

# Optical spatial solitons: historical overview and recent advances

Zhigang Chen<sup>1,2</sup>, Mordechai Segev<sup>3</sup> and Demetrios N Christodoulides<sup>4</sup>

<sup>1</sup> Department of Physics and Astronomy, San Francisco State University, San Francisco, CA 94132, USA

<sup>2</sup> TEDA Applied Physics School, Nankai University, Tianjin 300457, Peoples's Republic of China

<sup>3</sup> Physics Department and Solid State Institute, Technion—Israel Institute of Technology, Haifa 32000, Israel

<sup>4</sup> College of Optics and Photonics-CREOL, University of Central Florida, 4000 Central Florida Blvd., Orlando, FL 32826, USA

Received 2 April 2012, in final form 6 June 2012

Published

Online at [stacks.iop.org/RoPP/75](http://stacks.iop.org/RoPP/75)

## Abstract

Solitons, nonlinear self-trapped wave packets, have been extensively studied in many and diverse branches of physics such as optics, plasmas, condensed matter physics, fluid mechanics, particle physics and even astrophysics. Interestingly, over the past two decades, the field of solitons and related nonlinear phenomena has been substantially advanced and enriched by research and discoveries in nonlinear optics. While optical solitons have been vigorously investigated in both spatial and temporal domains, it is now fair to say that much of the soliton research has been driven mainly by the work on optical spatial solitons. This is partly due to that, although temporal solitons as realized in fiber optic systems are fundamentally one-dimensional entities, the high dimensionality associated with their spatial counterparts has opened up altogether new scientific possibilities in soliton research. Another reason is related to the response time of the nonlinearity. Unlike temporal optical solitons, spatial solitons have been realized by employing a variety of noninstantaneous nonlinearities, ranging from the nonlinearities in photorefractive materials and liquid crystals to the nonlinearities mediated by the thermal effect, thermophoresis and the gradient force in colloidal suspensions. Such a diversity of nonlinear effects has given rise to numerous soliton phenomena that could otherwise not be envisioned, because for decades scientists were under the mindset that solitons must strictly be the exact solutions of the cubic nonlinear Schrödinger equation as established for ideal Kerr nonlinear media. As such, the discoveries of optical spatial solitons in different systems and associated new phenomena have stimulated broad interest in soliton research. In particular, the study of incoherent solitons and discrete spatial solitons in optical periodic media not only led to advances in our understanding of fundamental processes in nonlinear optics and photonics, but also had a very important impact on a variety of other disciplines in nonlinear science. In this paper, we provide a brief overview of optical spatial solitons. This review will cover a variety of issues pertaining to self-trapped waves supported by different types of nonlinearities, as well as various families of spatial solitons such as optical lattice solitons and surface solitons. Recent developments in the area of optical spatial solitons, such as 3D light bullets, subwavelength solitons, self-trapping in soft condensed matter and spatial solitons in systems with parity–time symmetry will also be discussed briefly.

AQ1 (Some figures may appear in colour only in the online journal)

This article was invited by P Chaikin.

## Contents

<b>1. Introduction to optical spatial solitons</b>	<b>2</b>	<b>2. Spatial solitons supported by different types of non-linearities</b>	<b>4</b>
1.1. <i>Optical spatial solitons</i>	2	2.1. <i>Photorefractive solitons</i>	4
1.2. <i>How did optical spatial solitons emerge into optics?</i>	2	2.2. <i>Quadratic solitons</i>	4
1.3. <i>Why are spatial solitons so interesting?</i>	3		

2.3. <i>Spatial solitons in nonlocal and other nonlinear media</i>	5	4.3. <i>Discrete surface solitons</i>	10
<b>3. Different families of spatial solitons</b>	<b>5</b>	4.4. <i>Discrete solitons in other material systems</i>	11
3.1. <i>Dark solitons and optical vortex solitons</i>	5	<b>5. Recent developments in optical spatial solitons</b>	<b>12</b>
3.2. <i>Vector (multi-component) spatial solitons</i>	6	5.1. <i>New families of spatial solitons</i>	12
3.3. <i>Incoherent solitons</i>	6	5.2. <i>Filamentation and light bullets</i>	13
3.4. <i>Spatiotemporal solitons</i>	7	5.3. <i>Subwavelength spatial plasmon solitons</i>	14
3.5. <i>Dissipative spatial solitons and cavity solitons</i>	8	5.4. <i>Spatial solitons in soft condensed matter</i>	14
<b>4. Spatial solitons in discrete systems</b>	<b>8</b>	5.5. <i>Spatial solitons in systems with PT symmetry</i>	15
4.1. <i>Discrete solitons in 1D waveguide arrays</i>	8	<b>6. Concluding remarks</b>	<b>15</b>
4.2. <i>Discrete solitons in 2D photonic lattices</i>	9	<b>Acknowledgments</b>	<b>15</b>
		<b>References</b>	<b>15</b>

AQ2

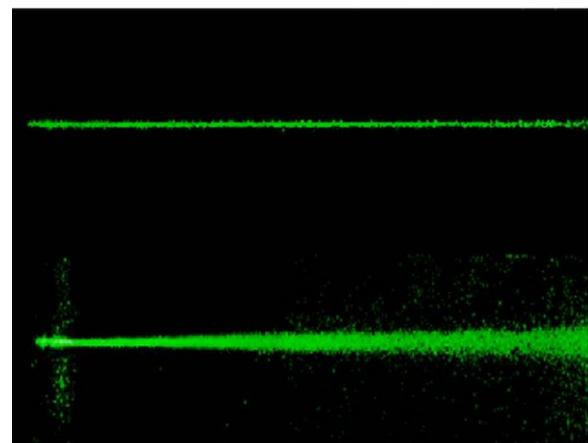
## 1. Introduction to optical spatial solitons

Optical spatial solitons, self-trapped optical beams, have been the subject of intense research in nonlinear optics especially over the past two decades. In this section, we give a brief introduction to the history and properties of optical spatial solitons. Other aspects associated with some of their fascinating characteristics will also be discussed.

### 1.1. Optical spatial solitons

Solitons are localized wave entities that can propagate in nonlinear media while maintaining a constant shape. They ubiquitously occur in many branches of physics including hydrodynamics, plasma physics, nonlinear optics and Bose–Einstein condensates. In optics, an optical wave-packet (a pulse or a beam) has a natural tendency to spread as it propagates in a medium, either due to chromatic dispersion or as a result of spatial diffraction. Most often, when this natural broadening is eliminated through a nonlinear process, a stable self-localized wave-packet forms. Such a self-trapped wave-packet, whether in time or space or both, is known as an optical soliton. *Optical spatial solitons* are self-trapped optical beams that propagate in a nonlinear medium without diffraction, i.e. their beam diameter remains invariant during propagation [1, 2]. Intuitively, a spatial soliton represents an exact balance between diffraction and nonlinearly induced self-lensing or self-focusing effects. It can also be viewed as an optical beam that induces a waveguide that, in turn, guides itself throughout propagation as if it were confined in an optical fiber. A top-view photograph showing the sharp transition between nonlinear self-trapping and linear diffraction for an optical beam propagating through a photorefractive crystal is displayed in figure 1, where the self-trapped optical beam represents a typical example of a two-dimensional (2D) optical spatial soliton. Likewise, in a way fully analogous to the spatial case, *optical temporal solitons* are non-spreading optical pulses formed when the group velocity dispersion is totally counteracted by nonlinear self-phase modulation effects [3, 4].

Over the years, there has been some debate on the use of the word ‘soliton’ in actual physical settings. Historically, the notion ‘soliton’ emerged from mathematics and it was strictly reserved for optical self-trapped wavepackets that happen to obey integrable nonlinear partial differential equations. In



**Figure 1.** Experimental demonstration of an optical spatial soliton propagating through a 5 mm long nonlinear photorefractive crystal. Top: side-view of the soliton beam from scattered light; bottom: normal diffraction of the same beam when the nonlinearity is ‘turned off’ [28–29].

nonlinear optics, the so-called nonlinear Schrödinger equation (NLSE) represents such an example. The one-dimensional (1D) NLSE that governs wave propagation in ideal Kerr nonlinear media can be fully solved (or integrated) using the ‘inverse scattering theory’ [5, 6], leading to soliton solutions, which remain invariant even after a collision event. In reality, however, most nonlinear physical systems of importance are either non-Kerr or involve other types of nonlinearity and hence are described by non-integrable evolution equations. Initially, self-trapped entities in non-integrable systems were referred to as ‘solitary waves’ in order to distinguish the interaction and collision properties of these waves from those associated with perfect ‘solitons’ in integrable systems. Yet, given that in many cases ‘solitary waves’ display behavior akin to actual solitons, this nomenclature distinction is no longer used in today’s literature. Thus, all self-trapped beams are now loosely called optical spatial solitons, regardless of the actual nonlinearity used to form them.

### 1.2. How did optical spatial solitons emerge into optics?

The idea that an optical beam can induce a waveguide and guide itself in it was first suggested by Askar’yan as early as 1962 [7]. A few years later, optical beam self-focusing was

first observed in materials with *Kerr* nonlinearities [8]. In order to investigate this effect, the wave equation in nonlinear *Kerr* media was analyzed in both one and two transverse dimensions, and spatial self-trapping of optical beams was proposed by Chiao *et al* [9]. However soon after, Kelley found that the 2D soliton solutions of the NLSE undergo catastrophic collapse and are thus unstable [10]. Even 1D solitons are also unstable in a 3D bulk nonlinear medium, since they can break up into multiple filaments due to transverse instabilities. Thus, stable spatial solitons in *Kerr* media can only exist in configurations where one of the two transverse dimensions is redundant, i.e. where diffraction is arrested in one dimension by some other means such as using a slab waveguide. A few years after Kelley's paper, Dawes and Marburger found numerically that *saturable* nonlinearities are capable of 'arresting' this catastrophic collapse and can lead to stable 2D spatial solitons [11]. However, these ideas have been largely ignored for 20 years. It was not until in the early 1990s, with the discovery of photorefractive solitons and subsequently the observation of quadratic solitons, that the idea of using saturable nonlinearities for stable formation of 2D solitons has started to be explored in experimental systems. Essentially, the very idea of a 'saturable nonlinearity', associated with materials in which the magnitude of the nonlinear index change has an upper bound (saturates with increasing intensity), turned out to be key to many of the new families of solitons discovered in the 1990s.

The first experiment on optical spatial solitons was reported in 1974 by Ashkin and Bjorkholm [12]. They used a 2D circularly symmetric beam in a nonlinear medium (a cell filled with sodium vapor) and observed self-trapping at higher powers. This is the classical signature of a spatial soliton. The nonlinearity encountered in the sodium vapor system was not of the classical *Kerr*, but rather of the saturable self-focusing type that exists near an electronic resonance in a two-level system. The saturable nature of this latter nonlinearity arises because intense fields tend to reduce the population difference between the two energy levels and hence no additional change in the refractive index occurs with any further increase in intensity. This saturable nature of the nonlinearity is essential because, as predicted by theory, 2D spatial solitons are only stable under saturable nonlinear conditions. However, at the same time, this atomic system also exhibits considerable loss, because it occurs at the close proximity of an atomic resonance. The loss inevitably limits the propagation distance and the ability to observe soliton dynamics. One way or the other, Ashkin and Bjorkholm did not continue to pursue research on solitons, and the field was deserted without experiments for another decade. The next experiment on spatial solitons was carried out in 1985, when self-trapping of an optical beam in a 1D planar waveguide filled with liquid CS<sub>2</sub> was observed (as the first 1D *Kerr* spatial soliton) by Barthelemy *et al* [13]. This soliton experiment opened the way to subsequent 1D soliton demonstrations in a variety of materials displaying a *Kerr*-type nonlinearity, including glass, semiconductors and polymers [14–17].

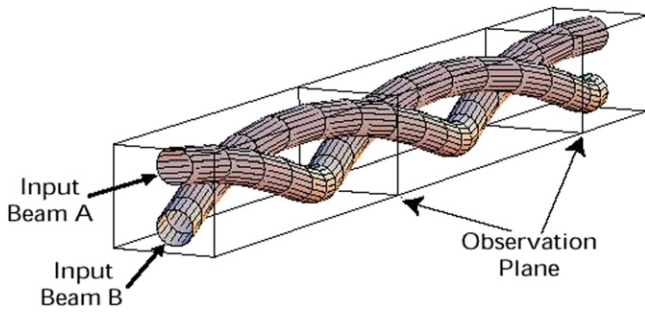
Despite these efforts, most spatial soliton experiments were carried out in 1D configurations using *Kerr*-type

nonlinear media, partly because exact analytical solutions could only be found for the 1D *Kerr* nonlinear NLSE, and partly because researchers were under the (wrong) impression that other kinds of nonlinear mechanisms would not be able to support the formation of stable solitons. In fact, apart from the Ashkin and Bjorkholm's first soliton experiment, it was never fully successful to generate a 2D circular soliton propagating as a long needle of light without diffraction. This is mainly for the following two reasons. The first reason is the requirement for exceedingly high powers needed to observe spatial solitons through the optical *Kerr* effect. This is because *Kerr*-type nonlinearities are of electronic origin and are therefore very weak. The second reason, as mentioned above, is even more fundamental. For non-saturable *Kerr* nonlinearities, light-induced lensing tends to become stronger and stronger as the beam intensity increases, thus causing catastrophic self-focusing and breakup of the beam. As we shall elaborate later, the first successful demonstrations of new classes of spatial solitons that do not necessarily rely on *Kerr* nonlinearities were crucial for the development of this field. In this respect, there were photorefractive solitons that led the way, followed by quadratic solitons, nematicons in nonlinear liquid crystals and spatial solitons in nonlocal nonlinear media.

### 1.3. Why are spatial solitons so interesting?

Optical spatial solitons are not only interesting in themselves, but also because they exhibit many fascinating features such as particle-like interactions during collisions. Apart from fundamental aspects, spatial solitons have also been suggested for a variety of applications, including for example waveguiding and beam-splitting, optical interconnects, frequency conversion, image transmission, gateless computing and soliton-based navigation.

The equivalence between solitons and particles was suggested in 1965 [18] when soliton collisions were first investigated. It was found that any collision between solitons involves 'forces': solitons interact very much like real particles, exerting attraction and repulsion forces on one another [19]. Such particle-like collisions were subsequently demonstrated with 1D *Kerr* spatial solitons in glass waveguides by Aitchison and co-workers [20, 21]. They showed that two in-phase *Kerr* solitons attract whereas two out-of-phase solitons repel each other. More complex soliton interactions involving coherent four-wave mixing were reported by Shalaby *et al* [22]. These two experiments demonstrated some of the basic collision properties of conventional *Kerr* solitons. Under standard launching conditions, soliton collisions occurring in 1D domain are fully elastic and hence the number of solitons is always conserved. Yet, energy exchange between solitons, in addition to attraction and repulsion, could also occur under different initial conditions. Soon after the discovery of photorefractive solitons [23–31], this situation drastically changed since a much broader class of collision experiments could be accessed experimentally. For example, the inelastic collision between two spatial solitons originating from a saturable nonlinearity was found to lead to soliton fusion or fission, with particle-like annihilation or birth of new solitons (see, e.g., [1, 2] for a detailed review).



**Figure 2.** An illustration of spiraling collision of interacting spatial solitons as demonstrated in the experiment with photorefractive solitons [31]. The arrows indicate the initial launching directions of the two soliton beams.

Another interesting aspect of this class of solitons is that spatial solitons are by nature (2+1)D entities (i.e. the two transverse dimensions plus one propagation coordinate), whereas temporal solitons are typically associated with a (1+1)D space–time evolution. With the ability to generate stable 2D solitons, a variety of new phenomena became possible; among them are 3D interactions between solitons, vortex solitons and angular momentum effects, and rotating dipole solitons. The fact that the spatial domain involves a higher dimensionality leads to a host of interesting phenomena and processes, which cannot be demonstrated with 1D Kerr spatial solitons and have no analog whatsoever in the temporal case. These include full vector solitons. Perhaps, one of the most intriguing examples is the 3D collision of interacting 2D spatial solitons spiraling around each other in a helical DNA-like trajectory, as illustrated in figure 2 [31].

Over the past two decades, the interest in optical spatial solitons has surged. As in any other research area, there have been certain key ideas and experiments that have defined this field and the directions it has taken. In this review we discuss a selection of such papers and their significance to the development of this field. Given that the activity that took place in this area is already enormous, this overview can be by no means complete.

## 2. Spatial solitons supported by different types of nonlinearities

Before early 1990s, nearly all experimental demonstrations of spatial solitons employed Kerr or Kerr-like nonlinearities. The discovery of new classes of solitons in material systems different from those with standard Kerr-nonlinearities opened up new possibilities in this area. The two major new classes of solitons discovered, at least in terms of nonlinear mechanisms, are photorefractive solitons and quadratic solitons. In what follows, we briefly discuss these two types of spatial solitons.

### 2.1. Photorefractive solitons

The discovery of photorefractive solitons [23–26] was, for many reasons, a radical turning point in the development of the field of spatial solitons. These solitons are formed due to multiple physical effects associated with the nonlinear

*photorefractive effect*, by which an index change can be established in a photorefractive material through a non-uniform light illumination. In essence, this process involves the absorption of light and subsequent charge generation, the motion of charge under the influence of electric fields, and the consequent establishment of local space-charge fields, which lead to an index change  $\Delta n$  via the electro-optic effect. The nonlinear response is typically non-local (due to the charge migration over macroscopic distances) and non-instantaneous (due to the dielectric relaxation and charge recombination). As such, the photorefractive nonlinearity is also inherently saturable, and can occur even with low-power nonuniform illumination. Because of these unique features of the photorefractive nonlinearity, photorefractive materials have provided a convenient and ideal platform for the observation of a variety of spatial soliton phenomena, including 2D soliton interaction and spiraling and, subsequently, incoherent solitons and discrete solitons.

Photorefractive solitons were first predicated and observed in the quasi-steady-state regime in 1992 by Segev and colleagues [23, 24]. Shortly thereafter, steady-state photorefractive self-focusing was reported [27]. Soon after, the formation of the so-called steady-state photorefractive ‘screening solitons’ was predicted by Segev *et al* [25] and Christodoulides and Carvalho [26]. These solitons were soon successfully demonstrated in a series of experiments [28–30, 32]. Figure 1 shows a typical example of a 2D screening soliton propagating through a biased photorefractive strontium barium niobate (SBN) crystal [28, 29]. In addition to steady-state screening solitons, many other families of photorefractive solitons (e.g. photovoltaic solitons) were subsequently reported [33, 34], all emerging from the rich behavior of the photorefractive effects [35].

The discovery of photorefractive solitons was important for many reasons. For example, the power necessary to generate these solitons can be as low as a few microwatts (insensitive to the absolute intensity), thus enabling many experiments to be carried out with weak CW laser beams. Another unique feature of photorefractive solitons is that a weak soliton beam can induce a waveguide that can be used in turn to guide other more intense beams at wavelengths that are photorefractively less sensitive [36–38]. This makes them attractive for waveguiding and steering applications. In particular, in a series of experiments it has been demonstrated that photorefractive soliton-induced waveguides can be used for device applications such as directional couplers and high-efficiency frequency converters [39, 40].

### 2.2. Quadratic solitons

Another general class of spatial solitons that has been experimentally demonstrated was that of quadratic solitons. Quadratic solitons were predicted in the 1970s in quadratic nonlinear media [41], and were experimentally demonstrated in 1995 [42]. In this case, the beam trapping mechanism arises because of the energy exchange between the fundamental and second harmonic, as described by the usual coupled mode equations for second harmonic generation (SHG). The initial

AQ3

experiment used type II phase-matching in KTP, i.e. involved two orthogonally polarized fundamental input beams. The crystal geometry was such that the extra-ordinary fundamental wave and the second harmonic ‘walked away’ from the ordinary fundamental beam. That is, the group velocities of the interacting beams were not collinear. It was demonstrated, however, that once the soliton was formed on and near phase-matching conditions, the fundamental and generated second-harmonic fields were mutually trapped as a result of the strong nonlinear coupling which counteracted both beam diffraction and beam walkoff. Furthermore, even though a steady-state quadratic soliton consists of in-phase fundamental and harmonic fields, they can also be generated during the SHG process using a fundamental beam input only. The importance of this particular type of interaction is that it demonstrated experimentally that nonlinear wave mixing itself can lead to soliton formation. In fact, it was shown that any parametric process involving the product of two finite beams could lead to mutual self-focusing, and presumably spatial solitons.

About the same time, (1+1)-dimensional quadratic solitons were demonstrated in LiNbO<sub>3</sub> waveguides [43, 44]. What distinguishes this experiment from previous work is the amount of second harmonic generated. In this case the level of second harmonic was small, since the geometry chosen was far from the phase-matching condition. This limit is often called the cascading or Kerr limit in which only a small amount of second harmonic is needed to impart a nonlinear phase shift to the fundamental beam proportional to its intensity. This leads to beam self-focusing, and quadratic soliton formation can take place similar to that expected in the Kerr case. This 1D experiment also indicated that the effective nonlinearity also depends on the phase mismatch, which can thus be tunable both in sign and magnitude. In addition, the geometry can be flexible because stringent phase-matching conditions need not be imposed. It is these concepts that ultimately led to quadratic solitons in which temporal spreading was also arrested, at least in one dimension. For a detailed review about quadratic solitons, please refer to [45].

### 2.3. Spatial solitons in nonlocal and other nonlinear media

We briefly mention here that, in addition to the basic Kerr (cubic), photorefractive and quadratic nonlinearities already discussed, there are many other families of such nonlinearities via which optical spatial solitons can form. Some typical examples of such nonlinear mechanisms include resonant nonlinear effects in atomic vapors [46], nonlinear upconverted photobleaching in dye-doped polymers [47, 48], quadratic electro-optic effects in paraelectric nonlinear crystals [49], orientational enhanced photorefractive in organic nonlinear materials [50, 51], orientational [52] and thermal nonlinear effects [53, 54] in liquid crystals and pyroelectric effects [55].

Among these, an important class of nonlinearities is that associated with nonlocal processes which may occur, for example, in liquid crystals and thermal nonlinear media. Nonlocality in nonlinearity has profound effects on the dynamics of optical solitons, since it can lead to a counterintuitive behavior, when compared with traditional

Kerr-type solitons [56–64]. For instance, nonlocal solitons can overcome repulsion between two out-of-phase bright solitons or two in-phase dark solitons. Nonlocal solitons can also form bound states, as observed in both 1D and 2D settings. In addition, it has been demonstrated that nonlocality can lead to new families of waves and soliton interaction dynamics that would have been otherwise impossible in local isotropic nonlinear media, including solitons in nonlinear media with an infinite range of nonlocality, long-range interactions between solitons and nonlocal surface solitons. Some of these newly studied soliton phenomena will be mentioned in section 5.

## 3. Different families of spatial solitons

Regardless of the origin of nonlinearity, one can classify spatial solitons into broader categories based on their inherent characteristics. Such broad categories may include, for example, that of bright and dark solitons, single- and multi-component solitons, coherent and incoherent solitons, spatial and spatiotemporal solitons, traveling wave and cavity solitons, as well continuous and discrete solitons. In this section, we provide a brief review of a few such different families of spatial solitons. The general area of discrete solitons will be discussed separately in the next section.

### 3.1. Dark solitons and optical vortex solitons

Dark spatial solitons represent self-trapped optical beams involving a dark ‘notch’ (in the 1D case) or a ‘hole’ (in the 2D case) in an otherwise bright background. A 1D dark soliton is typically characterized by a  $\pi$  phase shift across the dark line where the field is zero, while a 2D dark soliton is a vortex soliton manifested by an azimuthal  $2m\pi$  phase shift around the dark hole. As was theoretically shown in 1973, 1D dark solitons should also exist in materials with self-defocusing nonlinearities, in stark contrast to bright solitons [65]. 2D dark vortex solitons, studied even earlier within the context of super-fluidity [66], were also predicted to occur in nonlinear optics [67].

The first experiments on dark solitons were performed around 1990 and 1991 [68, 69]. These experiments employed a variety of media, including those with thermal and semiconductor nonlinearities, all of which were of the saturable type. In the early work of Schwartzlander and co-workers, spatial dark-soliton stripes and grids were experimentally observed in the transverse plane of a laser beam propagating in a self-defocusing nonlinear medium. Shortly after this work, stable propagation of a 2D dark vortex soliton was also observed in self-defocusing media with thermal nonlinearities [70]. The vortex was experimentally created using a quasi-helical phase mask, and it propagated without any change in shape under self-defocusing conditions, except for a phase rotation around its center of symmetry as imposed by its azimuthal phase. In addition to dark-soliton stripes and vortices, gray spatial solitons (with a phase change across the notch of less than  $\pi$ ) that have a finite transverse velocity were also demonstrated [71, 72].

After the first prediction and experimental observation of bright photorefractive solitons [23, 24], steady-state *dark*

photorefractive screening [25, 26] and photovoltaic [33] solitons were also predicted. The first attempt to observe 1D dark photorefractive solitons and 2D vortex solitons was made by Duree *et al* in a bulk photorefractive crystal [73]. These dark solitons were of the nonlocal type, and were observed in quasi-steady state. Shortly after, steady-state 1D dark screening solitons [30, 74, 75] and dark photovoltaic solitons [76] were also observed. While 2D dark vortex solitons were anticipated to exist and were observed indeed in isotropic self-defocusing media [70, 77], it was not immediately obvious that the anisotropic photorefractive nonlinearity would also support isotropic structures such as dark soliton grids and circular vortex solitons. In the experiments of [78, 79], steady-state photorefractive vortex solitons were successfully demonstrated by employing the screening nonlinearity in a biased SBN crystal as well as the photovoltaic nonlinearity in an unbiased LiNbO<sub>3</sub> crystal, despite the inherent anisotropy of the photorefractive nonlinearity. Interestingly, pairs of optical vortex solitons were also generated from instability and breakup of a dark soliton stripe in both isotropic [80] and anisotropic [81] saturable self-defocusing nonlinear media.

Dark and vortex solitons have a unique place in nonlinear optics [82, 83]. The fact that plane waves are unstable in self-focusing media and therefore can disintegrate into bright solitons is perhaps not surprising. However, plane waves are stable in self-defocusing media. Therefore, an extra ‘ingredient’ (phase jump or phase singularity) is needed to form 1D dark solitons or 2D vortex solitons, as shown in all experimental demonstrations in homogeneous nonlinear media. This phase feature associated with dark solitons has played an important role in soliton phenomena and in particular in vortex soliton dynamics in discrete nonlinear systems.

### 3.2. Vector (multi-component) spatial solitons

Vector solitons refer to solitons involving multiple components that are not only coupled together but those that are absolutely necessary in order to maintain their shapes during propagation. From a theoretical perspective, the existence of a vector soliton was first predicted in 1974 by Manakov [84], who studied two degenerate solitons that are polarized along two orthogonal axes and are coupled through equal self-phase and cross-phase modulation effects. An experiment on two-component vector solitons was carried out by Shalaby and Barthelemy in 1992, who demonstrated that a bright–dark spatial soliton pair can propagate in a nonlinear material such as CS<sub>2</sub> [85]. This composite structure involved two different soliton components coupled through cross-phase modulation between the two different wavelengths. Although vector solitons were also suggested in the temporal domain (i.e. in fibers) [86–88] and later on for spatial solitons from a different perspective [89], it was not until 1996 that the experimental field of multi-component vector solitons actually blossomed.

The first experimental demonstration of vector solitons in the form proposed by Manakov was performed in AlGaAs waveguides in 1996 [90]. When the electric field vectors are polarized parallel to AlGaAs 110 and 001 crystalline axes, it so happens that the self- and cross-phase

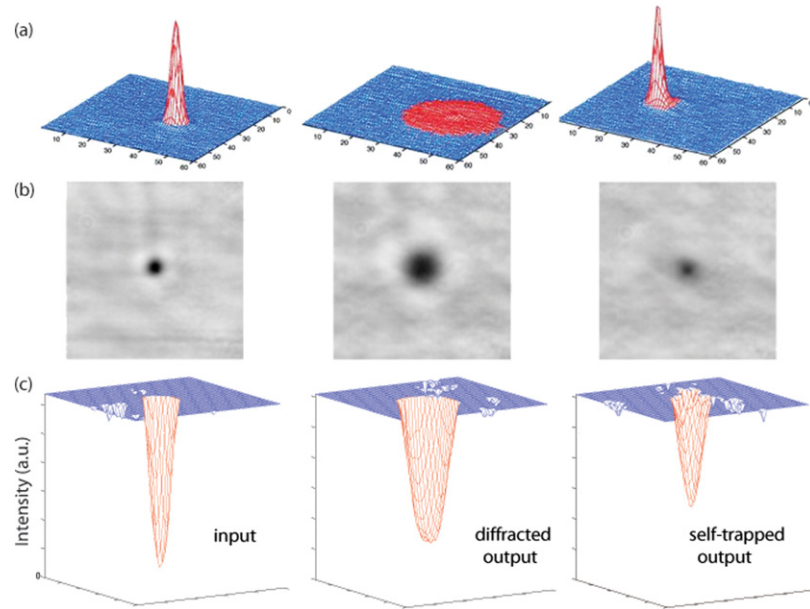
modulation terms are approximately equal, thus satisfying the requirement for Manakov solitons. In that experiment, the two polarizations were passed through two separate optical systems with different group velocity dispersions in order to eliminate the temporal coherence between them. Over the years, two additional methods (other than those relying on orthogonal polarizations) were suggested to realize multi-component solitons. The first one assumes that the two soliton components are widely separated in the frequency scale [91], whereas the second one considers components which are mutually incoherent with respect to each other [92]. The latter method proves particularly useful in terms of implementing  $N$ -component Manakov-type solitons. This is possible in materials with non-instantaneous nonlinearities (as in photorefractive crystals) even when all components share the same wavelength and polarization. This method was employed with photorefractive solitons, first to demonstrate two-component bright–bright, dark–dark and dark–bright soliton pairs [93, 94], and later on to demonstrate 1D multi-mode/multi-hump solitons [95] and 2D multimode solitons consisting of a bell-shaped component and a dipole mode [96, 97].

The general ideas behind multi-component vector solitons proved invaluable for later developments and in particular to the area of incoherent solitons discussed in the next section.

### 3.3. Incoherent solitons

Incoherent solitons are self-trapped partially coherent wave entities propagating in nonlinear media. Before 1996, incoherent solitons were thought to be impossible because solitons were considered to be exclusively coherent entities. In 1996, self-trapping of partially spatially incoherent light was first observed in experiment [98] by Segev’s group, followed by the observation of self-trapping of both temporally and spatially incoherent white light [99]. Yet in another experiment shortly after, self-trapping of 1D dark incoherent beams along with 2D dark incoherent beams (with 2D ‘voids’ nested in spatially incoherent beams) was also demonstrated [100]. These experimental demonstrations completely changed the commonly held belief that solitons could only be formed from coherent waves [101, 102]. Figure 3 shows the experimental results concerning white light bright solitons and incoherent dark solitons.

From a waveguide viewpoint, an incoherent soliton forms when its time-averaged intensity induces a multimode waveguide and then traps itself in it by populating the corresponding guided modes in a self-consistent fashion. These random-phase and weakly correlated self-trapped entities exhibit a host of unique properties that have no analog in the coherent regime. Moreover, their existence is related to many other areas of physics, in which nonlinearities, stochastic behavior and statistical averaging are involved. To describe the formation of incoherent solitons in non-instantaneous nonlinear materials, several theoretical approaches were developed, including the coherent density theory, the modal theory and the mutual coherence theory [103–107]. Meanwhile, the theory describing the self-trapping



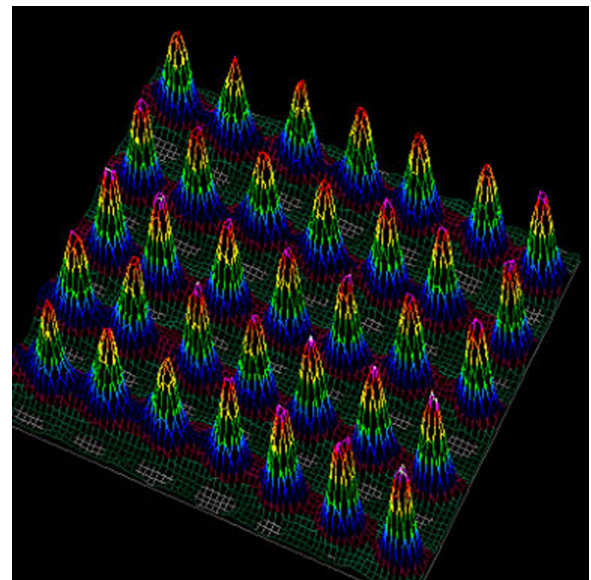
**Figure 3.** Experimental demonstration of self-trapping of (a) a 2D incoherent white-light beam and (b), (c) of a partially incoherent dark (vortex) beam. Top panels (a) show measured 3D intensity patterns of a white light beam at input, linear output and nonlinear output positions, and middle and bottom panels show 2D transverse patterns and 3D intensity plots of an incoherent vortex beam [99, 100].

of incoherent dark beams was also established, revealing that incoherent dark solitons have an inherent grayness because they consist of both bound and radiation modes [108, 109].

These experimental and theoretical studies clearly showed that bright and dark solitons can exist with both spatial and temporal coherence, and laid the basis for subsequent research on incoherent solitons and incoherent nonlinear wave dynamics in general. A host of new phenomena mediated by incoherent waves were subsequently uncovered. These include, for example, dark soliton splitting and ‘phase memory’ effects [110], anti-dark incoherent soliton states [111], and incoherent modulational instability [112–114]. Following this, a number of experiments were carried out demonstrating the effects of partial coherence on various soliton-related nonlinear wave phenomena. In particular, spontaneous clustering of solitons in partially coherent wavefronts [115] was demonstrated. The clustering phenomenon is an outcome of the interplay between random noise, weak correlation and high nonlinearity, which has no counterpart with solitons in coherent systems. Another example is the formation of spatial soliton pixels from partially coherent light [116] as illustrated in figure 4, whereas closely spaced spatial solitons are difficult to realize with coherent light. The rapid progress in the new area of incoherent solitons also set up the basis for the later advancements in incoherent (random-phase) solitons in discrete nonlinear systems.

### 3.4. Spatiotemporal solitons

Spatiotemporal solitons are simultaneously confined wavepackets of radiation in both space and time, often termed as ‘optical bullets’. While both temporal and spatial solitons have been well known since the early 1970s, the possibility of optical bullets was not suggested until 1990 [117]. Since then, there



**Figure 4.** Experimental demonstration of spatial soliton arrays of partially incoherent light [116].

were many attempts in generating such optical bullets, but experimental demonstration was hampered due to the difficulty in finding the right material so that, for a given optical pulse width, the dispersion length in time matches the diffraction length in space as well as the nonlinear length. In fact, in ordinary materials such as glass, a precise balance of the nonlinear and linear effects is hard to maintain, as a slight variation in pulse or material parameters can easily destroy such a balance.

The first experimental work reporting a ‘quasi-bullet’ used clever schemes to control the GVD along one spatial axis [118]. In that experiment, the spatiotemporal solitons observed were 2D in nature: they were confined in time

and one transverse spatial dimension, but underwent linear diffraction in the second dimension. The experiment used quadratic soliton interactions in the cascaded limit in bulk LiIO<sub>3</sub> with highly elliptically shaped beams. The pulse width of 110 fs was used, with a grating engineered GVD, to match the dispersion length to the diffraction length in one transverse dimension. Ultimately, however, such solitons are unstable at high intensity levels due to lack of confinement along the second spatial dimension, and thus a collapse into a number of filaments occurs. This first experiment further stimulated experimental search for light bullets [119]. X waves were also proposed as a means to resist the effects of diffraction and dispersion but by their very nature are not localized (have an infinite norm) [120]. Nevertheless, recent advances in spatiotemporal solitons as well as intelligent beam engineering have opened the road to ‘true’ 3D optical bullets as we shall mention in section 5.

### 3.5. Dissipative spatial solitons and cavity solitons

Dissipative solitons are self-trapped structures occurring in non-conservative systems, which require a nonlinear balance between loss and gain in addition to that between nonlinearity and dispersion/diffraction [121]. Dissipative solitons may arise as bright spots in a 2D transverse plane orthogonal to the beam propagation direction, often in an optical pattern-forming system with or without feedback. They have been studied in a number of experimental settings [122–127].

A typical example of such an entity is the so-called cavity soliton, which appears as a single-peak localized structure trapped between reflecting surfaces in contrast to other solitons that are traveling waves. In semiconductor devices, cavity solitons have been predicted to exist as spatial soliton pixels [122] and have been experimentally observed [127] in broad-area vertical cavity surface emitting lasers (VCSELs) driven by injection of a coherent and homogeneous field as a holding beam. This introduces new properties and significantly widens the class of materials in which soliton phenomena may be explored. Although there have been a number of experiments reported on patterns in resonators, evidence of localization of structures as isolated solitons was reported with a passive semiconductor microresonator, one of the simplest nonlinear optical feedback systems [125]. Requirements for the formation of solitons or localized structures were investigated in the spectral range where the nonlinearity of the semiconductor material is dispersive and defocusing. Another experiment reported stable, controllable spatial solitons in Na vapor with a single feedback mirror [126]. However, a clear experimental demonstration of cavity solitons was carried out with VCSELs that were electrically pumped above transparency but slightly below the lasing threshold [127], in which the generated cavity solitons could be written, erased and manipulated as objects independent of each other and of the boundary.

The progress on cavity solitons, especially in systems that do not support traveling-wave states (e.g. dissipative systems), is expected to bring up new possibilities with spatial solitons. Some recently studied examples include cavity soliton lasers

[128, 129], Bloch cavity solitons [130] and cavity polariton solitons [131].

## 4. Spatial solitons in discrete systems

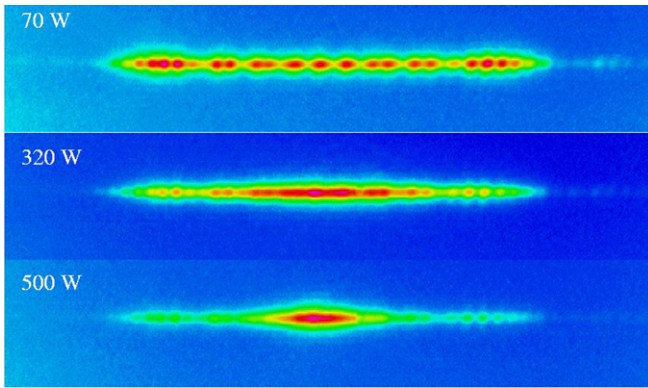
Linear and nonlinear discrete or periodic systems are abundant in nature. In optics, a typical example of such an arrangement is that of a closely spaced waveguide array, in which the collective behavior of wave propagation exhibits many intriguing and unexpected phenomena that have no counterpart in homogeneous media. In such discrete systems, another fascinating class of self-trapped states, spatial discrete solitons or lattice solitons, have been uncovered and have become the mainstream of soliton research in the past decade. These solitons arise from the interplay between discreteness and nonlinearity. Since the first prediction [132] and experimental demonstration [133] of 1D discrete solitons, this field has grown rapidly. For a detailed review, please refer to [134–136]. Below we just provide a brief overview of some key experimental works on discrete solitons carried out in 1D AlGaAs nonlinear waveguide arrays and 2D optically induced photonic lattices.

### 4.1. Discrete solitons in 1D waveguide arrays

A discrete soliton can form as a result of a balance between discrete diffraction and nonlinear self-focusing effects in periodic waveguide arrays, as first predicted by Christodoulides and Joseph in 1988 [132]. Unlike other families of spatial solitons, which are known to exist in homogeneous media, discrete solitons result from the collective behavior of the array as a whole. In reality they represent nonlinearity induced defect modes in a photonic crystal or a photonic lattice [134–136]. In the pioneering experiment of Eisenberg *et al* a 1D discrete soliton was observed in an AlGaAs nonlinear waveguide array [133], in which the light became trapped to only a few waveguides through the Kerr self-focusing nonlinearity, as opposed to linearly spreading laterally due to evanescent wave coupling (figure 5). This first observed discrete soliton existed in the so-called semi-infinite gap. This is because the nonlinear induced refractive index change the propagation constant of the soliton itself was locally elevated to the semi-infinite gap under the self-focusing nonlinearity. In subsequent experiments, Morandotti *et al* studied discrete soliton transport as well as self-focusing and defocusing dynamics in such 1D waveguide arrays [137, 138] based on the normal and anomalous diffraction properties of the system [139, 140]. Much of the earlier theoretical and experimental work on 1D waveguide arrays has been reviewed in [141, 142]. The bandgap structures and peculiar refraction and diffraction properties of discrete optical systems provided many new possibilities for spatial solitons that could not otherwise occur in continuous optical systems [136].

An example unique to solitons in discrete systems is that of a *gap soliton*. In optics, gap solitons are traditionally considered as temporal phenomena in 1D periodic media such as fiber gratings [143–145]. The existence of spatial gap





**Figure 5.** Experimental demonstration of a 1D discrete optical soliton in AlGaAs semiconductor waveguide arrays. Images are taken from the output facet of a 4 mm long sample for different powers. At low powers, the beam displays linear diffraction, but at high powers, a discrete soliton is formed [133].

solitons in waveguide arrays was suggested by Kivshar in 1993 [146]. In contrast to discrete solitons existing in the semi-infinite gap, these latter spatial gap solitons require a balance between anomalous diffraction and self-defocusing nonlinear effects. In this case, the nonlinear index change makes the soliton propagation constant to lie somewhere between the first and the second optical Bloch bands (within the first gap) near the edge of the first Brillouin zone (BZ) of a waveguide array. Such gap solitons have a ‘staggered’ phase structure, as first observed in 1D photonic lattices optically induced in a photorefractive nonlinear crystal by Segev’s group [147]. In subsequent experiments, 1D Floquet–Bloch gap solitons [148], reminiscent of those in Bragg gratings [143], and generation and steering of spatial gap solitons originating from higher Bloch bands [149, 150] were also demonstrated. Another process unique to solitons in discrete systems is that associated with discrete diffraction managed spatial solitons [151].

In addition to fundamental discrete solitons, other families of spatial solitons and related nonlinear phenomena studied in the bulk have also been investigated in discrete systems, including dark discrete solitons [138, 152], discrete modulation instability [153] and discrete vector solitons [154]. As we will see, the first experimental demonstration of 2D discrete solitons [155] opened up new opportunities in exploring the rich physics of nonlinear periodic systems.

#### 4.2. Discrete solitons in 2D photonic lattices

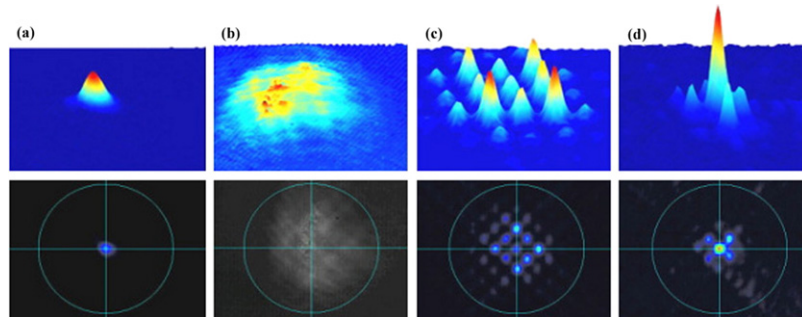
Although 1D waveguide arrays can serve as a test bench for studying many fascinating effects, it became increasingly clear that the opportunities offered by 1D arrays were rather limited, while rich soliton phenomena could arise in a high-dimensional environment. For instance, it has been suggested that, in 2D discrete waveguide networks, discrete solitons can be effectively routed or totally blocked using soliton collisions [156]. Yet, it has always been a challenge to fabricate 2D waveguide arrays with appropriate nonlinearity in bulk media.

The first experimental observation of a 2D optical discrete soliton was made in a biased photorefractive crystal by Segev’s

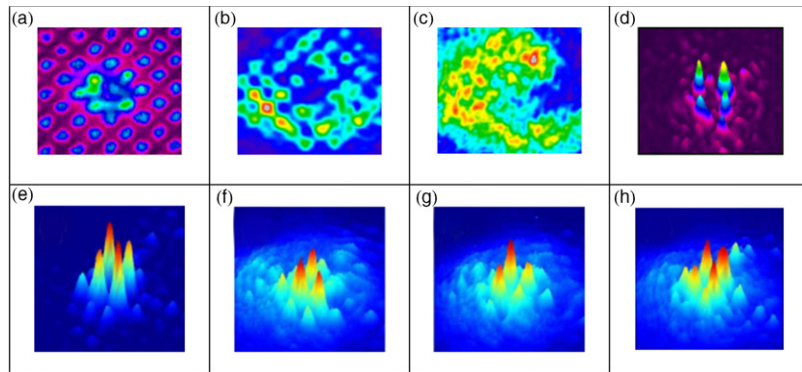
and Christodoulides’s groups [155]. Much of the success relied on a key idea, suggested by Efremidis *et al* from these same groups in 2002 [157], of optically inducing defect-free 2D nonlinear photonic lattices in biased photorefractive crystals such as SBN. The optical induction method suggested made use of the large electro-optic anisotropy of these nonlinear crystals and the associated photorefractive screening nonlinearity used earlier for the generation of photorefractive solitons [25–30, 32]. In such arrangements, the nonlinear index change induced and experienced by an optical beam depends on its polarization as well as on its intensity. Under appreciable bias conditions, this index change for an extraordinarily polarized beam can be 10 times larger or more than that for an ordinarily polarized beam, due to the large difference between the electro-optic coefficients in the nonlinear photorefractive crystal. Thus, if the lattice-inducing beam is *o*-polarized while the soliton-forming beam is *e*-polarized, the lattice beam would induce only a weak index change and could be considered as undergoing linear propagation, while the soliton-forming beam not only can experience the periodic optically induced lattice but also can exhibit nonlinear behavior. Using this mechanism, Fleischer *et al* created a 2D invariant photonic lattice in a SBN:75 crystal by interfering multiple plane-wave beams and observed the 2D discrete spatial solitons as well as 2D gap solitons [155].

The optical induction method was subsequently used by many other teams. In particular, Kivshar’s group observed twisted soliton modes in 1D optically induced gratings [158], and Chen’s group demonstrated discrete soliton-induced dislocations in 2D partially coherent lattices [159]. The latter established 2D photonic lattices based on the amplitude modulation of a partially coherent beam rather than multiple-beam interference, with active control of the *Talbot* effect of the lattice-inducing beam. In such partially coherent lattices, a clear transition from 2D discrete diffraction to discrete solitons was observed and recorded as the nonlinearity was gradually increased. Figure 6 shows typical experimental results of 2D discrete solitons observed in 2D photonic lattices induced with partially coherent light. The method of optical induction based on partially coherent light provided an effective way for inducing nonlinear photonic lattices [116] and later on for inducing various reconfigurable lattices with structured defects and surfaces [160, 161].

Apart from fundamental discrete solitons in 2D lattices, discrete vortex solitons with topological phase singularities were also proposed [162, 163] and soon after demonstrated experimentally in the same setting of such optically induced photonic lattices [164, 165]. These singly charged vortexing solitons have a nontrivial  $\pi/2$  step-phase structure, as confirmed experimentally by interference phase measurements (figure 7). Shortly after, higher band vortex gap solitons were also identified and observed in 2D induced lattices [166, 167], as well as higher charge [168, 169] and necklace-type [170] self-trapped vortex structures. In fact, using such a lattice reconfiguration under both self-focusing and self-defocusing nonlinearities [155, 171–173], a host of discrete soliton phenomena was demonstrated in various experimental



**Figure 6.** Experimental demonstration of a 2D discrete spatial soliton in an optically induced photonic lattice. (a) Input, (b) diffraction output without the lattice, (c) discrete diffraction at low nonlinearity and (d) discrete soliton formation at high nonlinearity. Top panels: 3D intensity plots; bottom panels: corresponding 2D transverse intensity patterns [159].



**Figure 7.** Experimental demonstration of discrete vortex solitons. (a)–(d) On-site vortex beam in which the singularity is centered on a waveguide site, where (a) shows input, (b) linear diffraction output, (c) interferogram formed by interfering the output beam (b) with a plane wave showing the characteristic  $0 \rightarrow 2\pi$  phase structure of a vortex and (d) nonlinear output of vortex-ring lattice soliton. (e)–(h) Off-site vortex lattice soliton in which the singularity is located between sites, where (e) shows the soliton output, and (f)–(h) are interferograms formed when the phase of the plane wave is successively changed in steps of  $\pi/2$  [164, 165].

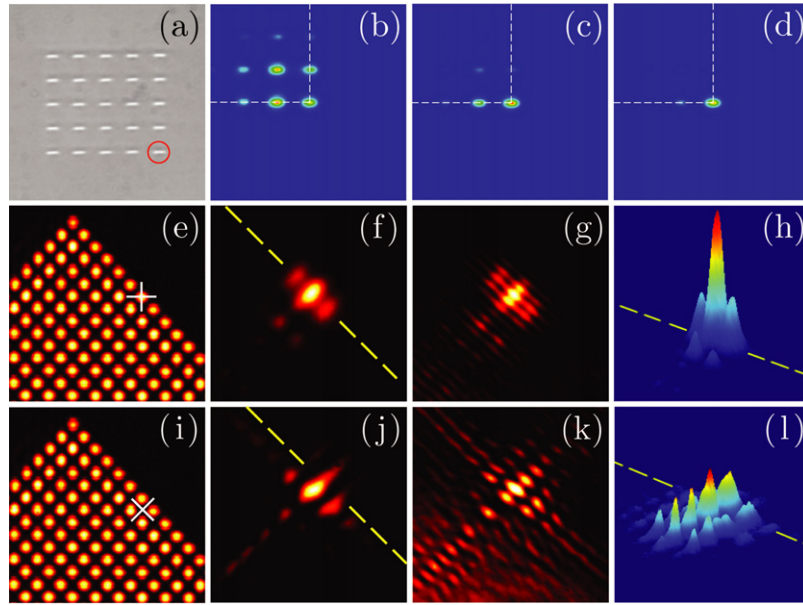
settings. These included, for example, 2D dipole and vector lattice solitons [174, 175], discrete soliton trains [174, 175], incoherent (random phase) lattice solitons [176, 177], rotary solitons in Bessel-type ring lattices [178, 179], solitons in quasi-crystal lattices and honeycomb lattices [180, 181], ‘reduced-symmetry’ solitons [182], in-gap and in-band (‘embedded’) soliton trains [183, 184] and discrete surface solitons, which we shall discuss separately in the section that follows. Other research conducted in 2D induced lattices includes, for example, the development of the Brillouin-zone spectroscopy method [185], spatial four-wave mixing and super-continuum generation [186, 187], bandgap guidance in lattices with low-index cores and structured defects [188–190], Bloch oscillations and Zener tunneling [191], and the seminal experiment on transport and Anderson localization in disordered photonic lattices [192].

Another major step in 2D photonic structures is the recent development of high-precision femtosecond-laser writing of waveguide arrays in silica, which also enabled the observation of discrete solitons, as reported by Szameit *et al* [193, 194]. This development is important not only because it provided a promising alternative for fabricating 2D arrays, but also because it led to a powerful platform for ‘fabricating’ photonic lattices of various configurations—virtually any periodic or aperiodic network of waveguide arrays could be realized using this method.

#### 4.3. Discrete surface solitons

Since Tamm and Shockley introduced electronic surface waves in the 1930s, surface wave phenomena have been of continuing interest in various areas of physics. In optics, nonlinear stationary surface states were actively considered in the early 1980s. However, progress in directly observing such nonlinear surface states was hampered by instabilities.

In the context of discrete systems, it is of course natural for one to ask as to whether optical self-trapped waves can exist at the surface of a photonic lattice or at the interface between two different photonic structures. Along these lines self-trapped surface waves (optical surface solitons) were soon proposed and experimentally demonstrated. Such states can be loosely interpreted as nonlinear defect modes with propagation constants (or eigenvalues) located within the forbidden optical bandgaps of a periodic structure. 1D in-phase surface solitons were first predicted to exist at the edge of nonlinear self-focusing waveguide lattices by Stegeman’s and Christodoulides’s groups [195], and demonstrated experimentally shortly after by the same group in AlGaAs arrays with a dominant Kerr nonlinear effect [196]. Meanwhile, 1D surface gap or ‘staggered’ solitons at the interface between a uniform medium and a self-defocusing waveguide array were also proposed [195, 197], and demonstrated in both quadratic [198] and saturable photorefractive nonlinear media [199, 200]. Unlike



**Figure 8.** Experimental demonstration of 2D surface solitons. (a) Microscope image of a laser-written array with an excited waveguide marked by a circle. (b)–(d) Output intensity distributions for progressively increasing input power levels. Observation of surface soliton (middle row) and surface gap soliton (bottom row) in optically induced photorefractive lattice. (e), (i) Lattice patterns with the waveguide excited by the probe beam marked by a cross. (f), (j) Surface soliton intensity patterns. (g), (k) Interference pattern between the soliton beam and a tilted plane wave. (h) 3D intensity plots of an in-phase surface soliton and (l) the corresponding pattern when its intensity is reduced significantly under the same bias condition. In all plots, dashed lines mark the interface [206, 207].

their in-phase counterparts, these latter entities have their propagation eigenvalues located in the first photonic bandgap (between the first and second Bloch bands) at the edge of the BZ. These studies extend the correspondence between optical surface waves and localized surface Tamm states into the nonlinear regime, since surface gap solitons can be viewed as the optical nonlinear analogs of Tamm states [201].

In the 2D domain [202–205], a direct experimental observation of 2D surface solitons remained a challenge due to experimental difficulties in fabricating 2D nonlinear lattices with sharp surfaces or interfaces. In 2007, two independent experimental demonstrations of 2D surface lattice solitons were reported using different materials and settings: one was accomplished by Wang *et al* in optically induced lattices in a photorefractive crystal [206], while the other was carried out by Szameit *et al* in femtosecond-laser-written waveguide arrays in bulk fused silica [207]. Figure 8 shows typical experimental results of 2D discrete surface solitons obtained from these two independent studies. In the fs-laser-written waveguide experiment, focused laser pulses were sent into bulk fused silica, which created a localized permanent increase in the refractive index of the material [194]. Consequently, when moving the sample transversely with respect to the beam, a longitudinal extended index modification (a waveguide) was written. A microscope image of the facet of such a laser-written  $5 \times 5$  waveguide array is shown in figure 8(a). While for low input peak powers a clear spreading of the light into the array was observed, for a high input peak power almost all of the light was localized in the excited waveguide [207]. In the photorefractive induction experiment, the lattice pattern was generated by a periodic modulation of a partially incoherent optical beam with an amplitude mask, which was then sent to an SBN crystal to induce a square lattice featuring sharp edges

or corners. With an appropriate high bias field, the spreading of a probe beam was suppressed at the lattice interface to form a discrete surface soliton and a surface gap soliton, while the beam at reduced intensity displayed significant diffraction under the same lattice conditions [206].

Over the last few years, considerable work has been devoted to discrete surface solitons [136, 201]. Much of the work theoretically investigated surface solitons of different types such as vector, vortex, incoherent, polychromatic and spatio-temporal surface solitons [203, 208–212]. Given that the geometries of the lattice surfaces and interfaces can vary considerably, discrete surface solitons were observed in a number of different settings including the interface between two dissimilar periodic media and superlattice surfaces [213–216]. In addition to nonlinear Tamm-like surface states, linear optical Shockley-like surface states were first introduced and observed in optically induced photonic superlattices [217, 218], and in subsequent experiments, transitions between Shockley-like and nonlinear Tamm-like surface states were also demonstrated. Furthermore, surface states that do not belong to the same family of Tamm or Shockley states have also been demonstrated as a new type of defect-free surface states in fs-laser-written curved waveguide arrays [219, 220].

#### 4.4. Discrete solitons in other material systems

In addition to the aforementioned intensity-dependent discrete media (AlGaAs semiconductors, optically induced photonic lattices and fs-laser-written waveguides), spatial discrete solitons were also studied and experimentally observed in many other material systems. In particular, Lederer's group theoretically investigated the formation of quadratic discrete

solitons based on the cascaded nonlinearity [221, 222], and subsequently, these solitons along with discrete modulation instability were observed by Stegeman's group in quadratic nonlinear waveguide arrays [223, 224]. Discrete solitons were also observed in nematic liquid crystal cells by Assanto's group [225, 226], and in defocusing photovoltaic LiNbO<sub>3</sub> waveguide arrays by Kip's group [227]. Other families of spatial solitons previously studied in continuous systems such as cavity solitons and dissipative solitons were also suggested in discrete nonlinear arrangements [228, 229]. Discrete solitons and nonlinear localized modes were studied in photonic crystal waveguides [230] and nonlinear coupled microcavities embedded in photonic crystals [231]. Many other theoretical and experimental studies considered discrete solitons in various other settings [134–136, 232, 233].

## 5. Recent developments in optical spatial solitons

Apart from a continuous growth of interest in spatial solitons in waveguide arrays and photonic lattices, other advances in optical spatial solitons are nowadays impacting fields like Bose–Einstein condensates, soft-condensed matter and plasmonics. Some recently discovered optical solitons or related novel phenomena include 3D light bullets, self-accelerating solitons propagating along curved trajectories, subwavelength plasmonic solitons in nonlinear metamaterials, soliton formation and self-induced transparency in nonlinear soft condensed matter, and spatial beam dynamics and optical solitons in nonlinear materials of parity–time (PT) symmetry.

### 5.1. New families of spatial solitons

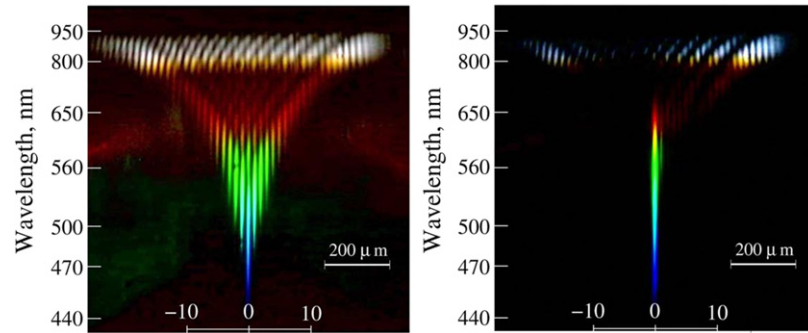
As previously indicated the field has been constantly enriching itself with new ideas. Even during the period of writing this review, there have been many such developments in spatial soliton phenomena reported in the literature. Here, we present just a few samples of such recent activities.

*Nonlocal incoherent solitons.* Incoherent optical spatial solitons are self-trapped beams having a multimode structure that varies randomly in time, which occur in nonlinear homogeneous [98] and periodic [176, 177] materials with *non-instantaneous* nonlinearities. Apart from incoherent spatial solitons supported by nonlinearities with a slow response time (much longer than the characteristic fluctuation time of the beam, thus named 'non-instantaneous'), it has been recently demonstrated by Segev's group that incoherent solitons could also exist in effectively *instantaneous* nonlocal nonlinear media [234, 235]. These solitons exhibit fundamentally new features: they propagate along random trajectories and they can be created in various nonlinear optical media as well as in other settings where the nonlinearity is nonlocal and very fast. This was somewhat surprising since it was believed that optical incoherent spatial solitons can exist only in noninstantaneous nonlinear media. The reason behind this was that only under such conditions can the nonlinear index change induced by a fluctuating beam become stationary in the propagation direction, which is necessary for guiding the multiple modes

comprising the incoherent beam. Nonlocal nonlinearities can overcome this obstacle, as the trapping process is mediated by a highly nonlocal nonlinear response, which yields a spatial averaging instead of the traditional temporal averaging provided by the noninstantaneous response. Using this approach, nonlocal incoherent solitons were demonstrated not only in bulk media, but also at the interface between a linear medium and a highly nonlocal effectively instantaneous nonlinear medium [236]. These incoherent nonlocal surface solitons exhibit features fundamentally different from their bulk counterparts or surface solitons created with local nonlinearities. Nonlocal incoherent solitons were also found to exist in spatially nonlocal nonlinear media with a logarithmic type of nonlinearity [237].

*Polychromatic solitons.* Polychromatic solitons are self-trapped optical beams involving multiple frequency components that are nonlinearly coupled together and propagate in the same direction. These solitons were previously studied theoretically as polychromatic partially spatially incoherent solitons in a noninstantaneous Kerr nonlinear medium, where a polychromatic incoherent soliton exists when its higher temporal frequency components are less spatially coherent than the lower frequency components [238]. Recently, there has been a great deal of interest in the study of multiple color beams, including the propagation of broad bandwidth and multi-color optical wavepackets in periodic photonic structures, under supercontinuum and white light generation conditions. In bulk nonlinear media, polychromatic or white-light solitons can only be supported by self-focusing nonlinearities. In photonic lattices, however, it was found that polychromatic solitons can exist under both self-focusing and -defocusing conditions. In particular, Neshev *et al* reported the experimental observation of discrete polychromatic solitons resulting from the localization of supercontinuum light through the nonlinear interaction of spectral components in extended periodic structures (figure 9) [239]. Polychromatic vortex solitons and discrete polychromatic surface solitons were demonstrated in subsequent experiments [240–242]. Meanwhile, in arrays of periodically curved waveguides, some fundamental processes occurring in solid-state physics such as dynamical localization have been introduced into optics and demonstrated in laser-written waveguide arrays [243–245]. In the nonlinear regime, polychromatic solitons have also been studied recently in curved waveguide arrays. When spectral components interact incoherently, the longitudinal modulation of the waveguide coupling can facilitate all optical switching of polychromatic light between two coupled waveguides, whereas in curved waveguide arrays nonlinearity leads to symmetry breaking and formation of polychromatic diffraction-managed solitons [246]. These results demonstrate new possibilities for tunable demultiplexing and spatial filtering of supercontinuum light.

*'Saddle' solitons and hybrid nonlinearity.* Periodic structures can exhibit several intriguing optical properties. In an optically induced 2D square lattice, for example, the high-symmetry X-point in the first Bloch band is akin to a 'saddle' point in the diffraction spectrum, where normal and anomalous diffractions co-exist along orthogonal directions. At this



**Figure 9.** Experimental demonstration of spectrally resolved nonlinear localization of the supercontinuum in a waveguide array. At low powers, the supercontinuum beam diffracts linearly (left), but self-trapping is achieved at high powers (right) [239].

X-point, a quasi-1D soliton train can be excited provided that an appropriate type of nonlinearity is used to balance beam diffraction in one particular direction, whereas in the orthogonal direction it is an extended plane wave. The propagation constant of such a 1D soliton train could reside within the first Bloch band, thus termed ‘in-band’ or ‘embedded’ solitons [184]. However, to simultaneously balance normal and anomalous diffractions in different directions, one needs orientation-dependent hybrid nonlinearity. Such nonlinearity was identified by Zhang *et al* in nonconventional biased photorefractive crystals [247–251], in which a co-existence of self-focusing and—defocusing nonlinearities at two orthogonal directions can lead to a controlled 1D soliton transition from different band edges or subband edges [252]. This new setting enables the reconfiguration of photonic structures and BZs for band-gap engineering and light manipulation, including Bragg reflection controlling the interplay between normal and anomalous diffraction/refraction under the same excitation conditions. Furthermore, with such hybrid nonlinearity, a new type of spatial gap solitons, namely, ‘saddle’ solitons, can be established. In fact, Hu *et al* observed such ‘saddle’ solitons by balancing the saddle-shaped bi-diffraction with hybrid focusing/defocusing in an optically induced 2D ionic-type lattice [253]. These ‘saddle’ solitons have a propagation constant residing in the Bragg reflection gap, but they differ from all previously observed solitons supported by a single focusing or defocusing nonlinearity.

*Self-accelerating Airy-type solitons.* Recently, a new class of nondiffracting beams, namely self-accelerating *Airy beams*, have been suggested by Christodoulides’ group [254–256]. In contradistinction with Bessel beams, Airy beams do not rely on simple conical superposition of plane waves. More importantly, they can self-accelerate during propagation in addition to being nondiffracting and self-healing. Over the past few years, considerable research has been devoted to Airy beams, ranging from their linear control to nonlinear self-trapping, covering fundamental aspects and demonstrations for proposed applications. In practice, all non-diffracting Airy beams must be truncated, to keep their power finite. Such truncated beams eventually diffract and lose their structure after a long enough propagation distance. To overcome this problem several teams considered methods by which nonlinear physical mechanisms could be used to nonlinearly self-trap Airy beams, very much as in the case of optical spatial solitons.

In particular, Chen’s group studied the behavior of Airy beams as they move from a nonlinear medium to a linear medium. It was shown that an Airy beam, initially driven by a self-defocusing nonlinearity, experiences anomalous diffraction and can maintain its shape in subsequent propagation. Conversely, its intensity pattern and acceleration cannot persist when driven by a self-focusing nonlinearity [257]. Fleischer’s group reported the experimental observation of self-trapping of Airy beams in nonlinear photorefractive media with diffusion nonlinearity [258]. In this case the self-trapped wave packet self-bends during propagation at an acceleration rate that is independent of the thermal energy associated with the diffusive nonlinearity, and they represent a typical example of Airy solitary-like wave formation using two-wave mixing. Quite recently, Segev’s group studied self-accelerating self-trapped beams in nonlinear optical media [259], exhibiting self-focusing and self-defocusing Kerr and saturable nonlinearities, as well as a quadratic response. In Kerr and saturable nonlinear media such beams are stable under self-defocusing and weak self-focusing, whereas for strong self-focusing the beams off-shoot solitons while their main lobe continues to accelerate. These self-trapped Airy-like accelerating beams in nonlinear media propagate along parabolic trajectories, different from all optical spatial solitons previously observed in continuum or discrete nonlinear media. Yet in more recent developments, self-accelerating beams have been extended to nonparaxial regimes, where one could expect solitons to travel along circular trajectory [260–263]. Under the action of nonlinearity, one could perhaps envision that ‘optical boomerang’ like beams might be realized originating from such nonparaxial self-accelerating beams.

## 5.2. Filamentation and light bullets

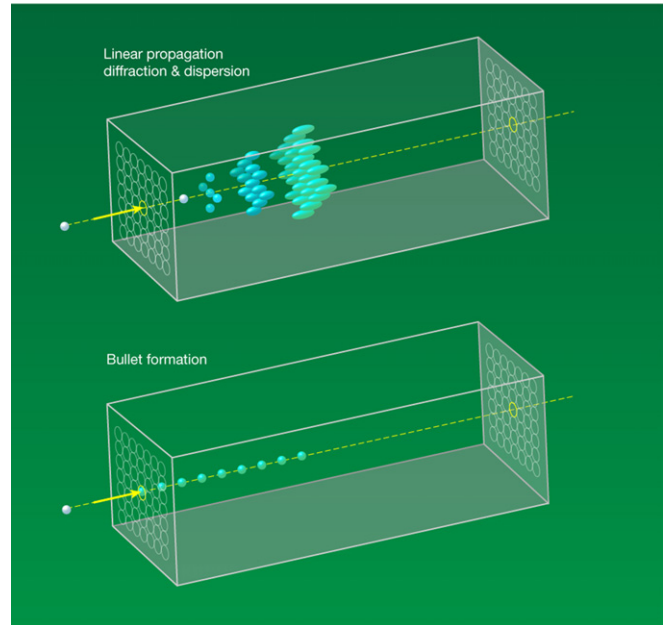
As mentioned before, there has been a considerable effort in generating wave packets that are localized in all 3D space–time dimensions (propagating free of diffraction and dispersion effects). Generating a light bullet typically requires a nonlinear mechanism that can simultaneously balance diffraction and dispersion during propagation, thus forming a *spatiotemporal soliton* as was first suggested by Silberberg [117]. Over the past two decades, researchers have explored a variety of alternative materials and techniques to generate light bullets. Quite recently, Chong *et al* explored a new approach

based on nondiffracting Bessel beam and non-dispersing 1D Airy pulse [246], thereby demonstrating Airy–Bessel wave packets as versatile linear light bullets [264]. For nonlinear light bullets, Burgess *et al* introduced a new form of stable spatiotemporal self-trapped optical packets stemming from the interplay of local and nonlocal nonlinearities [265], where self-trapped light beams in media with both electronic and molecular nonlinear responses can be established due to a decoupling of spatial and temporal effects for independent tuning. Panagiotopoulos *et al* proposed an approach to tailor the filamentation of intense femtosecond laser pulses with periodic lattices [266], where diffraction induced by the lattices provides a regularizing mechanism to the nonlinear self-action effects involved in filamentation, leading to a new propagation regime of intense lattice solitons.

It is also worth noting that, quite recently, Minardi and co-workers reported the first experimental observation of 3D light bullets excited by femtosecond pulses in a system featuring a quasi-instantaneous cubic nonlinearity and a periodic, transversally modulated refractive index [267]. A 40 mm long, closely spaced 2D hexagonal array of silica glass waveguides was fabricated, and 170 fs pulses at a wavelength of 1550 nm (for which the glass has anomalous dispersion) were launched into a single waveguide at the center of the array. As illustrated in figure 10 [268], an input pulse exhibited linear propagation in the array at low powers, spreading from the excited waveguide into several neighboring waveguides while the pulse profile broadened. However, at high powers, not only the pulse became localized in the original waveguide but also the output pulse duration became much shorter. These experimental results represent the first demonstration of light pulses simultaneously localized in space and time for distances longer than the characteristic lengths dictated by linear propagation.

### 5.3. Subwavelength spatial plasmon solitons

Spatial plasmon solitons were studied theoretically in Kerr slabs embedded between metal plates [269] and in discrete nonlinear metamaterials [270]. Solitons in nanoscale periodic structures consisting of metal and nonlinear dielectric slabs rely on a balance between tunneling of surface plasmon modes and nonlinear self-trapping. The evolution in such systems arises from the threefold interplay between periodicity, nonlinearity and surface plasmon polaritons, and is substantially different from that occurring in conventional nonlinear dielectric waveguide arrays. Later, Peleg *et al* [271] studied wave propagation in arrays of subwavelength waveguides with a sharp index contrast, and found a self-reviving soliton (*‘phoenix soliton’*) comprised of coupled forward- and backward-propagating light, originating solely from evanescent bands. In the linear regime, all Bloch waves comprising this wavepacket decay, whereas the nonlinearity can herd them into a propagating self-trapped beam. Recently, Ye *et al* [272] predicted theoretically that stable subwavelength plasmonic lattice solitons could be formed in arrays of metallic nanowires embedded in a nonlinear medium. The tight confinement of the guiding modes of the metallic



**Figure 10.** Illustration of 3D light bullets as observed in experiment with fs laser pulses propagating through a hexagonal array of glass waveguides. (top) A pulse undergoes spreading in time during propagation in the waveguide array. (bottom) Nonlinear effects cause the light packet to remain stable and compact as it moves [267, 268].

nanowires, combined with the strong nonlinearity induced by the enhanced field at the metal surface, can provide the main physical mechanisms for balancing the wave diffraction and the formation of plasmonic lattice solitons. Davoyan *et al* [273] suggested a method for using tapered waveguides for compensating the losses of surface plasmon polaritons in order to enhance nonlinear effects on the nanoscale. They studied nonlinear plasmon self-focusing in tapered metal–dielectric–metal slot waveguides and demonstrated stable propagation of spatial plasmon solitons. The study of subwavelength solitons as nonlinear self-trapped plasmon modes is just at the beginning, and any success is expected to have important applications to subwavelength nonlinear nanophotonics.

### 5.4. Spatial solitons in soft condensed matter

Artificial nonlinear materials consisting of liquid suspensions of sub-micrometer dielectric particles have been of strong interest since the early 1980s. Recently, a number of theoretical and experimental studies have considered nonlinear phenomena including modulation instability, spatial solitons and beam filamentation in nonlinear colloidal suspensions, and in soft condensed matter in general [274–279]. The effects leading to self-channeling of light in fluid suspensions can be classified into two general classes: effects relying on light scattering, i.e. optical gradient forces, and thermal effects relying on (weak) absorption. Using thermal effects in liquids, the refractive index typically decreases with increasing temperature. Therefore, such systems can only support dark solitons. Recently, Segev’s group predicted and observed a new type of self-trapped beams: a hot-particle soliton [280], formed in a nanoparticle suspension by virtue of

thermophoresis. These experiments manifest a new kind of interplay between light and fluid. On the other hand, using scattering effects, most studies so far have considered colloidal suspensions of the polystyrene–water type. In these latter arrangements, the particles have a refractive index that is higher than that of the liquid, i.e. the suspension has a *positive* polarizability. Interestingly, it has been recently suggested that beam propagation in nanosuspensions with negative polarizabilities (when the index of the particles is lower than that of the liquid) can be stable and can exhibit unusual nonlinear optical properties such as self-induced transparency [281]. In fact, experimental demonstrations of self-trapping and enhanced transmission of an optical beam in nonlinear nanosuspensions exhibiting *negative* polarizabilities were reported very recently [282]. While self-focusing was observed in colloidal nanosuspensions with both positive and negative polarizabilities, self-induced transparency of an optical beam was realized only in colloidal systems with negative polarizabilities. These studies bring about many possibilities for studying nonlinear phenomena in nanosuspensions and soft condensed matter in general.

### 5.5. Spatial solitons in systems with PT symmetry

In 1998, it was proposed by Bender and Boettcher that a wide class of Hamiltonians, even though non-Hermitian, can still exhibit real eigenvalue spectra provided that they obey the PT requirements or PT symmetry [283, 284]. The concept was introduced to optics ten years later by Christodoulides and co-workers, who investigated spatial beam dynamics in synthetic optical media with PT symmetries imposed by a balanced arrangement of gain or loss [285]. Experimental realizations of such PT systems have been reported recently, including observation of spontaneous PT symmetry breaking and power oscillations violating left–right symmetry in a PT optical coupled system involving a complex index potential [286], and observation of passive PT-symmetry breaking and phase transition that leads to a loss-induced optical transparency in specially designed pseudo-Hermitian guiding potentials [287]. These experimental results stimulated further interest in exploring new classes of PT-synthetic materials with intriguing and unexpected properties for fundamental research and novel applications. Nonlinear beam dynamics and spatial solitons in PT-symmetric potentials were also investigated theoretically. It was shown that a novel class of one- and 2D nonlinear self-trapped modes can exist in optical PT-synthetic lattices [288]. In fact, spatial solitons in PT symmetry systems have attracted a great deal of attention recently, fueled by continued interest in spatial solitons, discrete phenomena and PT-symmetric systems. For example, a number of theoretical studies have focused on the existence and stability of nonlinear localized modes and soliton dynamics supported by the PT-symmetric couplers or lattices under a variety of settings [289–293].

## 6. Concluding remarks

The field of optical spatial solitons, started really back in the 1970s and reached some maturity in the 1990s, is still

at the frontier of nonlinear optics and photonics. Although the formation mechanism and fundamental properties of spatial solitons under different nonlinearities have been studied extensively, much remains to be uncovered, especially with new developments in nonlinear synthetic materials including photonic bandgap materials, metamaterials, soft condensed matter and PT-symmetric materials. Judging from the current level of research activity, we believe that the area of spatial solitons will continue to flourish for many years to come.

## Acknowledgments

This work was supported in part by the US National Science Foundation and Air Force Office of Scientific Research. ZC and DC thank Arje Nachman at AFOSR for his continued support. The authors are indebted to many of their collaborators and many of their students and research fellows over the years. Thanks to Y Hu, P Zhang and D Gallardo for their help with this paper.

## References

- [1] Segev M and Stegeman G 1998 Self-trapping of optical beams: spatial solitons *Phys. Today* **51** 42–8
- [2] Stegeman G I and Segev M 1999 Optical spatial solitons and their interactions: universality and diversity *Science* **286** 1518–23
- [3] Hasegawa A 1990 *Optical Solitons in Fibers* 2nd edn (Berlin: Springer) xii, 79pp
- [4] Akhmediev N N and Ankiewicz A 1997 *Solitons: Nonlinear Pulses and Beams (Optical and Quantum Electronics Series)* (London: Chapman and Hall) xiii, p 335
- [5] Zakharov V E and Shabat A B 1972 Exact theory of 2-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media *Sov. Phys.—JETP* **34** 62
- [6] Ablowitz M J and Segur H 1981 *Solitons and the Inverse Scattering Transform (SIAM Studies in Applied Mathematics)* (Philadelphia: SIAM) x, 425pp, 2pp of plates
- [7] Askaryan G A 1962 Effects of the gradient of a strong electromagnetic beam on electrons and atoms *Sov. Phys.—JETP* **15** 1088–90
- [8] Hercher M 1964 Laser-induced damage in transparent media *J. Opt. Soc. Am.* **54** 563
- [9] Chiao R Y, Garmire E and Townes C H 1964 Self-trapping of optical beams *Phys. Rev. Lett.* **13** 479
- [10] Kelley P L 1965 Self-Focusing of optical beams *Phys. Rev. Lett.* **15** 1005
- [11] Dawes E L and Marburge Jh 1969 Computer studies in self-focusing *Phys. Rev.* **179** 862
- [12] Bjorkhol Je and Ashkin A 1974 CW self-focusing and self-trapping of light in sodium vapor *Phys. Rev. Lett.* **32** 129–32
- [13] Barthelemy A, Maneuf S and Froehly C 1985 Soliton propagation and self-confinement of laser-beams by Kerr optical non-linearity *Opt. Commun.* **55** 201–6
- [14] Aitchison J S *et al* 1990 Observation of spatial optical solitons in a nonlinear glass wave-guide *Opt. Lett.* **15** 471–3
- [15] Reynaud F and Barthelemy A 1990 Optically controlled interaction between 2 fundamental soliton beams *Europhys. Lett.* **12** 401–5
- [16] Aitchison J S *et al* 1992 Observation of spatial solitons in algaas wave-guides *Electron. Lett.* **28** 1879–80

AQ5

AQ6

- [17] Bartuch U *et al* 1997 Experimental investigations and numerical simulations of spatial solitons in planar polymer waveguides *Opt. Commun.* **134** 49–54
- [18] Zabusky N J and Kruskal M D 1965 Interaction of solitons in a collisionless plasma and recurrence of initial states *Phys. Rev. Lett.* **15** 240
- [19] Gordon J P 1983 Interaction forces among solitons in optical fibers *Opt. Lett.* **8** 596–8
- [20] Aitchison J S *et al* 1991 Experimental-observation of spatial soliton-interactions *Opt. Lett.* **16** 15–17
- [21] Aitchison J S *et al* 1991 Spatial optical solitons in planar glass wave-guides *J. Opt. Soc. Am. B* **8** 1290–7
- [22] Shalaby M, Reynaud F and Barthelemy A 1992 Experimental-observation of spatial soliton-interactions with a  $\pi$ -2 relative phase difference *Opt. Lett.* **17** 778–80
- [23] Segev M *et al* 1992 Spatial solitons in photorefractive media *Phys. Rev. Lett.* **68** 923–6
- [24] Duree G C *et al* 1993 Observation of self-trapping of an optical beam due to the photorefractive effect *Phys. Rev. Lett.* **71** 533–6
- [25] Segev M *et al* 1994 Steady-state spatial screening solitons in photorefractive materials with external applied-field *Phys. Rev. Lett.* **73** 3211–4
- [26] Christodoulides D N and Carvalho M I 1995 Bright, dark, and gray spatial soliton states in photorefractive media *J. Opt. Soc. Am. B* **12** 1628–33
- [27] Castillo M D I *et al* 1994 Spatial solitons in photorefractive BTO with drift mechanism of nonlinearity *Appl. Phys. Lett.* **64** 408–10
- [28] Shih M F *et al* 1995 Observation of 2-dimensional steady-state photorefractive screening solitons *Electron. Lett.* **31** 826–7
- [29] Shih M F *et al* 1996 Two-dimensional steady-state photorefractive screening solitons *Opt. Lett.* **21** 324–6
- [30] Chen Z G *et al* 1996 Steady-state dark photorefractive screening solitons *Opt. Lett.* **21** 629–31
- [31] Shih M F, Segev M and Salamo G 1997 Three-dimensional spiraling of interacting spatial solitons *Phys. Rev. Lett.* **78** 2551–4
- [32] Kos K *et al* 1996 One-dimensional steady-state photorefractive screening solitons *Phys. Rev. E* **53** R4330–3
- [33] Valley G C *et al* 1994 dark and bright photovoltaic spatial solitons *Phys. Rev. A* **50** R4457–0
- [34] Taya M *et al* 1995 Observation of dark photovoltaic spatial solitons *Phys. Rev. A* **52** 3095–100
- [35] Crosignani B *et al* 1998 Nonlinear optical beam propagation and solitons in photorefractive media *Riv. Nuovo Cimento* **21** 1–37
- [36] Morin M *et al* 1995 Wave-guides formed by quasi-steady-state photorefractive spatial solitons *Opt. Lett.* **20** 2066
- [37] Chen Z G, Mitchell M and Segev M 1996 Steady-state photorefractive soliton-induced Y-junction waveguides and high-order dark spatial solitons *Opt. Lett.* **21** 716–8
- [38] Shih M F *et al* 1997 Waveguides induced by photorefractive screening solitons *J. Opt. Soc. Am. B* **14** 3091–101
- [39] Lan S *et al* 1999 Directional coupler with soliton-induced waveguides *Opt. Lett.* **24** 475–7
- [40] Lan S *et al* 1999 Second-harmonic generation in waveguides induced by photorefractive spatial solitons *Opt. Lett.* **24** 1145–7
- [41] Hayata K and Koshiba M 1993 Multidimensional solitons in quadratic nonlinear media *Phys. Rev. Lett.* **71** 3275–8
- [42] Torruellas W E *et al* 1995 Observation of 2-dimensional spatial solitary waves in a quadratic medium *Phys. Rev. Lett.* **74** 5036–9
- [43] Schiek R, Baek Y and Stegeman G I 1996 One-dimensional spatial solitary waves due to cascaded second-order nonlinearities in planar waveguides *Phys. Rev. E* **53** 1138–41
- [44] Stegeman G I, Hagan D J and Torner L 1996  $\chi^{(2)}$  cascading phenomena and their applications to all-optical signal processing, mode-locking, pulse compression and solitons *Opt. Quantum Electron.* **28** 1691–740
- [45] Buryak A V *et al* 2002 Optical solitons due to quadratic nonlinearities: from basic physics to futuristic applications *Phys. Rep.—Rev. Sect. Phys. Lett.* **370** 63–235
- [46] Tikhonenko V, Christou J and LutherDavies B 1996 Three dimensional bright spatial soliton collision and fusion in a saturable nonlinear medium *Phys. Rev. Lett.* **76** 2698–701
- [47] Sarkisov S *et al* 1998 Optical branching in dye-doped polymeric waveguide *Opt. Commun.* **145** 265–73
- [48] Sarkisov S *et al* 1999 Optical channel waveguides formed by upconverted photobleaching of dye-doped polymer film in regime of dark spatial soliton *Opt. Commun.* **161** 132–40
- [49] DelRe E *et al* 1998 One-dimensional steady-state photorefractive spatial solitons in centrosymmetric paraelectric potassium lithium tantalate niobate *Opt. Lett.* **23** 421–3
- [50] Shih M F and Sheu F W 1999 Photorefractive polymeric optical spatial solitons *Opt. Lett.* **24** 1853–5
- [51] Chen Z G *et al* 2003 Self-trapping of light in an organic photorefractive glass *Opt. Lett.* **28** 2509–11
- [52] Assanto G and Peccianti M 2003 Spatial solitons in nematic liquid crystals *IEEE J. Quantum Electron.* **39** 13–21
- [53] Derrien F *et al* 2000 A thermal (2D+1) spatial optical soliton in a dye doped liquid crystal *J. Opt. A—Pure Appl. Opt.* **2** 332–7
- [54] Heninot J F *et al* 2004 Self-waveguiding in an isotropic channel induced in dye doped nematic liquid crystal and a bent self-waveguide *J. Opt. A—Pure Appl. Opt.* **6** 315–23
- [55] Safioui J, Devaux F and Chauvet M 2009 Pyroliton: pyroelectric spatial soliton *Opt. Express* **17** 22209–16
- [56] Mamaev A V, Saffman M and Zozulya A A 1997 Time-dependent evolution of an optical vortex in photorefractive media *Phys. Rev. A* **56** R1713–6
- [57] Nikolov N I *et al* 2004 Attraction of nonlocal dark optical solitons *Opt. Lett.* **29** 286–8
- [58] Dreischuh A *et al* 2006 Observation of attraction between dark solitons *Phys. Rev. Lett.* **96** 043901
- [59] Hutsebaut X *et al* 2004 Single-component higher-order mode solitons in liquid crystals *Opt. Commun.* **233** 211–7
- [60] Rotschild C *et al* 2006 Two-dimensional multipole solitons in nonlocal nonlinear media *Opt. Lett.* **31** 3312–4
- [61] Rotschild C *et al* 2005 Solitons in nonlinear media with an infinite range of nonlocality: first observation of coherent elliptic solitons and of vortex-ring solitons *Phys. Rev. Lett.* **95** 213904
- [62] Rotschild C *et al* 2006 Long-range interactions between optical solitons *Nature Phys.* **2** 769–74
- [63] Lopez-Aguayo S *et al* 2006 Stable rotating dipole solitons in nonlocal optical media *Opt. Lett.* **31** 1100–2
- [64] Alfassi B *et al* 2007 Nonlocal surface-wave solitons *Phys. Rev. Lett.* **98** 213901
- [65] Hasegawa A and Tappert F 1973 Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers: II. Normal dispersion *Appl. Phys. Lett.* **23** 171–2
- [66] Ginzburg V L and Pitaevskii L P 1958 On the theory of superfluidity *Sov. Phys.—JETP* **7** 858–61
- [67] Snyder A W, Poladian L and Mitchell D J 1992 Stable black self-guided beams of circular symmetry in a bulk Kerr medium *Opt. Lett.* **17** 789–91
- [68] Swartzlander G A *et al* 1991 Spatial dark-soliton stripes and grids in self-defocusing materials *Phys. Rev. Lett.* **66** 1583–6



- [69] Allan G R *et al* 1991 Observation of fundamental dark spatial solitons in semiconductors using picosecond pulses *Opt. Lett.* **16** 156–8
- [70] Swartzlander G A and Law C T 1992 Optical vortex solitons observed in Kerr nonlinear media *Phys. Rev. Lett.* **69** 2503–6
- [71] Luther-Davies B and Yang X P 1992 Wave-guides and Y-junctions formed in bulk media by using dark spatial solitons *Opt. Lett.* **17** 496–8
- [72] Bosshard C, Mamyshev P V and Stegeman G I 1994 All-optical steering of dark spatial soliton arrays and the beams guided by them *Opt. Lett.* **19** 90–2
- [73] Duree G *et al* 1995 Dark photorefractive spatial solitons and photorefractive vortex solitons *Phys. Rev. Lett.* **74** 1978–81
- [74] Castillo M D I *et al* 1995 (1+1)-Dimension dark spatial solitons in photorefractive  $\text{BI}_{12}\text{TiO}_{20}$  crystal *Opt. Commun.* **118** 515–9
- [75] Chen Z G *et al* 1997 Sequential formation of multiple dark photorefractive spatial solitons: experiments and theory *J. Opt. Soc. Am. B* **14** 1407–17
- [76] Taya M *et al* 1996 Y junctions arising from dark-soliton propagation in photovoltaic media *Opt. Lett.* **21** 943–5
- [77] Luther-Davies B, Powles R and Tikhonenko V 1994 Nonlinear rotation of 3-dimensional dark spatial solitons in a Gaussian laser-beam *Opt. Lett.* **19** 1816–8
- [78] Chen Z G *et al* 1997 Self-trapping of an optical vortex by use of the bulk photovoltaic effect *Phys. Rev. Lett.* **78** 2948–51
- [79] Chen Z G *et al* 1997 Steady-state vortex-screening solitons formed in biased photorefractive media *Opt. Lett.* **22** 1751–3
- [80] Tikhonenko V *et al* 1996 Observation of vortex solitons created by the instability of dark soliton stripes *Opt. Lett.* **21** 1129–31
- [81] Mamaev A V, Saffman M and Zozulya A A 1996 Vortex evolution and bound pair formation in anisotropic nonlinear optical media *Phys. Rev. Lett.* **77** 4544–7
- [82] Kivshar Y S and Luther-Davies B 1998 Dark optical solitons: physics and applications *Phys. Rep.—Rev. Sect. Phys. Lett.* **298** 81–197
- [83] Desyatnikov A S, Kivshar Y S and Torner L 2005 Optical vortices and vortex solitons *Progress in Optics* ed W Emil (Amsterdam: Elsevier) pp 291–391
- [84] Manakov S V 1974 On the theory of two-dimensional stationary self-focusing of electromagnetic waves *Sov. J. Exp. Theor. Phys.* **38** 248
- [85] Shalaby M and Barthelemy A J 1992 Observation of the Self-guided propagation of a dark and bright spatial soliton pair in a focusing nonlinear medium *IEEE J. Quantum Electron.* **28** 2736–41
- [86] Christodoulides D N and Joseph R I 1988 Vector solitons in birefringent nonlinear dispersive media *Opt. Lett.* **13** 53–5
- [87] Tratnik M V and Sipe J E 1988 Bound solitary waves in a birefringent optical fiber *Phys. Rev. A* **38** 2011–7
- [88] Haelterman M, Sheppard A P and Snyder A W 1993 Bound-vector solitary waves in isotropic nonlinear dispersive media *Opt. Lett.* **18** 1406–8
- [89] Snyder A W, Hewlett S J and Mitchell D J 1994 Dynamic spatial solitons *Phys. Rev. Lett.* **72** 1012–5
- [90] Kang J U *et al* 1996 Observation of Manakov spatial solitons in AlGaAs planar waveguides *Phys. Rev. Lett.* **76** 3699–702
- [91] Trillo S *et al* 1988 Optical solitary waves induced by cross-phase modulation *Opt. Lett.* **13** 871–3
- [92] Christodoulides D N *et al* 1996 Incoherently coupled soliton pairs in biased photorefractive crystals *Appl. Phys. Lett.* **68** 1763–5
- [93] Chen Z G *et al* 1996 Observation of incoherently coupled photorefractive spatial soliton pairs *Opt. Lett.* **21** 1436–8
- [94] Chen Z G *et al* 1996 Incoherently coupled dark-bright photorefractive solitons *Opt. Lett.* **21** 1821–3
- [95] Mitchell M, Segev M and Christodoulides D N 1998 Observation of multihump multimode solitons *Phys. Rev. Lett.* **80** 4657–60
- [96] Carmon T *et al* 2000 Observation of two-dimensional multimode solitons *Opt. Lett.* **25** 1113–5
- [97] Krolikowski W *et al* 2000 Observation of dipole-mode vector solitons *Phys. Rev. Lett.* **85** 1424–7
- [98] Mitchell M *et al* 1996 Self-trapping of partially spatially incoherent light *Phys. Rev. Lett.* **77** 490–3
- [99] Mitchell M and Segev M 1997 Self-trapping of incoherent white light *Nature* **387** 880–3
- [100] Chen Z G *et al* 1998 Self-trapping of dark incoherent light beams *Science* **280** 889–92
- [101] Segev M and Christodoulides D N 2001 Incoherent solitons: self-trapping of weakly-correlated wave-packets *Spatial Solitons* (Berlin: Springer) pp 87–123
- [102] Chen Z G, Segev M and Christodoulides D N 2003 Experiments on partially coherent photorefractive solitons *J. Optics A—Pure Appl. Opt.* **5** S389–97
- [103] Christodoulides D N *et al* 1997 Theory of incoherent self-focusing in biased photorefractive media *Phys. Rev. Lett.* **78** 646–9
- [104] Mitchell M *et al* 1997 Theory of self-trapped spatially incoherent light beams *Phys. Rev. Lett.* **79** 4990–3
- [105] Christodoulides D N *et al* 1998 Multimode incoherent spatial solitons in logarithmically saturable nonlinear media *Phys. Rev. Lett.* **80** 2310–3
- [106] Snyder A W and Mitchell D J 1998 Big incoherent solitons *Phys. Rev. Lett.* **80** 1422–4
- [107] Shkunov V V and Anderson D Z 1998 Radiation transfer model of self-trapping spatially incoherent radiation by nonlinear media *Phys. Rev. Lett.* **81** 2683–6
- [108] Christodoulides D N *et al* 1998 Theory of incoherent dark solitons *Phys. Rev. Lett.* **80** 5113–6
- [109] Coskun T H *et al* 1998 Dynamics of incoherent bright and dark self-trapped beams and their coherence properties in photorefractive crystals *Opt. Lett.* **23** 418–20
- [110] Coskun T H *et al* 1999 Dark incoherent soliton splitting and ‘phase-memory’ effects: theory and experiment *Phys. Rev. E* **59** R4777–80
- [111] Coskun T H *et al* 2000 Bright spatial solitons on a partially incoherent background *Phys. Rev. Lett.* **84** 2374–7
- [112] Soljacic M *et al* 2000 Modulation instability of incoherent beams in noninstantaneous nonlinear media *Phys. Rev. Lett.* **84** 467–70
- [113] Kip D *et al* 2000 Modulation instability and pattern formation in spatially incoherent light beams *Science* **290** 495–8
- [114] Klinger J, Martin H and Chen Z G 2001 Experiments on induced modulational instability of an incoherent optical beam *Opt. Lett.* **26** 271–3
- [115] Chen Z G *et al* 2002 Clustering of solitons in weakly correlated wavefronts *Proc. Natl Acad. Sci. USA* **99** 5223–7
- [116] Chen Z G and McCarthy K 2002 Spatial soliton pixels from partially incoherent light *Opt. Lett.* **27** 2019–21
- [117] Silberberg Y 1990 Collapse of optical pulses *Opt. Lett.* **15** 1282–4
- [118] Liu X, Qian L J and Wise F W 1999 Generation of optical spatiotemporal solitons *Phys. Rev. Lett.* **82** 4631–4
- [119] Koprnikov I G *et al* 2000 Self-compression of high-intensity femtosecond optical pulses and spatiotemporal soliton generation *Phys. Rev. Lett.* **84** 3847–50
- [120] Di Trapani P *et al* 2003 Spontaneously Generated X-shaped light bullets *Phys. Rev. Lett.* **91** 093904
- [121] Akhmediev N and Ankiewicz A 2005 *Dissipative Solitons (Lecture Notes in Physics vol 661)*

AQ7

AQ8

- [122] Brambilla M *et al* 1997 Spatial soliton pixels in semiconductor devices *Phys. Rev. Lett.* **79** 2042–5
- [123] Schreiber A *et al* 1997 Experimental investigation of solitary structures in a nonlinear optical feedback system *Opt. Commun.* **136** 415–8
- [124] Taranenko V B, Staliunas K and Weiss C O 1998 Pattern formation and localized structures in degenerate optical parametric mixing *Phys. Rev. Lett.* **81** 2236–9
- [125] Taranenko V B *et al* 2000 Patterns and localized structures in bistable semiconductor resonators *Phys. Rev. A* **61** 063818
- [126] Schapers B *et al* 2000 Interaction of localized structures in an optical pattern-forming system *Phys. Rev. Lett.* **85** 748–51
- [127] Barland S *et al* 2002 Cavity solitons as pixels in semiconductor microcavities *Nature* **419** 699–702
- [128] Tanguy Y *et al* 2008 Realization of a semiconductor-based cavity soliton laser *Phys. Rev. Lett.* **100** 013907
- [129] Genevet P *et al* 2008 Cavity soliton laser based on mutually coupled semiconductor microresonators *Phys. Rev. Lett.* **101** 123905
- [130] Staliunas K *et al* 2008 Bloch cavity solitons in nonlinear resonators with intracavity photonic crystals *Phys. Rev. Lett.* **101** 153903
- [131] Egorov O A *et al* 2009 Bright cavity polariton solitons *Phys. Rev. Lett.* **102** 153904
- [132] Christodoulides D N and Joseph R I 1988 Discrete self-focusing in nonlinear arrays of coupled wave-guides *Opt. Lett.* **13** 794–6
- [133] Eisenberg H S *et al* 1998 Discrete spatial optical solitons in waveguide arrays *Phys. Rev. Lett.* **81** 3383–6
- [134] Christodoulides D N, Lederer F and Silberberg Y 2003 Discretizing light behaviour in linear and nonlinear waveguide lattices *Nature* **424** 817–23
- [135] Kivshar Y S and Agrawal G P 2003 *Optical Solitons: From Fibers to Photonic Crystals* (Boston, MA: Academic)
- [136] Lederer F *et al* 2008 Discrete solitons in optics *Phys. Rep.—Rev. Sect. Phys. Lett.* **463** 1–126
- [137] Morandotti R *et al* 1999 Dynamics of discrete solitons in optical waveguide arrays *Phys. Rev. Lett.* **83** 2726–9
- [138] Morandotti R *et al* 2001 Self-focusing and defocusing in waveguide arrays *Phys. Rev. Lett.* **86** 3296–9
- [139] Eisenberg H S *et al* 2000 Diffraction management *Phys. Rev. Lett.* **85** 1863–6
- [140] Pertsch T *et al* 2002 Anomalous refraction and diffraction in discrete optical systems *Phys. Rev. Lett.* **88** 093901
- [141] Peschel U *et al* 2002 Optical discrete solitons in waveguide arrays: II. Dynamic properties *J. Opt. Soc. Am. B* **19** 2637–44
- [142] Eisenberg H S *et al* 2002 Optical discrete solitons in waveguide arrays: I. Soliton formation *J. Opt. Soc. Am. B* **19** 2938–44
- [143] Chen W and Mills D L 1987 Gap solitons and the nonlinear optical-response of superlattices *Phys. Rev. Lett.* **58** 160–3
- [144] Christodoulides D N and Joseph R I 1989 Slow Bragg solitons in nonlinear periodic structures *Phys. Rev. Lett.* **62** 1746–9
- [145] Eggleton B J *et al* 1996 Bragg grating solitons *Phys. Rev. Lett.* **76** 1627–30
- [146] Kivshar Y S 1993 Self-localization in arrays of defocusing wave-guides *Opt. Lett.* **18** 1147–9
- [147] Fleischer J W *et al* 2003 Observation of discrete solitons in optically induced real time waveguide arrays *Phys. Rev. Lett.* **90** 023902
- [148] Mandelik D *et al* 2003 Band-gap structure of waveguide arrays and excitation of Floquet–Bloch solitons *Phys. Rev. Lett.* **90** 053902
- [149] Mandelik D *et al* 2004 Gap solitons in waveguide arrays *Phys. Rev. Lett.* **92** 093904
- [150] Neshev D *et al* 2004 Controlled generation and steering of spatial gap solitons *Phys. Rev. Lett.* **93** 083905
- [151] Ablowitz M J and Musslimani Z H 2001 Discrete diffraction managed spatial solitons *Phys. Rev. Lett.* **87** 254102
- [152] Stepic M *et al* 2006 Observation of modulational instability in discrete media with self-defocusing nonlinearity *Opt. Lett.* **31** 247–9
- [153] Meier J *et al* 2004 Experimental Observation of Discrete Modulational Instability *Phys. Rev. Lett.* **92** 163902
- [154] Meier J *et al* 2003 Discrete vector solitons in Kerr nonlinear waveguide arrays *Phys. Rev. Lett.* **91** 143907
- [155] Fleischer J W *et al* 2003 Observation of two-dimensional discrete solitons in optically induced nonlinear photonic lattices *Nature* **422** 147–50
- [156] Christodoulides D N and Eugenieva E D 2001 Blocking and routing discrete solitons in two-dimensional networks of nonlinear waveguide arrays *Phys. Rev. Lett.* **87** 233901
- [157] Efremidis N K *et al* 2002 Discrete solitons in photorefractive optically induced photonic lattices *Phys. Rev. E* **66** 046602
- [158] Neshev D *et al* 2003 Spatial solitons in optically induced gratings *Opt. Lett.* **28** 710–2
- [159] Martin H *et al* 2004 Discrete solitons and soliton-induced dislocations in partially coherent photonic lattices *Phys. Rev. Lett.* **92** 123902
- [160] Chen Z G *et al* 2005 Formation of discrete solitons in light-induced photonic lattices *Opt. Express* **13** 1816–26
- [161] Chen Z and Yang J 2007 *Nonlinear Optics and Applications* (Trivandrum: Research Signpost) pp 103–49
- [162] Malomed B A and Kevrekidis P G 2001 Discrete vortex solitons *Phys. Rev. E* **64**02 026601
- [163] Yang J K and Musslimani Z H 2003 Fundamental and vortex solitons in a two-dimensional optical lattice *Opt. Lett.* **28** 2094–6
- [164] Neshev D N *et al* 2004 Observation of discrete vortex solitons in optically induced photonic lattices *Phys. Rev. Lett.* **92** 123903
- [165] Fleischer J W *et al* 2004 Observation of vortex-ring ‘discrete’ solitons in 2D photonic lattices *Phys. Rev. Lett.* **92** 123904
- [166] Manela O *et al* 2004 Two-dimensional higher-band vortex lattice solitons *Opt. Lett.* **29** 2049–51
- [167] Bartal G *et al* 2005 Observation of second-band vortex solitons in 2D photonic lattices *Phys. Rev. Lett.* **95** 053904
- [168] Kevrekidis P G *et al* 2004 Stable higher-order vortices and quasivortices in the discrete nonlinear Schrödinger equation *Phys. Rev. E* **70** 056612
- [169] Bezryadina A, Eugenieva E and Chen Z G 2006 Self-trapping and flipping of double-charged vortices in optically induced photonic lattices *Opt. Lett.* **31** 2456–8
- [170] Yang J *et al* 2005 Necklacelike solitons in optically induced photonic lattices *Phys. Rev. Lett.* **94** 113902
- [171] Efremidis N K *et al* 2003 Two-dimensional optical lattice solitons *Phys. Rev. Lett.* **91** 213906
- [172] Chen Z G *et al* 2004 Observation of two-dimensional lattice vector solitons *Opt. Lett.* **29** 1656–8
- [173] Yang J K *et al* 2004 Dipole solitons in optically induced two-dimensional photonic lattices *Opt. Lett.* **29** 1662–4
- [174] Chen Z G *et al* 2004 Anisotropic enhancement of discrete diffraction and formation of two-dimensional discrete-soliton trains *Phys. Rev. Lett.* **92** 143902
- [175] Kartashov Y V, Vysloukh V A and Torner L 2004 Soliton trains in photonic lattices *Opt. Express* **12** 2831–7
- [176] Buljan H *et al* 2004 Random-phase solitons in nonlinear periodic lattices *Phys. Rev. Lett.* **92** 223901
- [177] Cohen O *et al* 2005 Observation of random-phase lattice solitons *Nature* **433** 500–3
- [178] Kartashov Y V, Vysloukh V A and Torner L 2004 Rotary solitons in Bessel optical lattices *Phys. Rev. Lett.* **93** 093904
- [179] Wang X S, Chen Z G and Kevrekidis P G 2006 Observation of discrete solitons and soliton rotation in optically induced periodic ring lattices *Phys. Rev. Lett.* **96**

- [180] Freedman B *et al* 2006 Wave and defect dynamics in nonlinear photonic quasicrystals *Nature* **440** 1166–9
- [181] Peleg O *et al* 2007 Conical diffraction and gap solitons in honeycomb photonic lattices *Phys. Rev. Lett.* **98** 103901
- [182] Fischer R *et al* 2006 Reduced-symmetry two-dimensional solitons in photonic lattices *Phys. Rev. Lett.* **96** 023905
- [183] Lou C *et al* 2007 Nonlinear spectrum reshaping and gap-soliton-train trapping in optically induced photonic structures *Phys. Rev. Lett.* **98** 213903
- [184] Wang X S *et al* 2007 Observation of in-band lattice solitons *Phys. Rev. Lett.* **99** 243901
- AQ10 [185] Bartal G *et al* 2005 Brillouin zone spectroscopy of nonlinear photonic lattices *Phys. Rev. Lett.* **94**
- AQ11 [186] Bartal G, Manela O and Segev M 2006 Spatial four wave mixing in nonlinear periodic structures *Phys. Rev. Lett.* **97**
- [187] Manela O *et al* 2006 Spatial supercontinuum generation in nonlinear photonic lattices *Opt. Lett.* **31** 2320–2
- [188] Makasyuk I, Chen Z G and Yang J K 2006 Band-gap guidance in optically induced photonic lattices with a negative defect *Phys. Rev. Lett.* **96** 223903
- [189] Fedele F, Yang J K and Chen Z G 2005 Defect modes in one-dimensional photonic lattices *Opt. Lett.* **30** 1506–8
- [190] Wang X S, Chen Z G and Yang J K 2006 Guiding light in optically induced ring lattices with a low-refractive-index core *Opt. Lett.* **31** 1887–9
- [191] Trompeter H *et al* 2006 Bloch oscillations and Zener tunneling in two-dimensional photonic lattices *Phys. Rev. Lett.* **96**
- AQ12 [192] Segev M *et al* 2007 Transport and Anderson localization in disordered two-dimensional photonic lattices *Nature* **446** 52–5
- [193] Szameit A *et al* 2005 Discrete nonlinear localization in femtosecond laser written waveguides in fused silica *Opt. Express* **13** 10552–7
- [194] Szameit A *et al* 2006 Two-dimensional soliton in cubic fs laser written waveguide arrays in fused silica *Opt. Express* **14** 6055–62
- [195] Makris K G *et al* 2005 Discrete surface solitons *Opt. Lett.* **30** 2466–8
- [196] Suntsov S *et al* 2006 Observation of discrete surface solitons *Phys. Rev. Lett.* **96** 063901
- [197] Kartashov Y V, Vysloukh V A and Torner L 2006 Surface gap solitons *Phys. Rev. Lett.* **96** 073901
- [198] Siviloglou G A *et al* 2006 Observation of discrete quadratic surface solitons *Opt. Express* **14** 5508–16
- [199] Rosberg C R *et al* 2006 Observation of surface gap solitons in semi-infinite waveguide arrays *Phys. Rev. Lett.* **97** 083901
- [200] Smirnov E *et al* 2006 Observation of staggered surface solitary waves in one-dimensional waveguide arrays *Opt. Lett.* **31** 2338–40
- [201] Suntsov S *et al* 2007 Observation of one- and two-dimensional discrete surface spatial solitons *J. Nonlinear Opt. Phys. Mater.* **16** 401–26
- [202] Makris K G *et al* 2006 Surface lattice solitons *Opt. Lett.* **31** 2774–6
- [203] Kartashov Y V *et al* 2006 Surface vortex solitons *Opt. Express* **14** 4049–57
- [204] Kartashov Y V and Torner L 2006 Multipole-mode surface solitons *Opt. Lett.* **31** 2172–4
- [205] Vicencio R A *et al* 2007 Discrete surface solitons in two-dimensional anisotropic photonic lattices *Phys. Lett. A* **364** 274–6
- [206] Wang X S *et al* 2007 Observation of two-dimensional surface solitons *Phys. Rev. Lett.* **98** 123903
- [207] Szameit A *et al* 2007 Observation of two-dimensional surface solitons in asymmetric waveguide arrays *Phys. Rev. Lett.* **98** 173903
- [208] Hudock J *et al* 2005 Vector discrete nonlinear surface waves *Opt. Express* **13** 7720–5
- [209] Kartashov Y V, Ye F W and Torner L 2006 Vector mixed-gap surface solitons *Opt. Express* **14** 4808–14
- [210] Barak A *et al* 2007 Random-phase surface-wave solitons in nonlocal nonlinear media *Opt. Lett.* **32** 2450–2
- [211] Sukhorukov A A *et al* 2006 Polychromatic nonlinear surface modes generated by supercontinuum light *Opt. Express* **14** 11265–70
- [212] Mihalache D *et al* 2007 Spatiotemporal surface solitons in two-dimensional photonic lattices *Opt. Lett.* **32** 3173–5
- [213] Suntsov S *et al* 2007 Optical modes at the interface between two dissimilar discrete meta-materials *Opt. Express* **15** 4663–70
- [214] Suntsov S *et al* 2008 Optical spatial solitons at the interface between two dissimilar periodic media: theory and experiment *Opt. Express* **16** 10480–92
- [215] Szameit A *et al* 2008 Observation of two-dimensional lattice interface solitons *Opt. Lett.* **33** 663–5
- [216] Wang X S, Samodurov A and Chen Z G 2008 Demonstration of surface soliton arrays at the edge of a two-dimensional photonic lattice *Opt. Lett.* **33** 1240–2
- [217] Malkova N *et al* 2009 Observation of optical Shockley-like surface states in photonic superlattices *Opt. Lett.* **34** 1633–5
- [218] Malkova N *et al* 2009 Transition between Tamm-like and Shockley-like surface states in optically induced photonic superlattices *Phys. Rev. A* **80** 043806
- [219] Garanovich I L, Sukhorukov A A and Kivshar Y S 2008 Defect-free surface states in modulated photonic lattices *Phys. Rev. Lett.* **100**
- AQ13 [220] Szameit A *et al* 2008 Observation of defect-free surface modes in optical waveguide arrays *Phys. Rev. Lett.* **101**
- AQ14 [221] Darmanyan S, Kobayakov A and Lederer F 1998 Strongly localized modes in discrete systems with quadratic nonlinearity *Phys. Rev. E* **57** 2344–9
- [222] Peschel T, Peschel U and Lederer F 1998 Discrete bright solitary waves in quadratically nonlinear media *Phys. Rev. E* **57** 1127–33
- [223] Iwanow R *et al* 2004 Observation of discrete quadratic solitons *Phys. Rev. Lett.* **93** 113902
- [224] Iwanow R *et al* 2005 Discrete modulational instability in periodically poled lithium niobate waveguide arrays *Opt. Express* **13** 7794–9
- [225] Fratolocchi A *et al* 2004 Discrete propagation and spatial solitons in nematic liquid crystals *Opt. Lett.* **29** 1530–2
- [226] Fratolocchi A *et al* 2005 Discrete light propagation and self-trapping in liquid crystals *Opt. Express* **13** 1808–15
- [227] Chen F *et al* 2005 Discrete diffraction and spatial gap solitons in photovoltaic LiNbO<sub>3</sub> waveguide arrays *Opt. Express* **13** 4314–24
- [228] Peschel U, Egorov O and Lederer F 2004 Discrete cavity soliton *Opt. Lett.* **29** 1909–11
- [229] Ultanir E A, Stegeman G I and Christodoulides D N 2004 Dissipative photonic lattice solitons *Opt. Lett.* **29** 845–7
- [230] Mingaleev S F, Kivshar Y S and Sammut R A 2000 Long-range interaction and nonlinear localized modes in photonic crystal waveguides *Phys. Rev. E* **62** 5777–82
- [231] Christodoulides D N and Efremidis N K 2002 Discrete temporal solitons along a chain of nonlinear coupled microcavities embedded in photonic crystals *Opt. Lett.* **27** 568–70
- [232] Christodoulides C 2005 Discrete solitons in nonlinear optics *Opt. Express* **13** 1761 (special issue)
- [233] Kartashov Y *et al* 2009 Soliton shape and mobility control in optical lattices *Prog. Opt.* **52** 63–148
- [234] Rotschild C *et al* 2008 Incoherent spatial solitons in effectively instantaneous nonlinear media *Nature Photon.* **2** 371–6
- [235] Cohen O *et al* 2006 Incoherent solitons in instantaneous nonlocal nonlinear media *Phys. Rev. E* **73** 015601

- [236] Alfassi B, Rotschild C and Segev M 2009 Incoherent surface solitons in effectively instantaneous nonlocal nonlinear media *Phys. Rev. A* **80** 041808
- [237] Królikowski W, Bang O and Wyller J 2004 Nonlocal incoherent solitons *Phys. Rev. E* **70** 036617
- [238] Buljan H *et al* 2004 Polychromatic partially spatially incoherent solitons in a noninstantaneous Kerr nonlinear medium *J. Opt. Soc. Am. B* **21** 397–404
- [239] Neshev D N *et al* 2007 Nonlinear spectral-spatial control and localization of supercontinuum radiation *Phys. Rev. Lett.* **99** 1851–3
- [240] Neshev D N *et al* 2008 Observation of polychromatic vortex solitons *Opt. Lett.* **33** 1851–3
- [241] Sukhorukov A A *et al* 2008 Observation of polychromatic gap solitons *Opt. Express* **16** 5991–6
- [242] Neshev D N *et al* 2010 Supercontinuum generation with optical vortices *Opt. Express* **18** 18368–73
- [243] Longhi S 2009 Quantum-optical analogies using photonic structures *Laser Photon. Rev.* **3** 243–61
- [244] Longhi S *et al* 2006 Observation of dynamic localization in periodically curved waveguide arrays *Phys. Rev. Lett.* **96** 243901
- [245] Szameit A *et al* 2009 Polychromatic dynamic localization in curved photonic lattices *Nature Phys.* **5** 271–5
- [246] Qi X *et al* 2010 Polychromatic solitons and symmetry breaking in curved waveguide arrays *Opt. Lett.* **35** 1371–3
- [247] Zhang P *et al* 2007 Elliptical solitons in nonconventionally biased photorefractive crystals *Opt. Express* **15** 536–44
- [248] Zhang P *et al* 2008 *Elliptical discrete solitons supported by enhanced photorefractive anisotropy* *Opt. Express* **16** 3865–70
- [249] Zhang P *et al* 2008 Optically induced transition between discrete and gap solitons in a nonconventionally biased photorefractive crystal *Opt. Lett.* **33** 878–80
- [250] Zhang P *et al* 2009 Hybrid nonlinearity supported by nonconventionally biased photorefractive crystals *Appl. Phys. B* **95** 559–63
- [251] Zhang P *et al* 2010 Incomplete Brillouin-zone spectra and controlled Bragg reflection with ionic-type photonic lattices *Phys. Rev. A* **81** 041801
- [252] Hu Y *et al* 2009 Orientation-dependent excitations of lattice soliton trains with hybrid nonlinearity *Opt. Lett.* **34** 1114–6
- [253] Hu Y *et al* 2009 Saddle solitons: a balance between bi-diffraction and hybrid nonlinearity *Opt. Lett.* **34** 3259–61
- [254] Siviloglou G A and Christodoulides D N 2007 Accelerating finite energy Airy beams *Opt. Lett.* **32** 979–81
- [255] Siviloglou G A *et al* 2007 Observation of accelerating Airy beams *Phys. Rev. Lett.* **99** 213901
- AQ15 [256] Hu Y *et al* 2012 Generation and control of accelerating Airy beams *Nonlinear Photonics and Novel Phenomena*
- [257] Hu Y *et al* 2010 Persistence and breakdown of Airy beams driven by an initial nonlinearity *Opt. Lett.* **35** 3952–4
- [258] Jia S *et al* 2010 Diffusion-trapped Airy beams in photorefractive media *Phys. Rev. Lett.* **104** 253904
- [259] Kaminer I, Segev M and Christodoulides D N 2011 Self-accelerating self-trapped optical beams *Phys. Rev. Lett.* **106** 213903
- [260] Kaminer I *et al* 2012 Nondiffracting Accelerating Wave Packets of Maxwell's Equations *Phys. Rev. Lett.* **108** 163901
- [261] Courvoisier F, Mathis A, Froehly L, Giust R, Furfaro L, Lacourt P A, Jacquot M and Dudley J M 2012 Sending femtosecond pulses in circles: highly nonparaxial accelerating beams *Opt. Lett.* **37** 1736–8
- AQ16 [262] Chen Z 2012 Light bends itself into an arc *Am. Phys. Soc.* **5** 44
- [263] Zhang P *et al* 2012 Generation of linear and nonlinear nonparaxial accelerating beams *Opt. Lett.* **37**
- AQ17 [264] Chong A *et al* 2010 Airy–Bessel wave packets as versatile linear light bullets *Nature Photon.* **4** 103–6
- [265] Burgess I B *et al* 2009 Accessible light bullets via synergetic nonlinearities *Phys. Rev. Lett.* **102** 203903
- [266] Panagiotopoulos P *et al* 2010 Tailoring the filamentation of intense femtosecond laser pulses with periodic lattices *Phys. Rev. A* **82** 061803
- [267] Minardi S *et al* 2010 Three-dimensional light bullets in arrays of waveguides *Phys. Rev. Lett.* **105** 263901
- [268] Wise F W 2010 Viewpoint: generation of light bullets *Am. Phys. Soc.* **3** 107
- [269] Feigenbaum E and Orenstein M 2007 Plasmon-soliton *Opt. Lett.* **32** 674–6
- [270] Liu Y M *et al* 2007 Subwavelength discrete solitons in nonlinear metamaterials *Phys. Rev. Lett.* **99** 123903
- [271] Peleg O *et al* 2009 Nonlinear waves in subwavelength waveguide arrays: evanescent bands and the ‘Phoenix soliton’ *Phys. Rev. Lett.* **102** 163902
- [272] Ye F *et al* 2010 Subwavelength plasmonic lattice solitons in arrays of metallic nanowires *Phys. Rev. Lett.* **104** 203902
- [273] Davoyan A R *et al* 2010 Nonlinear nanofocusing in tapered plasmonic waveguides *Phys. Rev. Lett.* **105** 116804
- [274] Conti C, Ruocco G and Trillo S 2005 Optical spatial solitons in soft matter *Phys. Rev. Lett.* **95** 183902
- [275] Conti C *et al* 2006 Laser beam filamentation in fractal aggregates *Phys. Rev. Lett.* **97** 123903
- [276] El-Ganainy R *et al* 2007 Optical beam instabilities in nonlinear nanosuspensions *Opt. Lett.* **32** 3185–7
- [277] Reece P J, Wright E M and Dholakia K 2007 Experimental observation of modulation instability and optical spatial soliton arrays in soft condensed matter *Phys. Rev. Lett.* **98** 203902
- [278] Lee W M *et al* 2009 Nonlinear optical response of colloidal suspensions *Opt. Express* **17** 10277–89
- [279] Conti C and Delre E 2010 Optical supercavitation in soft matter *Phys. Rev. Lett.* **105** 118301
- [280] Lamhot Y *et al* 2010 Self-trapping of optical beams through thermophoresis *Phys. Rev. Lett.* **105** 163906
- [281] El-Ganainy R *et al* 2007 Soliton dynamics and self-induced transparency in nonlinear nanosuspensions *Opt. Express* **15** 10207–18
- [282] Man W *et al* 2012 *Enhanced Transmission in Engineered Colloidal Nano-suspensions* submitted
- AQ18 [283] Bender C M and Boettcher S 1998 Real spectra in non-Hermitian Hamiltonians having  $pt$  symmetry *Phys. Rev. Lett.* **80** 5243–6
- [284] Bender C M 2007 Making sense of non-Hermitian Hamiltonians *Rep. Prog. Phys.* **70** 947–1018
- [285] Makris K G *et al* 2008 Beam dynamics in  $PT$  symmetric optical lattices *Phys. Rev. Lett.* **100** 103904
- [286] Ruter C E *et al* 2010 Observation of parity–time symmetry in optics *Nature Phys.* **6** 192–5
- [287] Guo A *et al* 2009 Observation of  $PT$ -symmetry breaking in complex optical potentials *Phys. Rev. Lett.* **103** 093902
- [288] Musslimani Z H *et al* 2008 Optical solitons in  $PT$  periodic potentials *Phys. Rev. Lett.* **100** 030402
- [289] Abdullaev F K *et al* 2011 Solitons in  $PT$ -symmetric nonlinear lattices *Phys. Rev. A* **83** 041805
- [290] Hu S *et al* 2011 Solitons supported by complex  $PT$ -symmetric Gaussian potentials *Phys. Rev. A* **84** 043818
- [291] Suchkov S V *et al* 2011 Solitons in a chain of parity–time-invariant dimers *Phys. Rev. E* **84** 046609
- [292] He Y *et al* 2012 Lattice solitons in  $PT$ -symmetric mixed linear-nonlinear optical lattices *Phys. Rev. A* **85** 013831
- [293] Nixon S, Ge L and Yang J 2012 Stability analysis for solitons in  $PT$ -symmetric optical lattices *Phys. Rev. A* **85** 023822