(n) = \sqrt{(n+\chi n^2)/\chi}$, where χ is the so-called "deforming" parameter and n is the site position of any channel. We would like to mention that here the term χ represents another degree of freedom in nonlinearly altering the coupling coefficients between neighboring channels. Since the coupling constants between parallel waveguides depend exponentially with the separation distance d_n, $\kappa_{n,n+1} \sim \exp(-\gamma d_n)$ [11,12], the proposed n-functional dependence of the coupling constants can be established by simply adjusting the separation between waveguide elements. These waveguide arrays can be fabricated on semiconductor wafers by etching or in bulk silica by direct laser writing [11]. In fact, in the latter array system is easier to observe from the top light evolution using fluorescence effects.

2. Analysis and results

In the proposed system, the normalized modal field evolution can be described by the following set of coupled differential equations

$$i\frac{dE_0}{dZ} + f(1)E_1 = 0, \tag{1a}$$

$$i\frac{dE_n}{dZ} + f(n+1)E_{n+1} + f(n)E_{n-1} = 0, \tag{1b}$$

$$f(n) = \sqrt{\frac{n+\chi n^2}{\chi}}, \tag{1c}$$

where Eq. (1a) describes the field dynamics at the edge of the lattice (at $n = 0$) while Eq. (1b) at any other site $n > 0$. The normalized

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coordinate z is given by $Z = \kappa_1 z$, where z is the actual propagation distance and κ_1 stands for the coupling coefficient between sites 0 and 1.

In order to obtain the impulse response of this array system, i.e. when only one channel is excited, we consider the equation

$$i\frac{d\psi(x,Z)}{dZ} = -\left(A + A^{\dagger}\right)\psi(x,Z), \tag{2}$$

where the *nonlinear* "annihilation/creation" operators are respectively [13,14]

$$A = \sqrt{\frac{1+\chi}{\chi}}\, a\, f(n) = \sqrt{\frac{1+\chi}{\chi}} f(n+1)a, \quad A^{\dagger} = \sqrt{\frac{1+\chi}{\chi}} f(n)a^{\dagger} = \sqrt{\frac{1+\chi}{\chi}} a^{\dagger} f(n+1), \tag{3}$$

and $n = a^{\dagger}a$. The usual "annihilation/creation" operators are respectively defined by [15]

$$a = \frac{1}{\sqrt{2}}\left(x + \frac{d}{dx}\right), \tag{4.a}$$

$$a^{+} = \frac{1}{\sqrt{2}}\left(x - \frac{d}{dx}\right), \tag{4.b}$$

such that

$$a\psi_n(x) = \sqrt{n}\psi_{n-1}(x), \tag{5.a}$$

$$a^{\dagger}\psi_n(x) = \sqrt{n+1}\psi_{n+1}(x). \tag{5.b}$$

The eigenfunctions ψ_n are given by Gauss–Hermite functions

$$\psi_n(x) = \frac{1}{\sqrt{\pi^{1/2}2^n n!}} \exp\left(-x^2/2\right) H_n(x). \tag{6}$$

In a similar fashion, as in quantum theory, a Hamiltonian with the form of a harmonic oscillator can be written in terms of the deformed operators

$$H = \frac{\Omega^2}{2}\left(A^{\dagger}A + AA^{\dagger}\right). \tag{7}$$

By substituting Eqs. (1c) and (3), in Eq. (7) we find that

$$H = \left[n^2 + (n+1)^2 + \chi n^3 + \chi(n+1)^3\right], \tag{8}$$

where we have defined $\Omega^2/2 = \chi^2/(1+\chi)$. Notice that this Hamiltonian contains a cubic and a quadratic nonlinearity in the number operator.

Note that the set of functions $\psi_n(x)$ is complete and orthogonal; therefore, we can always expand any function in terms of $\psi_n(x, Z)$, e.g.

$$\psi(x,Z) = \sum_{n=0}^{\infty} E_n(Z)\psi_n(x). \tag{9}$$

In order to introduce Dirac's notation, we establish a correspondence between $\psi_n(x)$ and $|n\rangle$, ($\psi_n(x) \to |n\rangle$). Substitution of Eq. (9) into Eq. (2) then yields

$$i\sum_{n=0}^{\infty} \frac{dE_n(Z)}{dZ}|n\rangle = -\sum_{n=0}^{\infty} \left(f(n)E_n(Z)|n-1\rangle + f(n+1)E_n(Z)|n+1\rangle\right). \tag{10}$$

Using the orthonormality properties of the eigenfunctions $|n\rangle$, we finally derive Eq. (1b). In this manner, we have effectively obtained the equations governing light evolution in the system under investigation. Eq. (2) can be readily solved using the evolution operator

$$|\psi(z)\rangle = \exp\left[iZ\left(A + A^{\dagger}\right)\right]|\psi(0)\rangle. \tag{11}$$

Unlike the operators a and $a^{\dagger}$, operators A and $A^{\dagger}$ obey a more complicated commutation relation, $\left([A, A^{\dagger}] = \left(\frac{1+\chi}{\chi^2}\right)((n+1) + \chi(n+1)^3 - n^2 - \chi n^3)\right)$. Direct use of the Baker–Hausdorff formula is not allowed in factorizing the exponential in Eq. (11). Instead another way should be pursued. To show this, we define a new operator A_0 as follows

$$A_0 = n + \frac{1}{2\chi} + \frac{1}{2}, \tag{12}$$

such that, jointly these three operators, $(A, A^{\dagger}, A_0)$, provide an operator algebra where the commutation relations between them are closed and are given by

$$\left[A_0, A^{\dagger}\right] = A^{\dagger}, \quad [A_0, A] = -A, \quad \left[A, A^{\dagger}\right] = 2A_0. \tag{13}$$

Notice that these operators form an $SU(1,1)$ group and therefore the exponential in Eq. (11) may be factorized as follows [16]

$$\exp\left(iZ\left(A + A^{\dagger}\right)\right) = \exp\left(iF(Z)A^{\dagger}\right)\exp\left(G(Z)A_0\right)\exp(iH(Z)A). \tag{14}$$

By differentiating both sides of Eq. (14) with respect to Z, and by using the commutation relations, one can then show that F, G, and H are given by

$$F(Z) = H(Z) = \tanh(Z), \quad G(Z) = -2\ln[\cos(Z)]. \tag{15}$$

In this case, Eq. (11) takes the form:

$$|\psi(z)\rangle = \exp(i\tanh(Z))\exp(-2\ln(\tanh(Z)))\exp(i\tanh(Z))|\psi(0)\rangle. \tag{16}$$

To evaluate the field distribution in the mth waveguide when light is launched into a single lattice site at position k from the edge, the initial condition $|\psi(0)\rangle$ should be substituted by $|k\rangle$, and then perform the inner product $\langle m|\psi(z)\rangle$. In the particular case where the first waveguide is excited ($|k\rangle = |0\rangle$), the entire propagating lattice field configuration can be obtained and it corresponds to a classical analogue of a nonlinear coherent state. This is given by:

$$\left|\alpha = i\sqrt{\frac{1+\chi}{\chi}}\tanh(Z)\right\rangle = (\cosh(Z))^{-((1+\chi)/\chi)} \sum_{n=0}^{\infty} \frac{(\alpha)^n}{\sqrt{n!}}[f(n)]!\,|n\rangle. \tag{17}$$

In this realm, we are now in the position to emulate these quantum states by optical means, namely in specially designed waveguide arrays exhibiting the coupling rule of Eq. (1). The amplitude field distribution, at any distance Z, in the nth lattice site is provided by the expression

$$E_n(Z) = \frac{(\cosh(Z))^{-((1+\chi)/\chi)}}{\sqrt{n!}}\left(i\sqrt{\frac{1+\chi}{\chi}}\tanh(Z)\right)^n [f(n)]!, \tag{18}$$

where $[f(n)]! \equiv f(n)f(n-1)...f(1)$. Eq. (18) provides the evolution of these particular nonlinear coherent states with distance. In the quantum domain, such states can be thought as generalizations of the coherent states associated with the linear harmonic oscillator [17]. Furthermore, they have been used to model light modes in Kerr

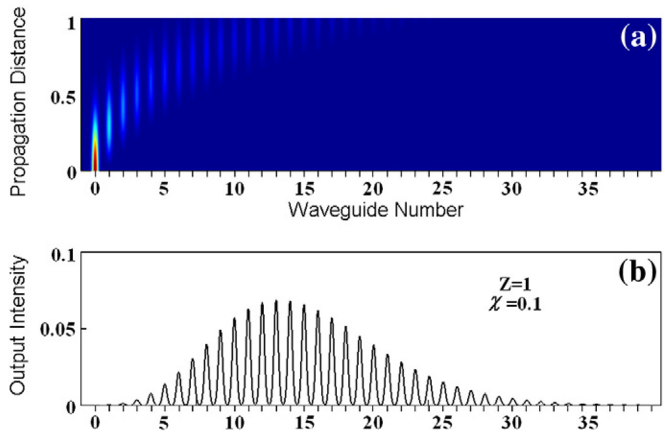

Fig. 1. Propagation dynamics for the classical nonlinear coherent state (a), and its corresponding output intensity profile (b) at Z = 1.

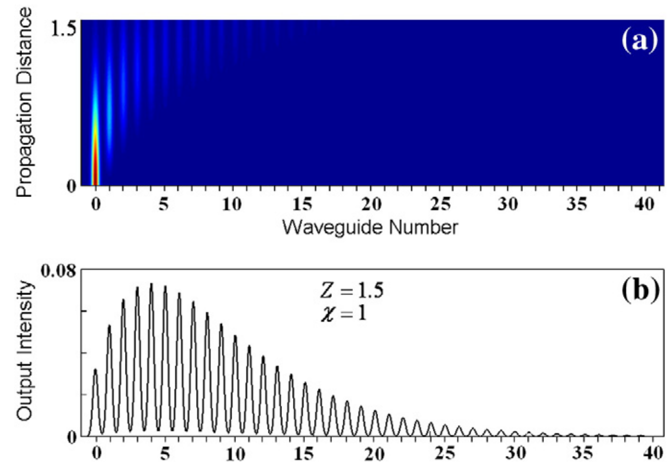

Fig. 2. Propagation dynamics for the classical nonlinear coherent state.

media, where the Hamiltonian of such systems is nonlinear in nature [18]. In addition, it has been pointed out that nonlinear coherent states exhibit many interesting properties such as squeezing, and sub-Poissonian behavior, to mention a few [19]. Figs. 1 and 2 depict the intensity evolution among waveguides when the 0th channel is initially excited. The corresponding output intensity profiles at two different propagation distances and for two different values of χ are also shown. As clearly indicated in these figures, the intensity distribution happens to be more localized around the first channel as the deforming parameter χ increases. In addition, this array performs very differently from its Glauber–Fock counterpart considered in Ref. [4]. To begin with, the χ factor cannot be totally eliminated or set to zero. Indeed, even at very low values of χ, the array response differs from the Poissonian output predicted in [4]. As Fig. 1 shows, this becomes more apparent when considering the right tails (high n values) of the intensity distribution. At higher deformations, the optical fields tend to persist around the first array channel. In all occasions, the intensity distribution at the output of the lattice resembles the photon probability distribution of a nonlinear coherent state shown for example in Ref. [20].

To monitor the nature of the intensity distribution, i.e. whether it is Poissonian, sub-Poissonian or super-Poissonian, we use Mandel's parameter $\left(Q = \left(\langle\sigma\rangle / \langle n\rangle\right) - 1 = R - 1\right)$, where σ is the variance or dispersion, and $\langle n\rangle$ is the "expectation value" of the intensity in the entire lattice [21]. Note that, Q can be any number between 0 and −1 when the process is "sub-Poissonian", and is "super-Poissonian" when it is greater than 0. If $Q = 0$, then the distribution is "Poissonian". Equivalently, we can consider the ratio:

$$R = \frac{\sigma}{\langle n\rangle}. \tag{19}$$

Depending upon the propagation distance, and the size of the deformation parameter χ, R can be equal, less than or greater than unity. For "states" with R in the range $0 \le R < 1$, the distribution function is sub-Poissonian, if $R > 1$, super-Poissonian, and if $R = 1$ is "coherent". In our case, σ is given by

$$\sigma = (\cosh(Z))^{-2\left(\frac{1+\chi}{\chi}\right)}\left[\sum_{n=0}^{N}\frac{|\alpha|^{2n}[f(n)!]^2 n^2}{n!} - (\cosh(Z))^{-2\left(\frac{1+\chi}{\chi}\right)}\left(\sum_{n=0}^{N}\frac{|\alpha|^{2n}[f(n)!]^2 n}{n!}\right)^2\right], \tag{20}$$

and the "expectation value "

$$\langle n\rangle = (\cosh(Z))^{-2\left(\frac{1+\chi}{\chi}\right)}\sum_{n=0}^{N}\frac{|\alpha|^{2n}[f(n)!]^2 n}{n!}, \tag{21}$$

where N is the total number of waveguides (where $N \to \infty$).

Fig. 3 shows that this classical nonlinear coherent state undergoes sub-Poissonian behavior for a propagation distance $Z \le 0.7$, when $\chi = 0.3$ (solid line) and for $Z \le 0.9$, $\chi = 1$ (dotted line). It then becomes "super-Poissonian" for the rest of the interval. We would like to emphasize once again that the field configurations resulting from this optical lattice are in fact classical in nature. We only used the mathematical formalism of quantum mechanics to investigate this system.

3. Conclusion

In this work we have shown that optical array lattices with specific coupling rules can support classical analogues of quantum nonlinear coherent states. The problem was solved analytically using operator algebra. We demonstrated that both classical sub-Poissonian and super-Poissonian intensity distributions can be generated over distance depending on the nonlinear deformation parameter used. Pertinent examples have been provided.

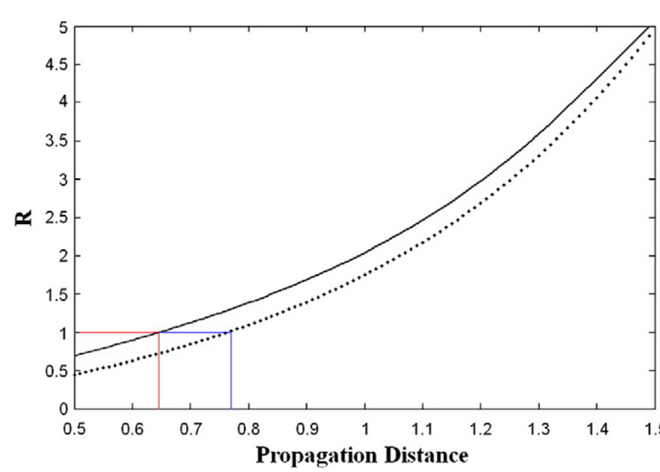

Fig. 3. Calculated ratio R for the classical nonlinear coherent state for $Z \in [0.5, 1.5]$ with two different values of χ, dotted line for $\chi = 1$, and solid line for $\chi = 0.3$.

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