Folded digital backward propagation for dispersion-managed fiber-optic transmission

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Abstract: In periodically dispersion managed long-haul transmission systems, waveform distortion is dominated by chromatic dispersion. As a result of the periodic waveform evolution, the nonlinear behavior also repeats itself in every dispersion period. It is shown that, under the weakly nonlinear assumption, nonlinear effects accumulated in a large number (K) of spans can be approximated by nonlinear effects accumulated in a single span with the same dispersion map and K times the nonlinearity. Thus, significant savings in computational load can be achieved in digital compensation of fiber nonlinearity using folded digital backward propagation (DBP). Simulation results show that the required computation for DBP of dispersion managed transoceanic transmission systems can be reduced by up to 2 orders of magnitude with negligible penalty using folded DBP.

OCIS codes: (060.2330) Fiber optics communications; (060.1660) Coherent communications; (190.4370) Nonlinear optics, fibers.

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1. Introduction

Optical signal is distorted by the joint effects of dispersion and nonlinearity during its propagation in optical fiber. In most of the installed long-haul systems, dispersion is compensated by periodically cascading two or more kinds of fiber with inverse dispersion parameters. With the advent of new inverse dispersion fibers (IDF), wide-band dispersion flatness has been obtained by compensating for both dispersion and dispersion slope while minimizing the total polarization mode dispersion (PMD) [1]. In the emerging coherent communication systems, dispersion can also be compensated using digital signal processing (DSP) [2]. As the technology of dispersion compensation matured, fiber nonlinearity effects including self phase modulation (SPM), cross phase modulation (XPM) and four-wave mixing (FWM) become the limiting factor to further increase the spectral efficiency and transmission distance of long-haul fiber communication systems [3–5].

In order to mitigate nonlinear effects, methods such as optimized dispersion management [6], large effective area fiber [1] and new modulation formats [7] have been investigated and employed. In addition to the above methods that mitigate nonlinearity, methods of compensating nonlinear impairments have been proposed. Nonlinear phase shift in dispersion-shifted fibers can be compensated with lumped nonlinear phase de-rotation based on the assumption that the intensity waveform remains unchanged throughout fiber propagation [8]. However, lumped nonlinearity compensation performs poorly where there is significant interaction between nonlinearity and dispersion. Additionally, nonlinearity pre-compensation at the transmitter side was proposed for direct-detection systems [9] and coherent optical OFDM systems [10].

Enabled by coherent detection, nonlinearity post-compensation via digital backward propagation (DBP) has attracted significant attention [11–13]. Recently, experimental demonstration of DBP for polarization-division multiplexed (PDM) wavelength-division multiplexing (WDM) transmission was reported [14].

DBP is typically implemented using the split-step method. In order for the split-step method to be accurate, a large number of steps are needed especially for inter-channel nonlinearity compensation of WDM systems, resulting in a very heavy computational load. Some efforts have been devoted to increase the computational efficiency of DBP. In comparison with solving the nonlinear Schrodinger equation (NLSE) for the total field of the WDM signal, solving the coupled NLSE was suggested because it requires a smaller step number and lower sampling rate [15,16]. The step number can be further reduced by factorizing the dispersive walk-off effects in the DBP algorithm [17] and using variable step size [18].

In this paper, we propose an efficient method of nonlinearity compensation for dispersion-managed fiber-optic transmission systems, taking advantage of the periodic behavior of the optical signal. In section 2, the theory of folded DBP is presented. In section 3, the

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effectiveness of this method is demonstrated with simulation of a long-haul fiber transmission system. The effect of residual dispersion is also investigated.

2. Theory

We focus on dispersion-managed fiber-optic transmission systems. Without loss of generality, we assume that each fiber span with a length of L is one period of the dispersion map. For long-haul fiber-optic transmission, an optimum power exists as a result of the tradeoff between optical signal to noise ratio (OSNR) and nonlinear impairments. The total nonlinear phase shift at the optimum power level is on the order of 1 radian [19]. Therefore, for transoceanic fiber transmission systems which consist of many (>100) amplified spans, the nonlinear effects in each span is weak. As a result, chromatic dispersion is the dominant factor that determines the evolution of the waveform within each span.

We can analyze the nonlinear behavior of the optical signal using a perturbation approach. The NLSE governing the propagation of the optical field, $A_j(z,t)$, in the j^{th} fiber span can be expressed as

$$\frac{\partial A_j(z,t)}{\partial z} = [D + \varepsilon \cdot N(\left|A_j(z,t)\right|^2)] \cdot A_j(z,t), \tag{1}$$

where $0 < z < L$ is the propagation distance within each span, D is the linear operator for dispersion, fiber loss and amplifier gain, $N(\left|A_j(z,t)\right|^2)$ is the nonlinear operator, ε (to be set to unity) is a parameter indicating that the nonlinear perturbation is small for the reasons given above. The boundary conditions are

$$A_1(0,t) = a(0,t), \tag{2}$$

$$A_j(0,t) = A_{j-1}(L,t) \text{ for } j \geq 2, \tag{3}$$

where $a(0,t)$ is the input signal at the beginning of the first span. We assume that the solution of Eq. (1) can be written as,

$$A_j(z,t) = A_{j,l}(z,t) + \varepsilon \cdot A_{j,nl}(z,t). \tag{4}$$

Substituting Eq. (4) into Eq. (1) and expanding the equation in power series of ε, we have

$$\frac{\partial A_{j,l}(z,t)}{\partial z} - D \cdot A_{j,l}(z,t) + \varepsilon \cdot [\frac{\partial A_{j,nl}(z,t)}{\partial z} - D \cdot A_{j,nl}(z,t) - N(\left|A_{j,l}(z,t)\right|^2) \cdot A_{j,l}(z,t)] + O(\varepsilon^2) = 0, \tag{5}$$

Equating to zero the successive terms of the series, we have

$$\frac{\partial A_{j,l}(z,t)}{\partial z} = D \cdot A_{j,l}(z,t), \tag{6}$$

$$\frac{\partial A_{j,nl}(z,t)}{\partial z} = D \cdot A_{j,nl}(z,t) + N(\left|A_{j,l}(z,t)\right|^2) \cdot A_{j,l}(z,t). \tag{7}$$

The boundary conditions are

$$A_{1,l}(0,t) = a(0,t), \tag{8}$$

$$A_{j,l}(0,t) = A_{j-1}(L,t) \text{ for } j \geq 2, \tag{9}$$

and

$$A_{j,nl}(0,t) = 0. \tag{10}$$

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First, we assume that dispersion is completely compensated in each span. As a result, at the end of the first span,

$$A_{1,l}(L,t) = a(0,t), \tag{11}$$

and

$$A_2(0,t) = A_1(L,t) = a(0,t) + \varepsilon \cdot A_{1,nl}(L,t), \tag{12}$$

where $A_{1,nl}(z,t)$ is the solution of Eq. (7) with $j = 1$. In the second span,

$$A_{2,l}(z,t) = A_{1,l}(z,t) + \varepsilon \cdot \bar{A}(z,t), \tag{13}$$

where the first and second terms are solutions to Eq. (6) with boundary conditions $A_{2,l}(0,t) = a(0,t)$ and $A_{2,l}(0,t) = \varepsilon \cdot A_{1,nl}(L,t)$, respectively, as a result of the principle of superposition. At the end of the second span, because of complete dispersion compensation,

$$A_{2,l}(L,t) = a(0,t) + \varepsilon \cdot A_{1,nl}(L,t). \tag{14}$$

The nonlinear distortion in the second span is governed by Eq. (7) with $j = 2$. Since

$$\left|A_{2,l}\right|^2 = \left|A_{1,l} + \varepsilon \cdot \bar{A}\right|^2 = \left|A_{1,l}\right|^2 + O(\varepsilon), \tag{15}$$

the differential equation and the boundary conditions for $A_{2,nl}(z,t)$ and $A_{1,nl}(z,t)$ are identical, so

$$A_{2,nl}(L,t) = A_{1,nl}(L,t). \tag{16}$$

As a result, the optical field at the end of the second span is given by

$$A_2(L,t) = A_{2,l}(L,t) + \varepsilon \cdot A_{2,nl}(L,t) = a(0,t) + \varepsilon \cdot 2A_{1,nl}(L,t). \tag{17}$$

That is, the nonlinear distortion accumulated in *2* spans is approximately the same as the nonlinear distortion accumulated in *1* span with the same dispersion map and *twice* the nonlinearity. It follows that, assuming weak nonlinearity and periodic dispersion management, the optical field after K spans of propagation can be written as

$$A_N(L,t) = a(0,t) + \varepsilon \cdot KA_{1,nl}(L,t), \tag{18}$$

which is the solution of the NLSE

$$\frac{\partial A_j(z,t)}{\partial z} = [D + \varepsilon \cdot KN(\left|A_j(z,t)\right|^2)] \cdot A_j(z,t). \tag{19}$$

The nonlinear term $N(\left|A_j(z,t)\right|^2)$ in Eq. (1) is proportional to the fiber nonlinear parameter γ. So the NLSE describing the optical propagation in a fiber span with the same dispersion map and K times the nonlinearity can be written as

$$\frac{\partial A_j(z,t)}{\partial z} = [D + \varepsilon \cdot K \cdot N(\left|A_j(z,t)\right|^2)] \cdot A_j(z,t). \tag{20}$$

The equivalence of Eqs. (19) and (20) suggests that DBP for K spans can be folded into a single span with K times the nonlinearity. Assuming that the step size for the split-step implementation of DBP is unchanged, the computational load for the folded DBP can be saved by the folding factor K.

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The above derivation is based on the assumption that waveform distortion due to fiber nonlinearity and the residual dispersion per span (RDPS) are negligible, and consequently the nonlinear behavior of the signal repeats itself in every span. This assumption is not exactly valid because first, fiber nonlinearity also changes the waveform, and secondly, dispersion is not perfectly periodic if the RDPS is non-zero or the dispersion slope is not compensated. These effects accumulate and as a result, the waveform evolutions are not identical between two spans that are far away from each other.

In order for the nonlinearity compensation to be accurate, it might be necessary to divide the entire long-haul transmission system into segments of multiple dispersion-managed spans so that the accumulated nonlinear effects and residual dispersion is small in each segment. Moreover, in order to minimize the error due to residual dispersion, folded DBP should be performed with a boundary condition calculated from lumped dispersion compensation for the first half of the segment. For a fiber link with $M \times K$ spans, the folded DBP is illustrated in Fig. 1.

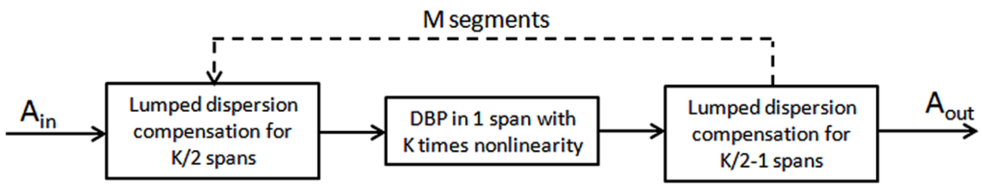

Fig. 1. Folded DBP for a periodically dispersion managed fiber link with $M \times K$ spans.

3. Simulation

We simulate a single polarization WDM system with quadrature phase-shift keying (QPSK) modulation at 56 Gb/s using VPItransmissionMaker. The simulation setup is shown in Fig. 2. 12 channels of NRZ (non-return-to-zero) QPSK signal are transmitted with 50 GHz channel spacing. The linewidth of the lasers is 100 KHz. The dispersion managed fiber link consists of 140 spans of 50 km of the OFS UltraWave SLA/IDF Ocean Fiber combination. In each span, the SLA fiber with a large effective area is used near the EDFA, followed by the IDF fiber with inverse dispersion and dispersion slope. The ASE noise is loaded at each EDFA module in the link and the noise figure is 4.5 dB. The loss, dispersion, relative dispersion slope and effective area of the SLA fiber are 0.188 dB/km, 19.5 ps/nm/km, 0.003/nm and 106 μm^2. The corresponding parameters for the IDF fiber are 0.23 dB/km, −44 ps/nm/km, 0.003/nm and 31 μm^2, respectively. The RDPS is determined by the proportion of SLA fiber to IDF fiber in each span. A piece of fiber at the receiver was used to compensate for the residual dispersion. After de-multiplexing and coherent detection, DSP was performed in Matlab.

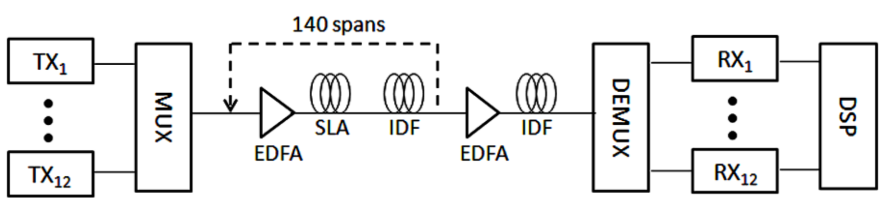

Fig. 2. Block diagram of the dispersion managed WDM system.

The DBP was performed as illustrated in Fig. 1. Without loss of generality, we solved the coupled scalar NLSE with the non-iterative asymmetric split-step Fourier method (SSFM) [13]. After matched filtering, phase estimation and clock recovery, the Q-value averages of the WDM channels were estimated.

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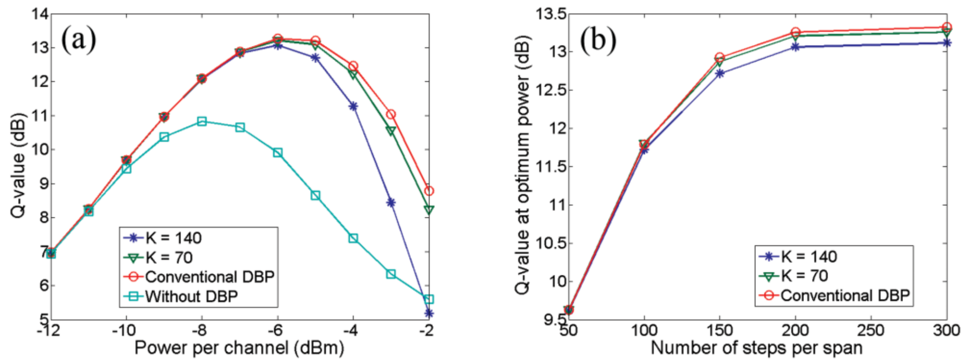

Fig. 3. (a) Q-value vs. launching power per channel and (b) Q-value at optimum power vs. step number per span with RDPS = 0 ps/nm.

We first simulated the transmission with full inline dispersion compensation, i.e., RDPS = 0. The Q-value as a function of the launching power is shown in Fig. 3(a). Without nonlinearity compensation, the maximum Q-value is 10.8 dB. With conventional DBP in all spans, the Q-value is increased to 13.3 dB as a result of nonlinearity compensation. With folded DBP with a folding factor of 140 (i.e., M = 1, K = 140), the maximum Q-value was 13.1 dB. The 0.2 dB Q-value penalty was due to the accumulated nonlinear waveform distortion which reduced the accuracy of nonlinearity compensation. There was almost no penalty when the folding factor was 70 (i.e. M = 2, K = 70).

In the split-step implementation DBP, the step size has to be small enough so that the dispersion and nonlinear effects can be properly de-coupled. In long-haul WDM fiber links, the step size is usually limited by dispersion [17,20]. Therefore in each fiber span, we use the same number of steps in SLA fiber and IDF fiber, so that the dispersion in each step is approximately the same. Figure 3(b) shows the Q-value as a function of step number per span. The folded DBP method does not require increased step number per span.

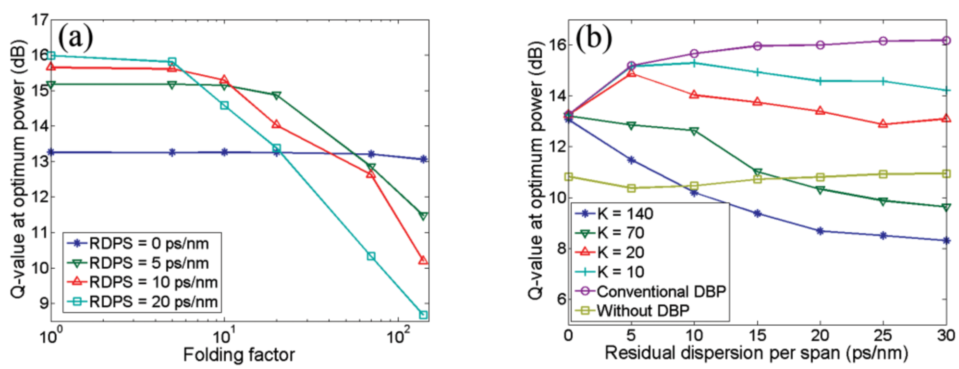

Fig. 4. (a) Q-value vs. folding factor *K* and (b) Q-value vs. RDPS.

The nonlinear impairments of a dispersion managed fiber link can be suppressed with inline residual dispersion [21,22]. But the non-zero RDPS can induce penalty in the folded DBP. Figure 4(a) shows the Q-values obtained at optimum power levels as functions of the folding factor. With a RDPS of 5 ps/nm (20 ps/nm), the maximum Q-value can be approached using a folding factor of 20 (5), which still represent significant computational savings. Figure 4(b) shows the Q-values as functions of the RDPS. With conventional DBP in all spans, the Q-value increases with RDPS and approaches the maximum value when RDPS is larger than 10 ps/nm. When folded DBP is used, the Q-value penalty increases with RDPS. It is shown that for a fiber link with non-zero RDPS, there is a tradeoff between computational load and system performance.

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4. Conclusion and discussion

In this paper, we proposed an efficient DBP method for periodically dispersion managed fiber system. With periodic dispersion management, the linear and nonlinear behavior of the signal repeats itself in every dispersion period. Taking advantage this periodic behavior, DBP of many fiber spans can be folded into one span. Polarization-mode dispersion does impact the effectiveness of DBP. We expect the effect of PDM on folded DBP to be similar to regular DBP described in [14].

We demonstrated the folded DBP at the receiver. However, this method of folding nonlinearity compensation of many spans into one span can also be applied to pre-compensation of fiber nonlinearity at the transmitter. In the current work, each amplified span contains one period of the dispersion map. The folding factor can be further increased by shortening the dispersion period, i.e., each amplified span may consist of several dispersion periods. Folded DBP can also be applied when each dispersion period consists of several amplified spans.

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