Observation of Bloch-like revivals in semi-infinite Glauber–Fock photonic lattices

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We report the first experimental implementation of Glauber–Fock oscillator lattices. Bloch-like revivals are observed in these optical structures in spite of the fact that the photonic array is effectively semi-infinite and the waveguide coupling is not uniform. This behavior is entirely analogous to the dynamics exhibited by a driven quantum harmonic oscillator. Our observations are in excellent agreement to the analytical results obtained in this fully integrable lattice system.

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Lattice systems play a ubiquitous role in diverse areas of physics and science in general. In solid state and atomic physics, crystalline structures govern the transport dynamics of charged carriers by establishing a sequence of allowed energy bands and forbidden bandgaps [1,2]. On the other hand, in the optical domain, such discrete arrangements can be readily realized in arrays of evanescently coupled waveguides [3]. As indicated in several studies [4–7], this family of optical structures can serve as an ideal environment where one can directly generate and observe a wide range of physical processes [4]. In this regard, in previous works we have suggested and successfully observed classical analogues of displaced Fock (number) states in fully integrable Glauber–Fock photonic lattices [8,9]. The classical realization of these displaced oscillator eigenstates was performed by establishing a correspondence between the number state $|n\rangle$ and classical light launched into the nth waveguide of the array and the coupling coefficients obeying a square root law distribution between nearest neighbors. If in addition a transverse optical potential is linearly ramped along the waveguides, the resulting Glauber–Fock oscillator lattices [10] are among a handful of integrable lattice models that can lead to Bloch-like oscillations and dynamic delocalization [11–14]. What makes this even more impressive is the fact that these revivals are possible in a semi-infinite arrangement. Figure 1 depicts a schematic view of a Glauber–Fock oscillator array and its corresponding refractive index profile. Yet as of now, no experimental demonstration of such effects has been reported in the literature. In this Letter, we report the first experimental realization of a discrete Glauber–Fock oscillator by employing evanescently coupled waveguide arrays. It is shown that the evolution of classical light in this type of quantum inspired lattices can give rise to intensity profiles that emulate the probability number distributions expected from a quantum harmonic oscillator when driven by a constant external force. Both periodic collapses and revivals are observed in the intensity evolution, at intervals that are entirely independent of the excitation site. The light dynamics in this novel class of arrays is described in closed form, from where one can deduce the associated turning points. In general, the propagation of the modal optical fields $\{E_n\}_{n=0}^{\infty}$ in a semi-infinite Glauber–Fock oscillator lattice is governed by the following set of coupled equations:

$$i\frac{dE_0}{dZ}+f(Z)CE_1=0,$$
$$i\frac{dE_n}{dZ}+f(Z)C\left(\sqrt{n+1}E_{n+1}+\sqrt{n}E_{n-1}\right)+\alpha nE_n=0, \quad (1)$$

where C denotes the coupling between the first two guides, $f(Z)$ is an arbitrary function of the propagation distance, the integer $n\geq 1$, and α is the ramping constant, which can be either positive or negative. Here we assume

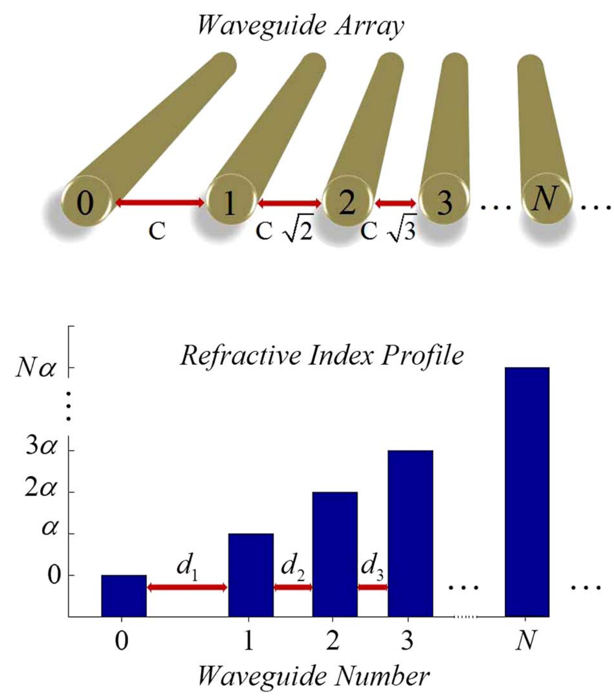

Fig. 1. (Color online) Glauber–Fock oscillator lattice and the associated effective refractive index profile.

that the small index ramping does not affect the coupling coefficients. In Eq. (1) the last term represents the linear gradient of the refractive index, which in a quantum oscillator system corresponds to a constant external driving force. As found in [10], Eq. (1) can be solved analytically for any arbitrary function $f(Z)$. Here, in this experimental study we restrict our attention to the special case $f(Z) = 1$. In addition, each waveguide in the array is supposed to be single-moded. In our optical system, the oscillator eigenstate $|k\rangle$ corresponds to the classical excitation of the kth guide, whereas the transition probability amplitude between the states $|k\rangle$ and $|n\rangle$ is represented by the field amplitude at the nth waveguide. In this case, one can directly show that the impulse response of this system is given by:

$$E_{k,n} = \begin{cases} \sqrt{\frac{n!}{k!}}\exp(A)B^{k-n}L_n^{k-n}(\Phi), & n \le k \\ \sqrt{\frac{k!}{n!}}\exp(A)(-B^*)^{n-k}L_k^{n-k}(\Phi), & n \ge k \end{cases} \quad (2)$$

where $L_n^m(\Phi)$ represents associated Laguerre polynomials, and

$$A(Z) = \frac{C^2}{\alpha^2}(\exp(i\alpha Z) - 1 - i\alpha Z) + i\alpha k Z,$$
$$B(Z) = \frac{C}{\alpha}(1 - \exp(-i\alpha Z)),$$
$$\Phi(Z) = \frac{2C^2}{\alpha^2}[1 - \cos(\alpha Z)]. \quad (3)$$

In deriving Eq. (2), we have assumed that only the kth waveguide was excited with unity amplitude, corresponding to a preparation of the oscillator in a single eigenstate [10]. Under such single site excitation conditions, the associated intensity distribution exhibits revivals at regular intervals $\hat{Z}$, that are given by $\hat{Z} = 2\pi m/\alpha$, where m is an integer. In Fig. 2 we demonstrate this process for a Glauber–Fock oscillator lattice having a ramping constant $\alpha = 0.5$ and $C = 1$, when the third channel is excited $(n = 2)$. In this case the revival distance is $\hat{Z} = 4\pi$. Given that the $n = 2$ channel is excited, the intensity evolution features three maxima (like a displaced Fock state [9]) midway in the cycle, while the intensity pattern collapses back into the initially excited waveguide at $\hat{Z} = 4\pi$ [10]. On the other hand, it can be shown

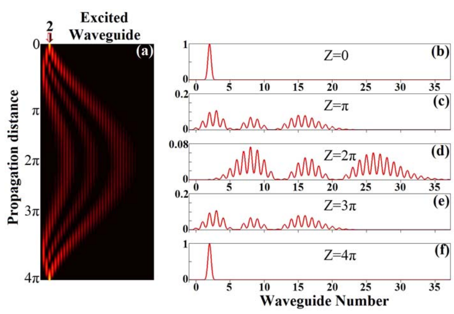

Fig. 2. (Color online) (a) Intensity evolution in a Glauber–Fock oscillator when the third $(n = 2)$ waveguide is initially excited. The beam develops three peaks and eventually revives at $\hat{Z} = 4\pi$.

that the response of this system is different for a positive or negative ramping $(\pm\alpha)$ if the array is excited with a wavepacket. This is clearly shown in Figs. 3(a) and 3(b) using a discrete Gaussian beam at the input when $\alpha = \pm 0.5$, respectively. In order to experimentally demonstrate these effects, we have fabricated such Glauber–Fock oscillator lattices in polished bulk fused silica glass by using femtosecond laser writing technology [15]. The required Glauber–Fock coupling distribution, $C_n = C\sqrt{n}$, (C_n denoting the coupling between adjacent guides n and $n - 1$), was accomplished by judiciously varying the waveguide separation d_n. In the weak coupling regime the coupling coefficients vary exponentially with the separation distance [16], that is $C_n = C\exp(-[d_n - d_1]/s)$ where s is a characteristic distance. In that case $d_n = d_1 - (s/2)\ln(n)$ imposes the desired distribution. In our system, at a wavelength of $\lambda = 808$ nm, the array parameters were chosen to be $C = 0.24\ \mathrm{cm}^{-1}$, $s = 8.4\ \mu$m, $d_1 = 33.7\ \mu$m. The transverse size of the lattices ($N = 27$ elements) was chosen such that the light never reaches the far end at $n = N - 1$. Hence, the lattice effectively acts as a semi-infinite system. The gradient for the refractive index was achieved by varying the waveguide writing velocity [15]. Here, we chose a velocity of 80 mm/min for the central waveguide $(n = 13)$ and imposed linear gradients $\Delta v \equiv v_n - v_{n-1} = 0$, 0.5, 1 and 2 mm/min on the individual lattices. Thereby, the case $\Delta v = 0$ yields a regular Glauber–Fock lattice, whereas lattices with increasing $\Delta v > 0$ feature an approximately linear, increasingly negative index gradient α. The output intensity patterns after propagating over the total length of $L = 10$ cm were imaged onto a CCD.

In Fig. 4 we present our experimental observations of the output intensities for the various lattices and the input sites $k = 0$ to 4. The lengths of the revival periods $\hat{Z}$ are fitted to the experimental data from a comparison with the analytic behavior expected from Eq. (2). Figures 4(a) and 4(b) depict the output intensities for a conventional Glauber–Fock lattice $(\Delta v = 0)$, which exhibit $k + 1$ maxima for an excitation of the kth site, as expected for the eigenstates of the displaced harmonic oscillator [8,9]. The other panels show the output distributions of the lattices with an increasing velocity gradient. We find revival periods of $\hat{Z} = 5L$ [Figs. 4(c) and 4(d)], $\hat{Z} = 3.3L$ [Figs. 4(e) and 4(f)] and $\hat{Z} = 2L$ [Figs. 4(g) and 4(h)] in the investigated arrays. Hence, oscillations up to half a Bloch-period are observed in these lattices. The good agreement between the experimental data and the calculations suggests that the refractive index gradient is indeed approximately linear. The observed oscillations are a direct outcome of coherent

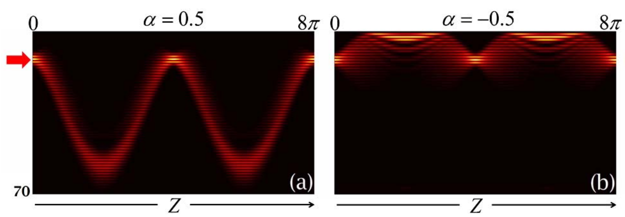

Fig. 3. (Color online) Evolution of a discrete Gaussian beam in a Glauber–Fock oscillator lattice when the ramping parameter is (a) positive $(\alpha = 0.5)$ and (b) negative $(\alpha = -0.5)$.

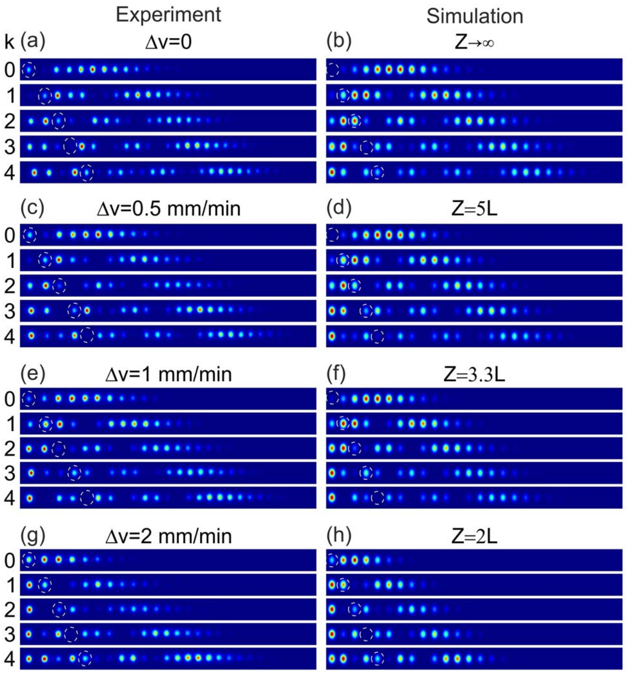

Fig. 4. (Color online) Experimentally observed and theoretically anticipated intensity patterns emerging at the output of a Glauber–Fock oscillator. (a), (b) Without index gradient ($\Delta v = 0$, $\hat{Z} \to \infty$); (c)–(f) increasing index gradient ($\Delta v > 0$, $\hat{Z} < \infty$). The sites $k = 0$ to 4 have been excited, as indicated by the dashed circles. Each image has been rescaled to its respective maximum.

interference caused by the photonic tunneling among waveguides and the boundary itself. What is remarkable here is that these Bloch-like oscillations are always possible and this without any light escaping into the array as is the case of a Glauber–Fock lattices [9]. In conclusion, we have directly observed Bloch-like oscillations in one-dimensional semi-infinite Glauber–Fock oscillator lattices. These fully integrable discrete dynamic systems are shown to display optical revivals akin to those taking place in driven quantum harmonic oscillators. What is also surprising here is that this periodic behavior is possible in spite of the fact that the array is asymmetric and semi-infinite. A generalization of the system to binary and nonlinear lattices promises the optical emulation of the Jaynes–Cummings model, and thereby should provide new insights into the dynamics of coupled atom-cavity systems [17,18]. Finally, observing the periodic evolution of quantum states in such waveguide lattices could be a topic of interest in quantum optics.

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