Adaptive Frequency-Domain Equalization for Mode-Division Multiplexed Transmission

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***Abstract*— We propose single-carrier adaptive frequency-domain equalization (SC-FDE) for mode-division multiplexed transmission. A two-mode fiber long haul transmission system is simulated. The performances of both FDE and time-domain equalization (TDE) are verified and compared. The FDE approach reduces the computational complexity significantly compared to the TDE while maintaining the same performance. For a two-mode transmission of 2000 km, FDE decreases the complexity by a factor of as much as 77 compared with TDE at the expense of a large memory length of 2048 that may require higher hardware complexity. The dynamic response of the adaptive equalizer is simulated by using a mode scrambler. FDE achieves the same tracking speed as TDE.**

***Index Terms*— Adaptive equalizers, multiplexing, multiple-input-multiple-output (MIMO), optical fiber communication.**

I. Introduction

To ACHIEVE ultra-high capacity beyond the nonlinear Shannon limit of the single-mode fiber (SMF), mode-division multiplexed transmission (MDM) in few-mode fibers (FMF) is rapidly gaining attraction. Ideally, a MDM system can increase the capacity by a factor of the number of modes. Moreover, FMF has much larger effective area and lower nonlinearity which further improve the capacity of the system. On the other hand, linear impairments such as differential mode group delay (DMGD) and mode coupling severely impact the transmission performance. To compensate/mitigate those impairments, multiple-input multiple-output (MIMO) equalization is required. The computational complexity of the equalizer grows as the DMGD increases. In order to make long-distance FMF transmission with large DMGD practical, the complexity of the equalizer has to be manageable. DMGD can be considered as a generalized form of polarization-mode dispersion (PMD). Therefore, DMGD equalization can be performed using similar algorithms as in PMD compensation except for more dimensions and longer filter length. So far, adaptive time-domain equalization (TDE) with data-aided least mean squared (DA-LMS) algorithm has been applied inmost of reported single-carrier transmission experiments [1], [2]. However, the computational complexity of TDE depends linearly on the total DMGD of the link which makes TDE unfeasible for long-haul MDM transmission [3]. For OFDM transmission, training symbol enabled MIMO algorithm was used in [4], in which the amount of compensated DMGD is limited by the length of prefix. Recently, we proposed to utilize single-carrier adaptive frequency-domain equalization (SC-FDE) [5] which leads to low computational complexity while maintaining similar transmission performance and adaptive properties as TDE. In this letter, details of the proposed SC-FDE are presented.

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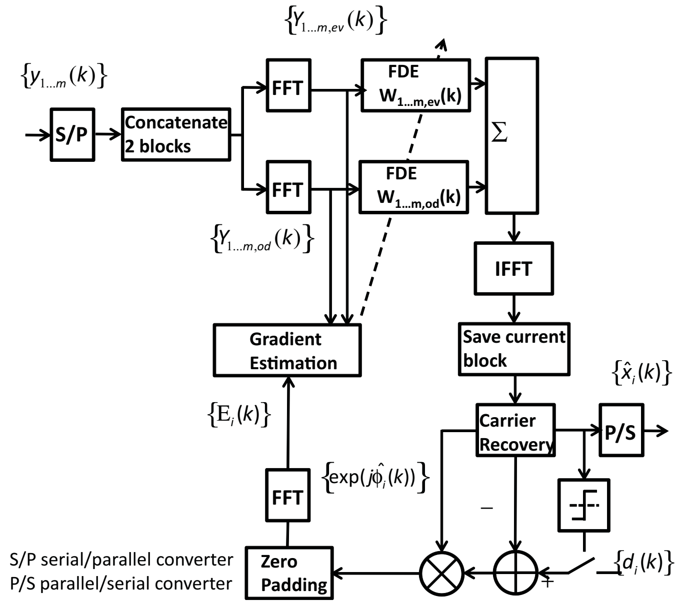

Fig. 1. Block diagram of the proposed FDE for one-mode channel.

II. Theory

For an MDM system transmitting m spatial-mode channels, an $m \times m$ MIMO algorithm is required. The equalizer contains an $m \times m$ adaptive linear frequency domain filter matrix. For simplicity, Fig. 1 shows the basic scheme of the proposed SC-FDE for one mode channel.

$\{y_i(k)\}$ and $\{\hat{x}_i(k)\}$ represent the input signal and the output signal vectors of mode i, $1 \leq i \leq m$. The overlap-and-save method was used to implement FDE. We choose an overlap rate of 0.5 because it is simple to implement. After parallelization, two consecutive blocks are concatenated. To perform an equivalent half symbol spacing FIR filtering process, the input signal streams are divided into even and odd tributaries which are multiplied by sub-equalizers separately after fast Fourier transform (FFT) [6]. The filtered signal is the sum of outputs of the even and odd equalizers. $W_{i,ev/od}(k)$ is an inversed channel filter for the even or odd tributaries over i$^{\text{th}}$ mode.

To compensate DMGD and mode crosstalk completely, the equalization filter length should be larger than the impulse response spread. In order to mitigate laser phase noise, carrier recovery is applied in the time domain. The module calculates the estimated laser phase as well as the recovered signal. The error block then is generated by comparing the processed signal and the reference which could be a training sequence (data-aided mode) or the data after hard decision (decision-directed mode) in the time domain. The gradient for updating the filter for the q$^{\text{th}}$ input mode and the p$^{\text{th}}$ output mode is computed as follows:

$$\nabla_{pq}^{e,o}(k) = \mathrm{E}_p(k)\left(Y_q^{e,o}(k)\right)^* \tag{1}$$

where $\mathrm{E}_p(k)$ is the error block from the p$^{\text{th}}$ mode in the frequency domain and $\left(Y_q^{e,o}(k)\right)^*$ is the conjugated input signal block from the q$^{\text{th}}$ mode. Since $Y_q^{e,o}(k)$ is contaminated by the laser phase noise, to compute the gradient without the impact of the phase noise, the error block is multiplied by an estimated phase fluctuation $\exp\left(j\hat{\varphi}_p(k)\right)$ in the time domain

$$e_p(k) = \left(d_p(k) - \hat{x}_p(k)\right)\exp\left(j\hat{\varphi}_p(k)\right). \tag{2}$$

By doing so, the phase fluctuation factor in $\left(Y_q^{e,o}(k)\right)^*$ can be canceled in equation (1). The incremental adjustment $\Delta W_{pq}^{e,o}(k)$ then can be computed from the estimated gradient by

$$\Delta W_{pq}^{e,o}(k) = \mu\nabla_{pq}^{e,o}(k) \tag{3}$$

where μ is the convergence step size. The inverse channel filter weights converge to the optimum solution after a training process. Gradient constraint condition is applied to enforce an accurate calculation of linear convolution [7]. To track the temporal variation of the channel, the algorithm is switched to the decision-directed mode after initial convergence. In contrast to TDE, calculations such as correlation and convolution can be simplified to be multiplication in FDE. Additional computation for conversion between the time and the frequency domain can be implemented using FFT algorithm whose complexity grows logarithmically with block size.

The total complexity of the algorithm can be measured by the number of complex multiplications per symbol per mode. We first consider TDE using an FIR filter matrix adapted by the LMS algorithm. The delay length is half symbol period and tap length is N_f which is chosen to be the same as the length of impulse response of the FMF. Thus, N_f equals to $2\Delta\tau LR_s$ where $\Delta\tau$ is the DMGD of the fiber, L is the link distance, R_s is the symbol rate. To compute each output symbol from one mode channel, we need mN_f multiplications where m is the number of modes used for transmission. Updating the filter coefficients requires $mN_f/2$ multiplications. Therefore, the complexity for TDE without carrier recovery can be expressed as

$$C_{TDE} = 3m\Delta\tau LR_s. \tag{4}$$

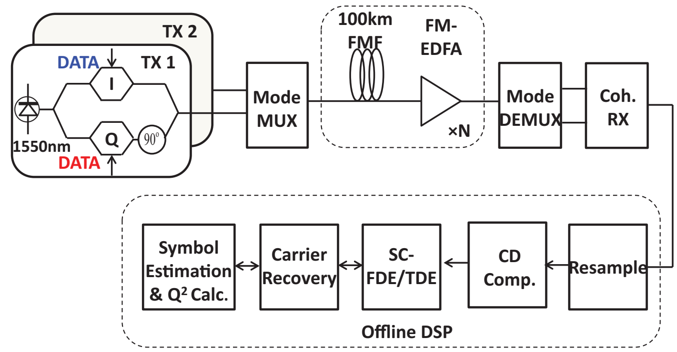

Fig. 2. Configuration of the simulation system.

For the proposed FDE, to obtain $N_f/2$ output symbols per mode, we need $2mN_f$ multiplications. Updating both even and odd equalizers requires another $2mN_f$. The total number of FFT/IFFT per mode is $4+4m$ containing 2 FFT for the input, a pair of FFT/IFFT in the updating loop and $4m$ FFT/IFFT for executing gradient constraint in the gradient estimation block [7]. FFT is assumed to be implemented by the radix-2 algorithm requiring $N\log(N)/2$ complex multiplications to execute FFT of N complex number. Thus the complexity for the proposed FDE can be expressed as

$$C_{FDE} = (4+4m)\log_2(2\Delta\tau LR_s) + 8m. \tag{5}$$

III. Simulation

A. Fiber Link Model

To verify the effectiveness of the proposed FDE, a MDM system is simulated. Fig. 2 shows the configuration of the simulated link. Without loss of generality, the transmission FMF supports only two modes, LP_{01} and LP_{11}. At the transmitter, two CW lasers with a 100 kHz line-width operating at 1550 nm were separately modulated by two 28 GBaud QPSK signals which were combined and coupled to the FMF by a mode-multiplexer (MUX). The fiber link included N spans of 100 km FMF and N FM-EDFAs with noise figure of 5 dB to compensate the span losses for both modes. At the receiver, a mode-demultiplexer (DEMUX) extracted two mode channels, which were fed into the coherent receivers. Digital signal processing was then applied on the received data to recover the signals.

Multi-section field propagation model was used to simulate two-mode transmission in FMF [8]. The section length was set to be 1km. Mode scattering coefficient defined in [8] was set to be −30 dB/km which is slightly higher than the fiber used in [9] (−34.4 dB/km). Loss and dispersion coefficient were 0.2 dB/km and 18 ps/nm/km for both modes, the same as the FMF used in [2]. DMGD was set to be 27 ps/km which was also aligned with [2]. At both ends of a single span of FMF, −22 dB inter-mode crosstalk was assumed from mode MUX/DEMUX or splicing.

The received signal was resampled to 2 samples per symbol. The chromatic dispersion of the link was compensated by two frequency domain equalizers. Two signal tributaries then enter the adaptive equalizer. To ensure the best performance,

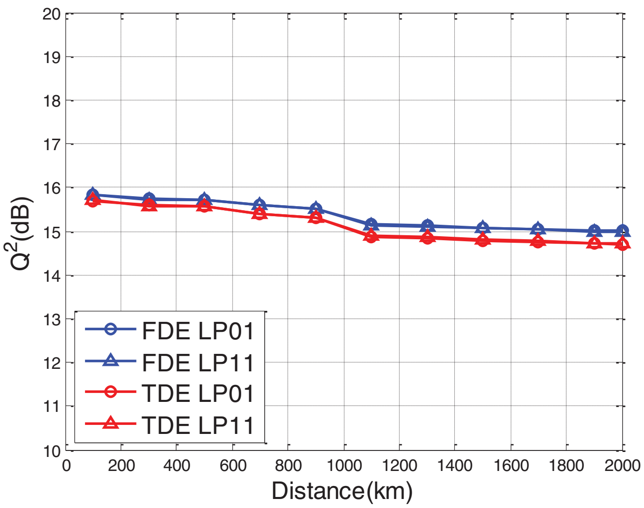

Fig. 3. Q^2 factor versus link distance (OSNR = 16 dB).

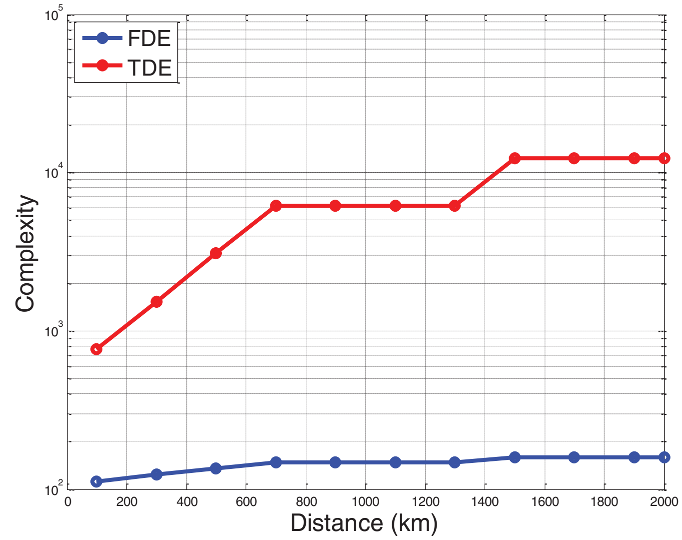

Fig. 4. Complexity versus distance.

two carrier recovery stages were used. One was inside the adaptive loop applying DA-LMS phase estimation with training sequence and M^{th} power phase estimation with transmitted data. The other stage located at the output of the adaptive equalizer for decision directed-LMS phase estimation to further mitigate the laser phase noise. Although the two-stage phase estimation only provides 0.2 dB improvement over single stage setup in terms of Q^2 factor for QPSK in our simulation, it is expected to deliver higher improvements for higher-order constellation or larger laser linewidth. After carrier recovery, hard-decision symbols estimation is followed by Q^2 factor calculation.

B. Results

To evaluate the performance of SC-FDE, transmissions with different link distances from 100 km to 2000 km were simulated. To make a fair comparison, filter size, convergence step size and initial values of filter weights were chosen to be the same for both FDE and TDE. The filter length was larger than the total DMGD of the link. The amount of filter taps was selected to be an integer power of 2 to facilitate efficient FFT implementation. Before the coherent receivers, variable noise was loaded to ensure a fixed OSNR level of 16 dB for different transmission distances. The first 10^5 symbols were used as training sequence followed by 9×10^5 test symbols. Fig. 3 showed the Q^2 factor as a function of distance for both FDE and TDE.

According to Fig. 3, both FDE and TDE effectively mitigate the inter-mode cross talk and show similar performance. Based on equations (4–5), the computational complexity was also plotted in Fig. 4. The complexity does not continuously grow as the distance increases, for instance, from 700 km to 1300 km. This is due to the fact that the filter tap lengths were chosen to be an integer power of 2.

As the transmission distance grows, the accumulated DMGD increases leading to larger filter sizes. The complexity of FDE increases much slower than TDE due to the fact that the complexity of FDE scales logarithmically with the total DMGD instead of linearly. At a transmission distance of 2000 km, FDE reduces complexity by a factor of as much as 77 compared to TDE.

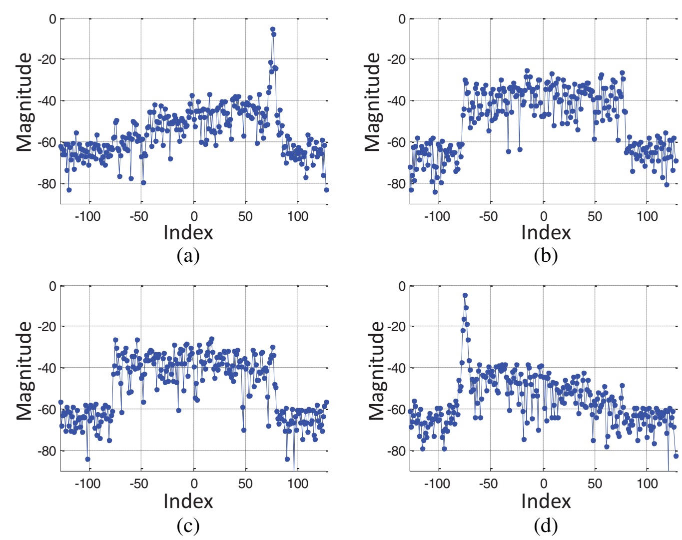

Fig. 5. Magnitude of the even FDE sub-filter coefficients in the time-domain for a two-span transmission. (a) $|W_{11}|^2$ (dB). (b) $|W_{12}|^2$ (dB). (c) $|W_{21}|^2$ (dB). (d) $|W_{22}|^2$ (dB).

The magnitude of FDE sub-filter coefficientsfor the even samples in the time domain after convergence was plotted in Fig. 5 for 2 ×100 km MDM transmission. The total DMGD is 5.4 ns and the sub-filter contains 256 taps. The each diagonal filter element compensates multipath interference (MPI) for each mode while each off-diagonal filter element mitigates crosstalk between 2 mode channels. The dominant peaks in the diagonal filters correspond to original signal. The relative delay between them coincides with the DMGD of 2 spans of FMF. The tap weights in the off-diagonal filter form a pedestal caused by distributed mode coupling through the fiber.

In our proposed FDE algorithm, a training process was used for initial convergence of the filter coefficients. In Fig. 6, the Q^2 factor is plotted as a function of the training length when

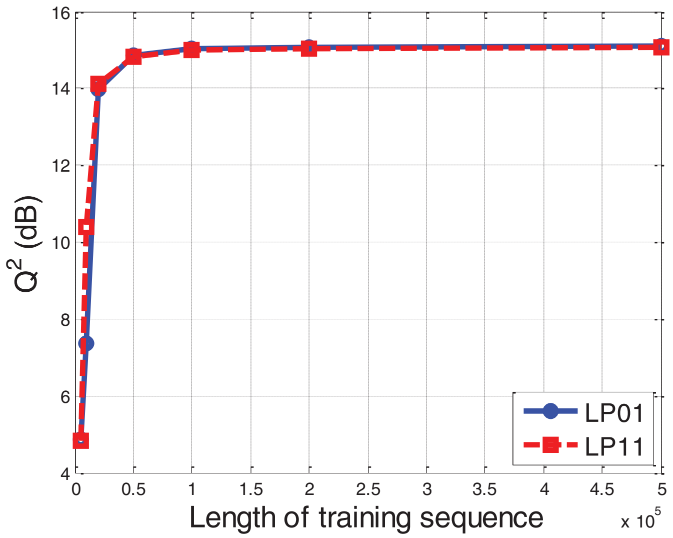

Fig. 6. Q^2 factor versus training symbol length for the proposed FDE algorithm.

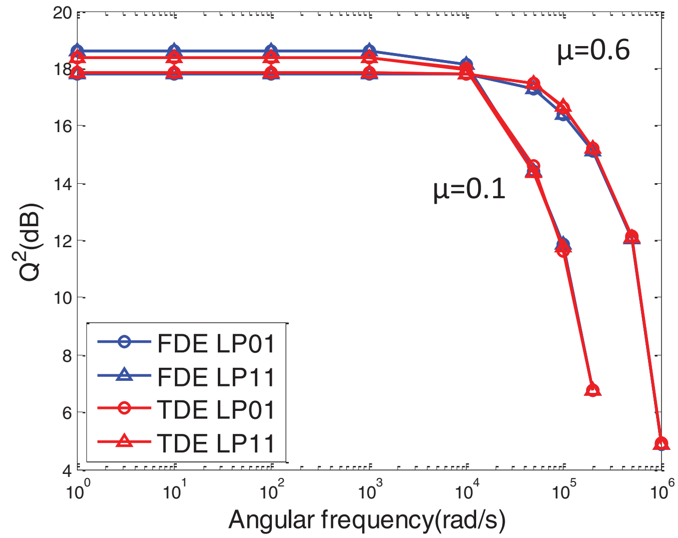

Fig. 7. Q^2 versus rotational angular frequency (OSNR = 19 dB).

the tap length of the frequency domain sub-equalizer is 2048 and the total length of the data sequence is 1×10^6. According to Fig. 6, when 2×10^4 training symbols are used, the Q penalty is about 0.9 dB. The minimum length of training symbols for our approach should be on that order.

The simulation results above assumed that the mode coupling was static. However, in practice, especially for long-haul transmission, temporal variation of environmental conditions leads to time-variant mode coupling. One of the advantages of an adaptive equalizer is that it can continuously track the temporal variation of the system. To verify the dynamic response of SC-FDE, a mode scrambler was inserted between the FMF and the mode DEMUX for the single-span transmission. The mode scrambler provided endless mode rotation with a time-dependent rotation matrix of angular frequency Ω.

$$J_s = \begin{bmatrix} \cos(\Omega t) & \sin(\Omega t) \\ -\sin(\Omega t) & \cos(\Omega t) \end{bmatrix}. \tag{6}$$

In Fig. 7, Q^2 factor is shown as a function of the angular frequency of mode rotation for different convergence step sizes. The sub-filter size is 128 taps for the single span transmission. The Q^2 factor remains a constant until the variation is too fast to be tracked. The convergence property of the algorithm can be adjusted by tuning μ. The maximum Q^2 for $\mu = 0.1$ is slightly higher than $\mu = 0.6$ due to lower mis-adjustment. When $\mu = 0.6$, FDE only suffers a 0.4 dB drop from the maximum in terms of Q^2 factor when the mode rotation is operated at 50 krad/s. It should be noted that in a practical environment, the speed as well as the coupling strength is much smaller than in this simulation. Besides, FDE shows the same tracking capability as TDE when μ is equal. Although FDE updates in the period of a block, the error signal is permitted to vary at the symbol rate which determines the effective updating rate. Therefore, FDE and TDE have the same convergence property [7]. It should be mentioned that the updating loop for FDE contains operations such as FFT and phase estimation which may slow down the tracking speed. In the simulation, the delay caused by those operations is not included.

IV. Conclusion

An SC-FDE MIMO algorithm is proposed and simulated for FMF transmission. It is shown that the proposed FDE significantly reduces the computational complexity compared to TDE while maintaining the similar equalizing and tracking performances. For two-mode transmission of 2000 km, the complexity (in terms of number of complex multiplications per symbol per mode) of FDE is a factor of 77 less than that of TDE, at the expense of a large memory length of 2048 which may require higher hardware complexity in future ASIC design.

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