# Accelerating finite energy Airy beams

**Georgios A. Siviloglou and Demetrios N. Christodoulides**

*College of Optics/CREOL, University of Central Florida, Orlando, Florida 32816, USA*

Received December 21, 2006; revised January 19, 2007; accepted January 20, 2007;
posted January 25, 2007 (Doc. ID 78349); published March 19, 2007

We investigate the acceleration dynamics of quasi-diffraction-free Airy beams in both one- and two-dimensional configurations. We show that this class of finite energy waves can retain their intensity features over several diffraction lengths. The possibility of other physical realizations involving spatiotemporal Airy wave packets is also considered. © 2007 Optical Society of America

*OCIS codes:* 050.1940, 260.2030, 350.5500.

Diffraction-free beams are by definition localized optical wave packets that remain invariant during propagation. Perhaps the best known example of such a diffraction-free wave is the Bessel beam first predicted theoretically and experimentally demonstrated by Durnin *et al.* in 1987.[1,2] Other such nondiffracting wave configurations include, for example, higher-order Bessel beams, Mathieu beams and their higher-order counterparts, as well as waves based on parabolic cylinder functions.[3,4] In systems that exhibit bidiffraction (normal diffraction in one direction and anomalous in the other) such as photonic crystals and lattices, nondiffracting X-waves and Bessel-like beams are also possible.[5–7] Strictly speaking, these solutions convey infinite power, and for this very reason they are "impervious" to diffraction. If, on the other hand, these diffraction-free beams pass through a finite aperture (are truncated), diffraction eventually takes place.[8] Yet, in such cases, the rate of diffraction can be considerably slowed down depending on the degree of truncation, i.e, how large is the limiting amplitude aperture with respect to the features of the beam. In the case of finite Bessel beams, such effects were first theoretically analyzed by Gori *et al.*[9]

An important aspect associated with such diffraction/dispersion-free wave packets is their dimensionality. In fact all the above-mentioned solutions exist only in (2+1)D and (3+1)D configurations. The problem becomes more involved in the lowest dimension [e.g., in (1+1)D], which is known to describe the diffraction of planar optical beams or pulse propagation in dispersive optical fibers. Yet, even in this case, dispersion-free Airy wave packets are possible, as first predicted by Berry and Balazs within the context of quantum mechanics.[10] This interesting class of Airy structures is unique in the sense that these beams lack parity symmetry and tend to *accelerate* during propagation. The acceleration process associated with these beams was later interpreted by Greenberger on the basis of the equivalence principle.[11] We emphasize that even in this latter case the Airy wave packet is again associated with an infinite energy. In addition, by its very nature, the Airy beam is "weakly confined," since its oscillating tail decays very slowly, i.e., $Ai(-x) \approx \pi^{-1/2}x^{-1/4}\sin[(2/3)x^{3/2}+(\pi/4)]$ as $x \to +\infty$.[12] Therefore, for all practical purposes, it will be rather difficult to synthesize such beams unless of course they are amplitude truncated. Finite energy (exponentially decaying) diffractionless Airy planar beams in nonlinear unbiased photorefractive crystals have been predicted as a result of thermal diffusion.[13] Yet, so far, to our knowledge the propagation behavior of finite power Airy wave packets has never been investigated under linear conditions.

In this paper we investigate the acceleration dynamics of quasi-diffraction-free finite energy Airy beams. We show that even in this case these Airy waves can retain their intensity features over several diffraction lengths and can still accelerate in the transverse direction. The propagation evolution of both one- and two-dimensional Airy beam configurations is investigated in detail. The possibility of other physical realizations involving spatiotemporal Airy wave packets is also examined.

We begin our analysis by considering the (1+1)D paraxial equation of diffraction that governs the propagation dynamics of the electric field envelope $\phi$ associated with planar optical beams:

$$i\frac{\partial \phi}{\partial \xi} + \frac{1}{2}\frac{\partial^2 \phi}{\partial s^2} = 0. \qquad (1)$$

In Eq. (1) $s=x/x_0$ represents a dimensionless transverse coordinate, $x_0$ is an arbitrary transverse scale, $\xi=z/kx_0^2$ is a normalized propagation distance (with respect to the Rayleigh range), and $k=2\pi n/\lambda_0$ is the wavenumber of the optical wave. Incidentally, this same equation is also known to govern pulse propagation in dispersive media.[6]

Here we study the dynamics of finite power Airy beams by considering their exponentially decaying version,

$$\phi(s,\xi=0) = Ai(s)\exp(as), \qquad (2)$$

at the input of the system ($\xi=0$). In Eq. (2) the decay factor $a>0$ is a positive quantity to ensure containment of the infinite Airy tail and can thus enable the physical realization of such beams. We note that the positive branch of the Airy function decays very rapidly, and thus the convergence of the function in Eq. (2) is guaranteed. Figure 1(a) depicts the field profile of such a beam at $z=0$; Fig. 1(b), its corresponding intensity. Of interest is the Fourier spectrum of this beam, which in the normalized $k$-space is given by

 

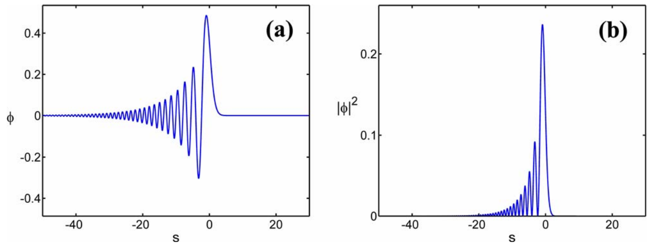


Fig. 1. (Color online) (a) Normalized field profile and (b) normalized intensity profile of a finite energy Airy beam when $a=0.1$.

$$\Phi_0(k)=\exp(-ak^2)\exp\left(\frac{i}{3}(k^3-3a^2k-ia^3)\right). \quad (3)$$

From Eq. (3) it becomes directly evident that the wave-packet power spectrum is Gaussian. From Parseval's theorem, the total power of this finite energy Airy wave packet can be directly obtained and is given by

$$\int_{-\infty}^{\infty} \mathrm{d}s|\phi(s,\xi=0)|^2=\sqrt{\frac{1}{8\pi a}}\exp\left(\frac{2a^3}{3}\right). \quad (4)$$

By directly solving Eq. (1) under the initial conditions of Eq. (2), we find that this Airy-like beam will evolve according to

$$\phi(\xi,s)=Ai[s-(\xi/2)^2+ia\xi]\exp(as-(a\xi^2/2)-i(\xi^3/12)+i(a^2\xi/2)+i(s\xi/2)). \quad (5)$$

Note that in the limit $a=0$ our solution reduces to the nondispersive wave packet found in Ref. 10. Figure 2(a) shows the propagation of such a planar Airy beam up to a distance of 1.25 m when $x_0=100$ $\mu$m and the decay parameter is $a=0.1$. The corresponding cross-sections of the intensity profiles at various distances are shown in Fig. 2(b). For these parameters, the intensity FWHM of the first lobe of this beam is 171 $\mu$m. We note that for a Gaussian beam of this same width its Rayleigh range would have been 13.25 cm at a wavelength of $\lambda_0=0.5$ $\mu$m. For this example the intensity features of this beam remain essentially invariant up to 75 cm, as clearly seen in Fig. 2. Evidently this wave endures because of the quasi-diffraction-free character of the Airy wave packet. We emphasize that for this same distance the front lobe of the beam would have expanded by at least 6 times. As Fig. 2(b) indicates, the beam starts to deteriorate first from the tail as a result of truncation. The last feature to disappear (around 100 cm) is the front lobe. After a certain distance (in this case 120 cm) the beam intensity becomes Gaussian-like, as expected from its Gaussian power spectrum in the Fraunhofer limit.

Even more importantly, in spite of its truncation (necessary for its realization), the Airy wave packet still exhibits its most exotic feature, i.e., its trend to freely *accelerate*. This characteristic is rather peculiar given the fact that it may occur in free space, e.g., in the absence of any index gradients from prisms, etc. This behavior is reflected in the term $s-(\xi/2)^2$ that appears in the argument of the Airy function in Eq. (5). These acceleration dynamics can be clearly seen in Fig. 2(a), where the beam's parabolic trajectory becomes evident. For the example discussed here, the beam will shift by 880 $\mu$m at $z=75.4$ cm.

These results can be readily generalized in two dimensions, i.e., when the initial field envelope is given by $\phi(x,y,z=0)=Ai(x/x_0)Ai(y/y_0)\exp[(x/w_1)+(y/w_2)]$. The intensity profile of such a 2D beam at $z=0$ and $z=50$ cm is shown in Figs. 3(a) and 3(b), respectively, when $x_0=y_0=100$ $\mu$m and $w_1=w_2=1$ mm. In this case, the 2D Airy beam remains almost invariant up to a distance of $z=50$ cm, and it accelerates in the same manner along the 45° axis in the $x$–$y$ system.

In addition, Airy beams in combination with other nondiffracting field configurations can also be used to describe multidimensional (3+1D) finite energy wave packets in the presence of diffraction and dispersion.

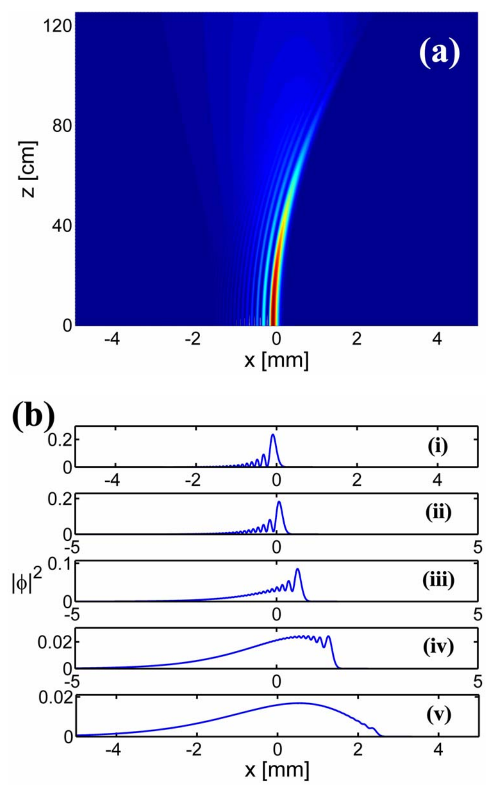


Fig. 2. (Color online) (a) Propagation dynamics of a finite energy Airy beam as a function of distance. (b) Cross-sections of the normalized beam intensity at (i) $z=0$ cm, (ii) 31.4 cm, (iii) 62.8 cm, (iv) 94.3 cm, and (v) 125.7 cm.

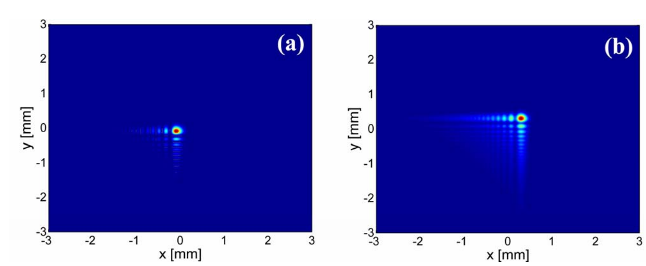


Fig. 3. (Color online) Two-dimensional finite energy Airy beam (a) at the input $z=0$ cm and (b) after propagating $z=50$ cm.

In such a case, the beam envelope in the spatiotemporal domain obeys[6]

$$i\frac{\partial\psi}{\partial Z}+\frac{1}{2}\left(\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}+\frac{\partial^2\psi}{\partial T^2}\right)=0, \tag{6}$$

where in Eq. (6), without any loss of generality, an anomalously dispersive system was assumed. For example, a localized Airy finite energy spatiotemporal wave packet can be obtained using Bessel–Gauss beams,[9] i.e., at the input $\psi=Ai(T)\exp(aT)J_0(r)\exp(-r^2/w_0^2)$, where $r=(X^2+Y^2)^{1/2}$ and $w_0$ is the "aperture" spot size of the beam. Under these initial conditions, using separation of variables we find that this wave evolves according to $\psi=\phi(Z,T)U(Z,X,Y)$, where $\phi(Z,T)$ is given by Eq. (5) and $U(Z,X,Y)$ is given by the solution of Gori *et al.*[9] Figure 4 depicts an isosurface plot of such an Airy–Bessel–Gauss wave packet at the input $Z=0$ [Fig. 4(a)] and after propagation at $Z=3$ [Fig. 4(b)]. Even in this case the wave accelerates forward and remains essentially invariant.

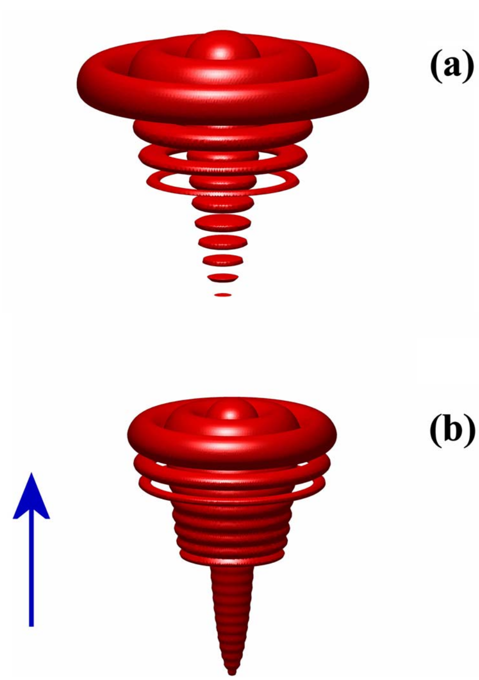


Fig. 4. (Color online) Isosurface intensity contour plot for a spatiotemporal Airy–Gauss–Bessel wave packet (with $a=0.15$, $w_0=9$) (a) at the input $Z=0$ and (b) after a normalized propagation distance of $Z=3$. The arrow depicts the direction of acceleration.

Accelerating Airy wave packets can also be implemented in dispersive optical fibers. Equation (3) suggests that in the temporal domain such an exponentially decaying Airy pulse can be produced by passing a transform-limited Gaussian pulse through a system with appreciable cubic dispersion.[14] A system of this sort can be implemented using another fiber at the zero dispersion point or by employing pulse shaping techniques.[15] Acceleration pulse dynamics can then be observed in a fiber with either normal or anomalous group velocity dispersion.

In conclusion, we have shown that freely accelerating finite energy Airy beams are possible in both one- and two-dimensional configurations. The possibility of observing this same process in the spatiotemporal domain was also considered.

## References

1. J. Durnin, J. Opt. Soc. Am. A **4**, 651 (1987).
2. J. Durnin, J. J. Miceli, and J. H. Eberly, Phys. Rev. Lett. **58**, 1499 (1987).
3. J. C. Gutiérrez-Vega, M. D. Iturbe-Castillo, and S. Chávez-Cerda, Opt. Lett. **25**, 1493 (2000).
4. M. A. Bandres, J. C. Gutiérrez-Vega, and S. Chávez-Cerda, Opt. Lett. **29**, 44 (2004).
5. J. Lu and J. F. Greenleaf, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **39**, 19 (1992).
6. D. N. Christodoulides, N. K. Efremidis, P. Di Trapani, and B. A. Malomed, Opt. Lett. **29**, 1446 (2004).
7. O. Manela, M. Segev, and D. N. Christodoulides, Opt. Lett. **30**, 2611 (2005).
8. J. W. Goodman, *Introduction to Fourier Optics*, 3rd ed. (Roberts & Company, 2005).
9. F. Gori, G. Guattari, and C. Padovani, Opt. Commun. **64**, 491 (1987).
10. M. V. Berry and N. L. Balazs, Am. J. Phys. **47**, 264 (1979).
11. D. M. Greenberger, Am. J. Phys. **48**, 256 (1980).
12. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, 1972).
13. D. N. Christodoulides and T. H. Coskun, Opt. Lett. **21**, 1460 (1996).
14. M. Miyagi and S. Nishida, Appl. Opt. **18**, 678 (1979).
15. A. M. Weiner, Rev. Sci. Instrum. **71**, 1929 (2000).