# Electromagnetic Properties of Metals Based on Transmission Measurements at 64 MHz

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## Abstract

A mathematical model is developed to determine the refractive and absorption indices of metals based on transmission measurements. Thin Ti sheets of different thicknesses are placed in a magnetic field of frequency 64 MHz, and the incident and transmitted magnetic field strengths are measured. Selected Ti sheets were treated with a laser beam to diffuse Pt into them. The transmission data were used to determine material properties based on the approach of inverse problems. The electrical conductivity of the laser Pt-diffused samples increased by 9.1% when compared to the as-received, untreated Ti sheets. The reflectivity of the treated Ti samples also increased by 0.2% compared to the untreated samples. These data indicate that lasers enable modifying the electromagnetic properties of materials.

**Key words:** inverse, electromagnetic properties, metal

## 1. Introduction

Several instruments, such as spectrum analyzers and network analyzers, are available for measuring the electrical and electromagnetic properties of materials at high radiofrequencies. For lower frequencies (e.g., 30 MHz, 100 MHz), however, there is a need for reliable instruments for measuring the electromagnetic response of materials to magnetic fields of high flux densities (e.g., 10-100 μT). There are numerous applications for such measurements including non-destructive evaluation of materials and new materials development for microelectronics, nanoelectronics and biomedical applications. This paper presents a model to determine the electromagnetic properties of Ti based on experimental data pertaining to the incident and transmitted magnetic field strength.

## 2. Mathematical model

A mathematical model is developed for the transmission of the magnetic field through the Ti sheets by considering a three-medium system as illustrated in Fig. 1. The magnetic field is assumed to propagate in the z direction and the three media are considered non-magnetic and homogeneous. The magnetic field generator, which is indicated by a current loop in Fig. 1, is placed at a distance $d_1$ from the front surface of the Ti sheet and its back surface is at a distance $d_2$ from the current loop. The incident magnetic field is partially reflected at both the air-Ti interfaces and absorbed inside the sheet. There are forward and backward propagating fields in medium 1 (air in front of the Ti sheet) and medium 2 (Ti sheet) due to reflection, and there is only a forward propagating field in medium 3 (air behind the sheet). $H_{in}$ and $H_r$ are the incident and reflected magnetic field strengths in medium 1, respectively, while $H_{2+}$ and $H_{2-}$ denote the forward and backward moving magnetic field strengths in the sheet, respectively. $H_{3+}$ is the forward moving magnetic field strength in medium 3.

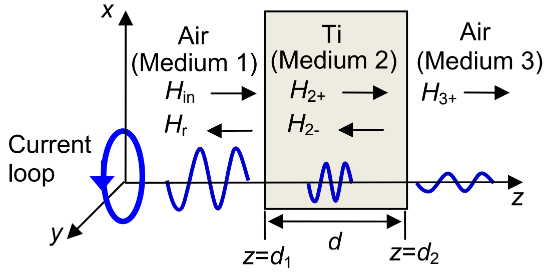


Figure1: Schematic of air-Ti sheet-air system.

Representing $t_{12}$ and $t_{23}$ as the transmission coefficients for the first interface between media 1 and 2 and the second interface between media 2 and 3, respectively, and denoting $r_{12}$ and $r_{23}$ as the corresponding reflection coefficients, the overall transmission coefficient of the Ti sheet, $t(d)$, can be written as [1]:

$$t(d)=\frac{H_{3+}(d)}{H_{in}(d_1)}=\frac{t_{12}t_{23}\exp(i\beta_2 d)}{1-r_{12}r_{23}\exp(i2\beta_2 d)} \tag{1}$$

based on the solutions of Maxwell's electromagnetic equations, where $\beta_2$, which is the propagation constant in the medium 2, is given by the following expression:

$$\beta_j=\frac{\omega}{c}\hat{n}_j \text{ for } j=1,2,3 \tag{2}$$

Here, $\hat{n}_j$ is the complex refractive index of medium $j$, $j$=1,2,3, which can be expressed in terms of the angular frequency of the magnetic field $\omega$, permittivity of air $\varepsilon_0$, relative permittivity $\varepsilon_{rj}$, electrical conductivity $\sigma_j$, refractive index $n_j$, and absorption index $k_j$ as:

$$\hat{n}_j=\sqrt{\varepsilon_{rj}+i\frac{\sigma_j}{\omega\varepsilon_0}}=n_j+ik_j \tag{3}$$

Based on the boundary conditions for the magnetic field, the transmission and reflection coefficients for each interface can be expressed as:

$$r_{lm}=\frac{Z_l-Z_m}{Z_l+Z_m} \text{ for } l,m=1,2,3 \tag{4}$$

$$t_{lm}=\frac{2Z_l}{Z_l+Z_m} \text{ for } l,m=1,2,3 \tag{5}$$

where $Z_j$, which is the impedance of medium $j$, for $j$=1,2,3, and is given by [2]:

$$Z_j(z)=-\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\hat{n}_j}\frac{\frac{-i}{\beta_j z}+\frac{1}{(\beta_j z)^2}}{\frac{-i}{\beta_j z}+\frac{1}{(\beta_j z)^2}+\frac{i}{(\beta_j z)^3}} \tag{6}$$

where $\mu_0$ is the permeability of air. The transmittance, $T(d)$, can be written as:

$$T(d)=\frac{\mathrm{Re}[Z_3(d_2)]}{\mathrm{Re}[Z_1(d_1)]}t(d)t^*(d) \tag{7}$$

Substituting Eqs. (1) to (6) into Eq. (7), the transmittance can be expressed as:

$$T(d)=\frac{16n_1n_3(u^2+v^2)}{p(d)^2+q(d)^2} \tag{8}$$

where $u$, $v$, $p(d)$ and $q(d)$ are defined by the following expressions in terms of $C(d)$, $D(d)$, $F(d)$, $G(d)$, $x_a(z)$, $y_a(z)$, $x_m(z)$ and $y_m(z)$:

$$u=n_2[x_a(d_1)x_m(d_1)-y_a(d_1)y_m(d_1)]-k_2[x_a(d_1)y_m(d_1)+x_m(d_1)y_a(d_1)] \tag{9}$$

$$v=k_2[x_a(d_1)x_m(d_1)-y_a(d_1)y_m(d_1)]+n_2[x_a(d_1)y_m(d_1)+x_m(d_1)y_a(d_1)] \tag{10}$$

$$p(d)=\exp\left(\frac{\omega}{c}k_2 d\right)\left[C\cos\left(\frac{\omega}{c}n_2 d\right)+D\sin\left(\frac{\omega}{c}n_2 d\right)\right]+\exp\left(-\frac{\omega}{c}k_2 d\right)\left[F\cos\left(\frac{\omega}{c}n_2 d\right)-G\sin\left(\frac{\omega}{c}n_2 d\right)\right] \tag{11}$$

$$q(d)=\exp\left(\frac{\omega}{c}k_2 d\right)\left[D\cos\left(\frac{\omega}{c}n_2 d\right)-C\sin\left(\frac{\omega}{c}n_2 d\right)\right]+\exp\left(-\frac{\omega}{c}k_2 d\right)\left[G\cos\left(\frac{\omega}{c}n_2 d\right)+F\sin\left(\frac{\omega}{c}n_2 d\right)\right] \tag{12}$$

$$\begin{aligned}C(d)=&\left[x_a(d_1)n_2-y_a(d_1)k_2+x_m(d_1)n_1\right]\left[x_m(d_2)n_3+x_a(d_2)n_2-y_a(d_2)k_2\right]\\&-\left[x_a(d_1)k_2+y_a(d_1)n_2+y_m(d_1)n_1\right]\left[y_m(d_2)n_3+x_m(d_2)k_2+y_m(d_2)n_2\right]\end{aligned} \tag{13}$$

$$\begin{aligned}D(d)=&\left[x_a(d_1)n_2-y_a(d_1)k_2+x_m(d_1)n_1\right]\left[y_m(d_2)n_3+x_a(d_2)k_2+y_a(d_2)n_2\right]\\&+\left[x_a(d_1)k_2+y_a(d_1)n_2+y_m(d_1)n_1\right]\left[x_m(d_2)n_3+x_m(d_2)n_2-y_m(d_2)k_2\right]\end{aligned} \tag{14}$$

$$\begin{aligned}F(d)=&\left[x_a(d_1)n_2-y_a(d_1)k_2-x_m(d_1)n_1\right]\left[x_m(d_2)n_3-x_a(d_2)n_2+y_a(d_2)k_2\right]\\&-\left[x_a(d_1)k_2+y_a(d_1)n_2-y_m(d_1)n_1\right]\left[y_m(d_2)n_3-x_m(d_2)k_2-y_m(d_2)n_2\right]\end{aligned} \tag{15}$$

$$\begin{aligned}G(d)=&\left[x_a(d_1)n_2-y_a(d_1)k_2-x_m(d_1)n_1\right]\left[y_m(d_2)n_3-x_a(d_2)k_2-y_a(d_2)n_2\right]\\&+\left[x_a(d_1)k_2+y_a(d_1)n_2-y_m(d_1)n_1\right]\left[x_m(d_2)n_3-x_m(d_2)n_2+y_m(d_2)k_2\right]\end{aligned} \tag{16}$$

$$x_a(z)=\frac{\left(\frac{\omega}{c}z\right)^4}{1-\left(\frac{\omega}{c}z\right)^2+\left(\frac{\omega}{c}z\right)} \tag{17}$$

$$y_a(z)=\frac{-\left(\frac{\omega}{c}z\right)}{1-\left(\frac{\omega}{c}z\right)^2+\left(\frac{\omega}{c}z\right)^4} \tag{18}$$

$$x_m(z)=\frac{\frac{\omega}{c}(n_2+k_2)z-\left(\frac{\omega}{c}\right)^2\left(n_2^{\,2}-2n_2k_2-k_2^{\,2}\right)z^2+\left[\frac{\omega}{c}k_2z-\left(\frac{\omega}{c}\right)^2\left(n_2^{\,2}-k_2^{\,2}\right)z^2\right]^2}{\left[\frac{\omega}{c}n_2z+2\left(\frac{\omega}{c}\right)^2n_2k_2z^2\right]^2+\left[1+\frac{\omega}{c}k_2z-\left(\frac{\omega}{c}\right)^2\left(n_2^{\,2}-k_2^{\,2}\right)z^2\right]^2} \tag{19}$$

$$y_m(z)=\frac{-\left[\frac{\omega}{c}n_2z+2\left(\frac{\omega}{c}\right)^2n_2k_2z^2\right]}{\left[\frac{\omega}{c}n_2z+2\left(\frac{\omega}{c}\right)^2n_2k_2z^2\right]^2+\left[1+\frac{\omega}{c}k_2z-\left(\frac{\omega}{c}\right)^2\left(n_2^{\,2}-k_2^{\,2}\right)z^2\right]^2} \tag{20}$$

Eq. (8) shows that the transmittance depends on two variables: refractive index $n_2$ and absorption index $k_2$ of Ti. After measuring the incident and transmitted magnetic field strengths for two Ti sheets of different thicknesses, and determining their transmittances, one can calculate $n_2$ and $k_2$ by the approach of inverse problems. Other electromagnetic properties of Ti such as the relative permittivity $\varepsilon_{r2}$, electrical conductivity $\sigma_2$, absorption coefficient $\alpha$ and reflectivity $R$ can be determined from the following expressions:

$$\varepsilon_{r2}=n_2^2-k_2^2 \tag{21}$$

$$\sigma_2=2\omega\varepsilon_0 n_2 k_2 \tag{22}$$

$$\alpha=2\frac{\omega}{c}k \tag{23}$$

$$R=\frac{\left(x_1n_2-y_1k_2-n_1x_{21}\right)^2+\left(x_1k_2+y_1n_2-n_1y_{21}\right)^2}{\left(x_1n_2-y_1k_2+n_1x_{21}\right)^2+\left(x_1k_2+y_1n_2+n_1y_{21}\right)^2} \tag{24}$$

Two sets of Ti sheets were used for determining the material properties. One set of samples was the as-received Ti sheets, while the second set was prepared by difussing Pt into Ti using a laser diffusion technique.

## 3. Experiments

High purity titanium sheets of thicknesses 25 and 50 μm were used to carry out laser Pt diffusion experiments. Square sheets of side 20 mm were used. The sheets were placed in a laser diffusion chamber as illustrated in Fig. 2. The chamber was filled with a platinum precursor that was prepared by dissolving $Pt(acac)_2$ [platinum(II) acetylacetonate, $Pt(C_5H_7O_2)_2$] in acetylacetone [$CH_3CHCH_2CHCH_3$] and heated inside a bubbler. A carrier gas, argon, was passed through the bubbler to transport the $Pt(acac)_2$ vapor to the chamber. A

Nd:YAG laser beam was used to heat the Ti sheet, resulting in the thermochemical decomposition of the precursor. This process produces Pt atoms which subsequently diffuse into the Ti sheet.

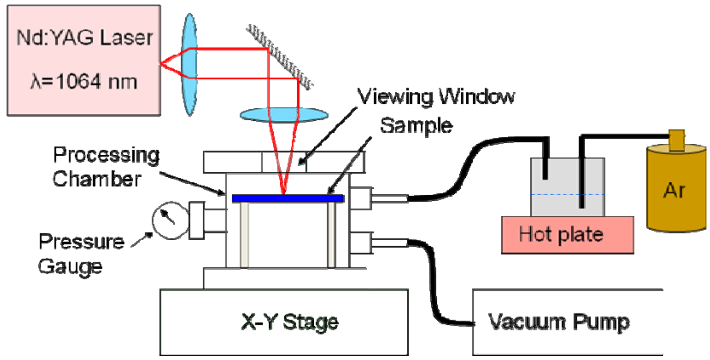


Figure 2: Setup for laser diffusion

Fig. 3 shows the setup for measuring the incident and transmitted magnetic field strengths. The Ti sheet is placed between two copper plates having rectangular windows so that the two surfaces of the sheet are exposed to the air. Two HP 11940A probes, one of which acts as a magnetic field source and the other as the field sensor, are placed on either side of the sheet as shown in Fig. 3 respectively. The magnetic field source was connected to a signal generator to create magnetic fields of strength 13.0 mA/m at 64 MHz. The field sensor was connected to a spectrum analyzer to measure the amplitude of the transmitted magnetic field.

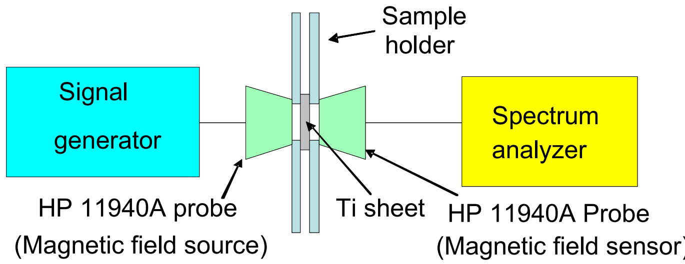


Figure 3: Setup for magnetic field strength measurement

## 4. Results and discussion

The measured data represent the incident and transmitted magnetic field strengths, $H_{in}^*$ and $H_{3+}^*$, respectively, in units of dBμV, and the spectrum analyser provides a scaling factor of $10^{(H^*+48.5)/20}$ to convert the field strengths from the measured unit to the unit of μA/m for $H_{in}$ and $H_{3+}$. The experimental values of transmittance are determined using the expression $T = (H_{3+}/ H_{in})^2$. These values are listed in Table I for different samples. Using Eqs. (21) to (24), the properties of Ti can be calculated as listed in Table II. Using the values of $n_2$ and $k_2$, the calculated transmittance, $T_{cal}$, is obtained from the mathematical model. The errors between the calculated and experimental transmittance are determined as:

$$E_r = \left|\frac{T_{cal} - T}{T}\right|, \tag{25}$$

which are also listed in Table II. The laser Pt-diffused sample has higher electrical conductivity, reflectivity and absorption coefficient. The reflectivity, *R*, of the Pt-diffused sample is 0.9484, while it is 0.9461 for the as-received sample. This increase in the reflectivity of the laser-treated sample is due to the increased conductivity from $2.30\times10^6$ $m^{-1}\Omega^{-1}$ of the as-received sample to $2.51\times10^6$ $m^{-1}\Omega^{-1}$ of the treated sample. The conductivity of the treated sample increases by 9.1% when compared to that of the as-received sample.

Table I. Experimental data for magnetic field strengths and transmittance for as-received and laser Pt-diffused Ti samples

| Sample | $d$ (μm) | $H_{in}$ (mA/m) | $H_{3+}$ (μA/m) | $T$ (% ) |
|---|---|---|---|---|
| As-received | 25 | 13.0 | 521.8 | 0.16 |
| | 50 | 13.0 | 310.8 | 0.06 |
| Laser Pt-diffused | 25 | 13.0 | 498.3 | 0.15 |
| | 50 | 13.0 | 263.0 | 0.04 |

Based on the conservation of energy, the amount of incident magnetic energy is equal to the sum of the absorbed and transmitted magnetic energies. For the 25 μm thick as-received Ti sheet, the reflectance is 0.9461 and the transmittance is 0.0016, indicating that the absorbance is 0.0523. For the 25 μm thick laser Pt-diffused Ti sheet, on the other hand, the absorbance is 0.0501. Therefore, this laser-treated Ti sheet absorbs less amount of the magnetic energy. The reduction in the absorbance is 4.2% of the absorbance of the as-received sample.

Table II. Calculated properties of Ti

| Sample | $n_2$ | $k_2$ | $\varepsilon_{r2}$ | $\sigma$ ($m^{-1}\Omega^{-1}$) | $R$ | $\alpha$ ($mm^{-1}$) | $E_r$ for 25μm sheet | $E_r$ for 50μm sheet |
|---|---|---|---|---|---|---|---|---|
| As-received | 17986 | 17987 | $-3.60\times10^4$ | $2.30\times10^6$ | 0.9461 | 24.1 | 23.8% | 23.6% |
| Laser Pt-diffused | 18798 | 18799 | $-3.76\times10^4$ | $2.51\times10^6$ | 0.9484 | 25.2 | 13.5% | 12.6% |

## 5. Conclusion

A mathematical model is presented to determine the electromagnetic properties of metals based on transmission measurements. Since the properties of interest were just for magnetic fields of low frequencies, such as 30 – 100 MHz, and instruments are not readily available for such frequency ranges, the inverse problem approach proved to be a good technique for determining the materials properties in this study. The properties of Ti sheets of thicknesses 25 μm and 50 μm were modified by diffusing Pt into the sheets using a laser diffusion technique. Using the model, the refractive and absorption indices of Ti were determined for the magnetic field of frequency 64 MHz. Other electromagnetic properties of Ti such as the relative permittivity, conductivity, absorption coefficient and reflectivity are also calculated. The conductivity of the Pt-diffused sample increased by 9.1% when compared to that of the as-received sample and, consequently, the reflectivity of the former was higher than the latter. Furthermore the absorbance of the Pt-diffused sample decreased by 4.2% when compared to that of the as-received sample.

## 6. References

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