# TEMPERATURE PROFILE IN THIN WIRES DURING LASER HEATING

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## Abstract

The temperature distribution in a moving thin wire heated by a rectangular profile laser beam is studied. The output Gaussian profile beam of a laser system is transformed to a rectangular profile beam and the corresponding laser irradiance is integrated in the transverse direction to obtain the one-dimensional heat flux in the wire. The typical radius of the wire is in the range of hundreds of micrometers and the length of the wire can be infinitely long. The local temperature can vary from a few hundred to several thousand Kelvin depending on the laser intensity and, therefore, temperature-dependent thermophysical properties need to be considered. The resulting non-linear heat conduction equation is solved analytically to express the temperature distribution as an integral equation. The integral was determined numerically and the solution adopted is computationally robust. The results are presented in terms of two dimensionless numbers Bi and Pe so that a graphical representation of the solution can be easily applied to determine the laser processing parameters for other wires.

## Introduction

Laser is used in many different forms of manufacturing processes. Due to its power density and maneuverability laser processers do have distinguish advantages over the other conventional manufacturing processes. As a manufacturing tool it is very versatile. Laser processing is based on the absorption of incident laser resulting localized heating of the material. This heat generation would give arise a temperature profile around the laser irradiated spot [1]. The end result of the laser heating is depend on the incident laser power and the temperature profile. When analyzing laser material processes the temperature profile is an important parameter. Depending on laser-matter interaction time the resulting temperature profile can be governed by equilibrium or non-equilibrium heat transfer [1,6]. Except for very short pulsed lasers, the heat transfer due to laser irradiation can be considered as equilibrium heat transfer. This equilibrium heat transfer can be pure conduction or heat transfer with phase change [6]. Laser heat treatment and laser surface treatment are pure conduction and laser cutting, welding, drilling and piercing phase change heat transfer.

To estimate the laser process parameters it is essential to have some theoretical modeling beforehand the experimental work. Over the years number of mathematical models were developed to capture the physical parameters behind laser heating [7,8,9]. Most of the models are capable of capturing the pure conduction based heat transfer and phase change heat transfer because they were mainly developed for applications involving phase change heat transfer [1,2,3]. On the other hand it is possible to apply the finite differences or finite element based approximations to evaluate the laser manufacturing processes. These approximations will work best when the spatial dimensions match one another [1].

Laser surface heat treatment is done on wires in order to change material parameters or to induce foreign material to change the surface parameters. This process is essentially one dimension process and it should be possible to model this with much simpler way than using the models with phase change heat transfer for large geometries, because laser heating, thermal behavior and the geometry all are one dimensional. When it comes to a situation where one dimension is much more lager than the other dimension these finite element base approximations face difficulties. When one dimension is in hundreds of micron and the length is in meters to capture the whole process correctly it is essential to have a very finer grid.

Here the goal is to come up with a simple one dimensional mathematical model which can predict the temperature profile under one dimensional conduction heat transfer along laser heated wires. The model was based on the first law of thermodynamics. The incident laser power has a two dimensional intensity profile. The model which is based on one dimensional heat transfer the effect of this two dimensional intensity has to be changed in to a one dimensional beam profile. This was achieved by considering the laser intensity on a cross section of the wire. With the integration the two dimensional laser energy input was transformed in to a one dimensional heat source term. Considering the heat balance on entire wire the governing equation was derived. It was assumed that the wire is infinitely long and the whole system can be modeled as an infinitely long fin. The governing equation was solved using successive integration over the temperature and quadratic form of temperature is used to make the numerical system stable.

### Mathematical Modelling

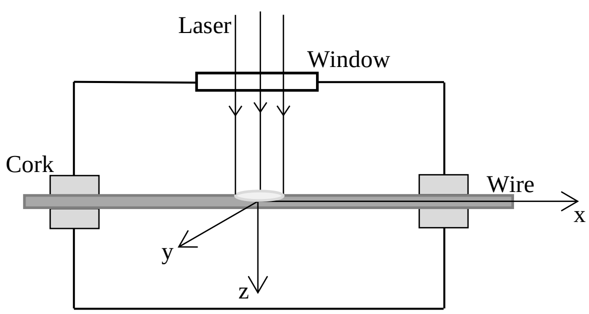


Fig 1. Schematic diagram of the process

The laser heating of the thin wire is modelled as an infinitely long circular fin with a heat source term due to the laser irradiation heating of the wire. The heat generation is only a fraction of the incident laser power due to the reflection of the laser from the wire surface. Typical metals reflect more than 90% of the incident laser power in the near infrared frequencies. The absorbed laser radiation will generate heat when the photons interact with metal surface, this heat is carried away from the surface to the interior of the material by the thermal conduction mechanism. Assume that the wire is moving in the positive '$x$' direction and the heat flow is positive in this direction. The incident laser intensity was assumed to be two dimensional profile with a rectangular shape.

$$I = I_0 e^{-\left(\frac{2x^2}{w_x^2}+\frac{2y^2}{w_y^2}\right)} \qquad (1)$$

Where $I$ is the incident laser intensity, $w_x$ and $w_y$ are the waist beam sizes. For this beam profile and considering the laser energy balance on the wire the value of the constant $I_0$ can be determined.

$$(1-R)P_i = I_0 \int_{-\infty}^{\infty} e^{-\left(\frac{2x^2}{w_x^2}\right)} \int_{-\infty}^{\infty} e^{-\left(\frac{2y^2}{w_y^2}\right)} dy\, dx \qquad (2)$$

Where $P_i$ is the incident laser power and $R$ is the reflectivity of the wire surface. Evaluating this double integral and solving for the $I_0$ leads to

$$I_0 = \frac{2(1-R)P_i}{\pi w_x w_y} \qquad (3)$$

To calculate the heat transfer effect on the wire as a one dimensional thin circular fin this two dimensional laser intensity has to be changed to a one dimensional field. This is achieved by integrating over the $y$ direction of the incident laser beam over a small length of $\Delta x$ along the wire.

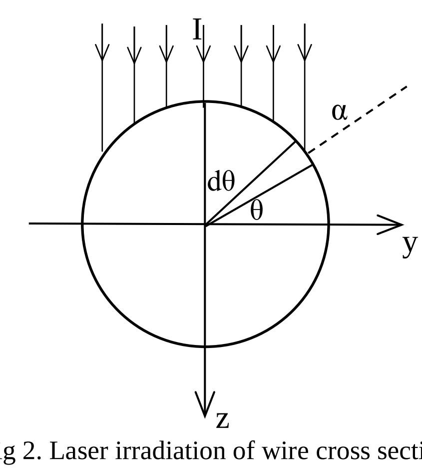


Fig 2. Laser irradiation of wire cross section

$$\int_{\theta=0}^{\pi/2} dP_w = 2\int_{\theta=0}^{\pi/2} I_0 e^{-\left(\frac{2x^2}{w_x^2}\right)} e^{-\left(\frac{2y^2}{w_y^2}\right)} \Delta x\, Cos\alpha\, r d\theta \quad (4)$$

Here $P_w$ the power absorbed with in the distance $\Delta x$ by the wire. And $r$ is the radius of the wire, $\alpha$ is the laser incident and the surface normal at angle $\theta$ on the wire cross section. This integral is evaluated on the $y$ direction and using the definitions of error functions it can be simplified to achieve a simple expression for one-dimensional laser intensity.

$$P_w = -\sqrt{\frac{2}{\pi}}\frac{P_a}{w_y} erf\left(\sqrt{2}\frac{r}{w_y}\right) e^{-\frac{2x^2}{w_x^2}} \Delta x \quad (5)$$

$$P_w' = -\sqrt{\frac{2}{\pi}}\frac{P_a}{w_y} erf\left(\sqrt{2}\frac{r}{w_y}\right) e^{-\frac{2x^2}{w_x^2}} \quad (5a)$$

$P_a = P_i(1-R)$, power absorbed by the wire and $\mathrm{erf}(x)$ denotes the standard error function. In order to find the temperature distribution of the wire energy balance was considered on a one dimensional infinitesimal length $\Delta x$. The system was considered to be in steady state.

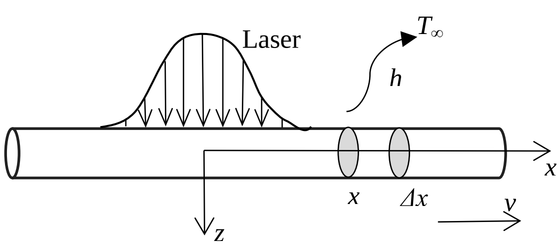


Fig 3. Wire energy balance

$$\text{Rate of energy input} = \text{Rate of energy output}$$

$$q''_{cond}(x)\pi r^2 + \rho C_p v\pi r^2 T(x) + P_w'\Delta x = q''_{cond}(x+\Delta x)\pi r^2 + \rho C_p v \pi r^2 T(x+\Delta x) + h[T(x) - T_\infty]2\pi r\Delta x \quad (6)$$

Combining with Fourier's law of heat conduction

$$\frac{d}{dx}\left[-k(T)\frac{dT}{dx}\right] + \rho C_p v\frac{dT}{dx} + \frac{2h}{r}(T - T_\infty) = \frac{P_w'}{\pi r^2} \quad (7)$$

$T$ is the wire temperature at point $x$, $k$ is the temperature dependent heat conductivity of the wire material, $\rho$ and $C_p$ are the density and the thermal capacity of wire material respectively. Convection heat transfer between wire surface and the environment is given as $h$. $v$ denotes the speed of the wire. $T_\infty$ is the environment temperature.

Due to the large temperature gradient present within this problem thermal conductivity can't be considered as a constant. The temperature dependent conductivity is approximated by first order dependency.

$$k(T) = k_0[1 + \beta(T - T_0)] \quad (8)$$

$k_0$ is the thermal conductivity at reference temperature $T_0$ and $\beta$ is the coefficient of linear thermal conductivity of the wire materila. The boundary conditions apply to this problem are

$$\begin{aligned} x = -\infty &\rightarrow T = T_0 \\ x = +\infty &\rightarrow T = T_0 \end{aligned} \quad (9)$$

The resulting boundary value problem is a nonlinear differential equation due to the presence of reference temperature values in convection term and in the conduction term. To solve this system it is required to introduce some non-dimensional parameters as follows

$$x^* = \frac{x}{l} \quad (10)$$

$$T^* = \frac{T - T_0}{2P_a l^2 / \pi w_x r^2 k_0} \quad (11)$$

Here $x^*$ and $T^*$ are the non-dimensional length and the non-dimensional time respectively. The characteristic length $l$ is defined as the following manner.

$$l = 2\sqrt{\alpha\tau} \qquad (11a)$$

$$\alpha = \frac{k_0}{\rho C_p} \qquad (11b)$$

$$\tau = \frac{2w_x}{v} \qquad (11c)$$

Here $\alpha$ is the thermal diffusivity of the wire material at the reference temperature and $\tau$ is the laser wire interaction time. The initial temperature is set as the same as surrounding temperature.

$$T_\infty = T_0 \qquad (12)$$

Using the above constants following numbers were defined to obtain the non-dimensional governing equation.

$$Bi = \frac{2hl^2}{k_0 r} \qquad (13)$$

$$Pe = \frac{\rho C_p v l}{k_0} \qquad (14)$$

$$\beta^* = \frac{2P_a l^2}{\pi^2 r^2 w_y k_0} \qquad (15)$$

$Bi$ is the Biot number, $Pe$ is the Peclet number and $\beta^*$ is the non-dimensional coefficient of linear thermal conductivity. The following constants $a$, $b$ is defined for the convenience of writing as the equation become dimension less.

$$a = -\sqrt{\frac{\pi}{2}}\frac{w_x}{w_y} erf\left(\sqrt{2}\frac{r}{w_y}\right)$$

$$b = \frac{2l^2}{w_x^2} \qquad (16)$$

The non-dimensional form of the governing differential equation is write as

$$-\frac{d}{dx^*}\left[1 + \beta^* T^* \frac{dT^*}{dx^*}\right] + Pe\frac{dT^*}{dx^*} + BiT^* = ae^{-bx^{*2}}$$

$$x^* = -\infty \;\rightarrow\; T^* = T_0$$
$$x^* = +\infty \;\rightarrow\; T^* = T_0 \qquad (17)$$

It was found that due to the presence of second derivative term of temperature this equation becomes numerically unstable. In order to overcome this numerical instability the governing equation is modified in the following manner.

$$\frac{d}{dx^*}\left[\frac{d}{dx^*}\left(T^* + \frac{\beta^*}{2}T^{*2}\right)\right] - Pe\frac{d}{dx^*}\left(T^* + \frac{\beta^*}{2}T^{*2}\right) - Bi\left(T^* + \frac{\beta^*}{2}T^{*2}\right)$$
$$= -ae^{-bx^{*2}} - Pe\beta^* T^* \frac{dT^*}{dx^*} - Bi\frac{\beta^*}{2}T^{*2} \qquad (18)$$

Now a quadratic temperature term is defined as

$$Q = T^* + \frac{\beta^*}{2}T^{*2} \qquad (19)$$

Now rearranging the governing equation obtain the final equation for boundary value problem.

$$\frac{d^2Q}{dx^{*2}} - Pe\frac{dQ}{dx^*} - BiQ = -ae^{-bx^{*2}} - Pe\beta^* T^* \frac{dT^*}{dx^*} - Bi\frac{\beta^*}{2}T^{*2}$$

$$x^* = -\infty \;\rightarrow\; Q = 0$$
$$x^* = +\infty \;\rightarrow\; Q = 0$$
$$(20)$$

Method of variation of parameters is used to solve this system of equation. The homogeneous solution to this equation is

$$Q_H(x^*) = A_1 e^{m_1 x^*} + A_2 e^{m_2 x^*} \qquad (21)$$

Here $m_1$ and $m_2$ are the roots of the characteristic equation.

$$m_1 = {}^1\!/_2 \left(Pe + \sqrt{Pe^2 + 4Bi}\right)$$
$$m_2 = {}^1\!/_2 \left(-Pe + \sqrt{Pe^2 + 4Bi}\right) \qquad (22)$$

Evaluating the Wronskian and subsequent simplification would yield particular integral for the quadratic temperature profile. A function $G(\eta)$ is define for convenience of writing as,

$$G(\eta) = -ae^{-b\eta^2} - Pe\beta^* T^* \frac{dT^*}{d\eta} - Bi\frac{\beta^*}{2} T^{*2} \qquad (23)$$

Now the particular integral can be written as

$$Q_P(x^*) = e^{-m_2 x^*} \int_0^{x^*} \frac{e^{m_1 \eta}}{-(m_1 + m_2) e^{(m_1+m_2)\eta}} G(\eta) d\eta - e^{m_1 x^*} \int_0^{x^*} \frac{e^{m_1 \eta}}{-(m_1 + m_2) e^{(m_1+m_2)\eta}} G(\eta) d\eta \qquad (24)$$

The final solution is the addition of $Q_H$ and $Q_P$ which gives the quadratic temperature profile of the wire.

$$Q(x^*) = Q_H(x^*) + Q_P(x^*) \qquad (25)$$

Here $\eta$ is a dummy variable. Then applying the boundary conditions appropriately to the previous equation will yield the final integrated form of the solution. From the equation it can be seen that temperature profile is still a part of the answer. To overcome this difficulty the answer is obtained by using numerical techniques.

$$Q(x^*) = \frac{ae^{m_1 x^*}}{(m_1 + m_2)} \int_{x^*}^{+\infty} e^{-(m_1\eta + b\eta^2)} d\eta + \frac{\beta^*(Bi + Pe.m_1) e^{m_1 x^*}}{2(m_1 + m_2)} \int_{x^*}^{+\infty} e^{-m_1 \eta} T^{*2} d\eta + \frac{ae^{-m_2 x^*}}{(m_1 + m_2)} \int_{-\infty}^{x^*} e^{-(m_2\eta + b\eta^2)} d\eta + \frac{\beta^*(Bi - Pe.m_2) e^{-m_2 x^*}}{2(m_1 + m_2)} \int_{-\infty}^{x^*} e^{m_2 \eta} T^{*2} d\eta \qquad (26)$$

This equation is solved by taking a simple one dimensional grid with successive approximation of temperature. The updated value for the non-dimensional temperature can be found as

$$T^*(x^*) = \frac{1}{\beta^*}\left(-1 + \sqrt{1 + 4Q(x^*)}\right) \qquad (27)$$

When the error between two successive iterations are smaller than the predefined error value iteration process was stopped and the temperature distribution of the last iteration is taken as the final answer.

Following data values were used for the numerical simulation. The temperature dependent thermal conductivity was calculated in the following manner.

Table 1. Variation of thermal conductivity with temperature

| Temperature ($^oC$) | Conductivity ($W \cdot m^{-2} \cdot K^{-1}$) |
|---|---|
| 21 | 11.2 |
| 93 | 12.7 |
| 204 | 15 |
| 316 | 17 |
| 427 | 19.2 |
| 538 | 21.3 |

| | |
|---|---|
| 649 | 23.4 |

When this data is plotted it can be seen that temperature dependent thermal conductivity can be approximated as a linear function of temperature.

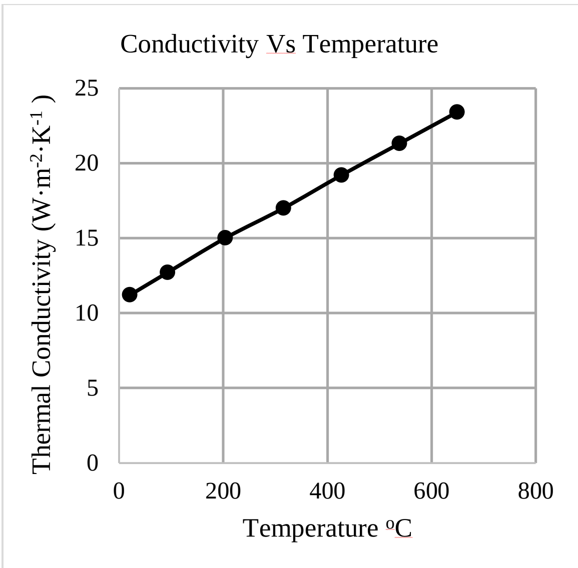


Fig 4. Thermal conductivity vs. temperature

Using above graph the temperature dependent thermal conductivity as in the equation (8) with the following constants.

$\boldsymbol{K_0 = 11.2}$ W·m$^{-2}$·K$^{-1}$

$\beta = 0.00174$ K$^{-1}$

$T_0 = 21$ $^{o}$C

Other data values used for the numerical simulation are as follows.

Density = 8430 kg·m$^{-3}$
Heat capacity = 502 J·kg$^{-1}$·K$^{-1}$
Laser power = 10 W
Reflectivity = 91 %
Length of laser beam = 1000 μm
Width of laser beam = 100 μm
Velocity of the wire = 9 mm·s$^{-1}$
Convection heat transfer coefficient = 250 W·m$^{-2}$·K$^{-1}$
Room temperature = 21 $^{o}$C

**Results and Discussion**

It can be seen that the computation technique with successive approximations to approximate the second order derivative converge to a stable solution.

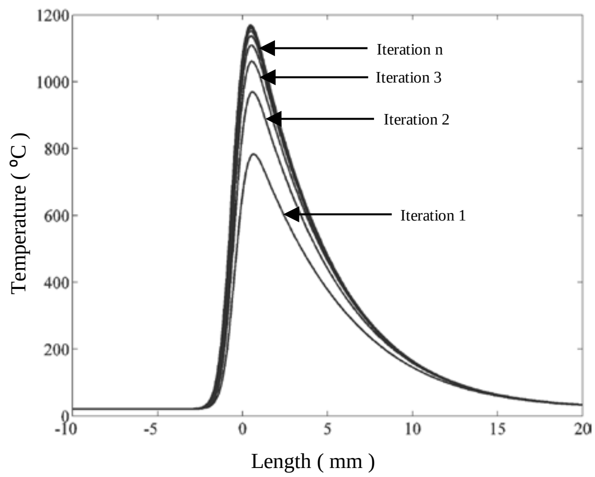


Fig 5. Change of solution with successive iterations

The final solution for a given values of parameters shows an unsymmetrical temperature profile along the wire. This happens because of the wire movement along a particular direction. The plot between the error norm and the iteration number shows the typical behaviour of convergence. It is interesting to note that the answer is reached from the bottom side that is first iteration gives a value much lower than the actual answer and each successive iteration yield values higher than the previous iteration.

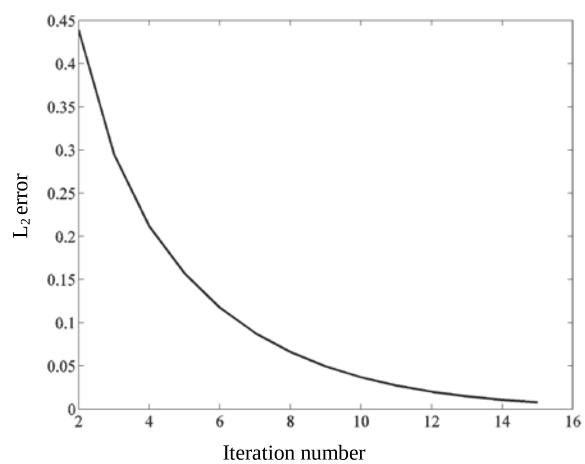


Fig 6. Convergence of the solution

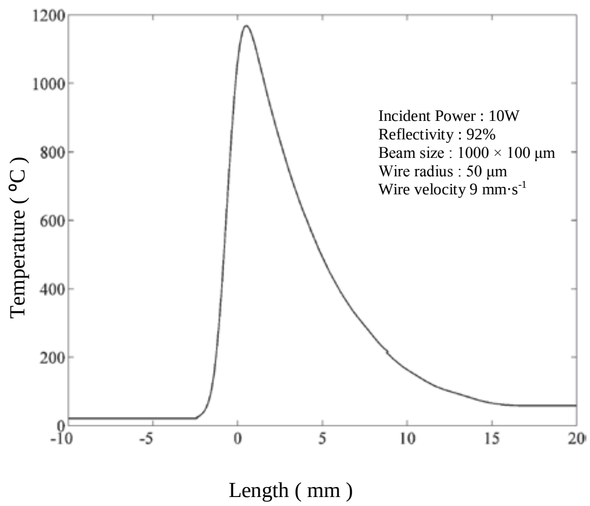


Fig 7. Temperature distribution along the wire

It can be seen that this temperature profile depend on the non-dimensional numbers $Pe$ and $Bi$. They can make the temperature profile a very narrow one or distributed one. The following graphs shows how the non-dimensional temperature distribution behaves with changing parameters. The following results show the wire temperature with the change in $Pe$ number and the $Bi$ number. These graphs were plotted with the non-dimensional length so that the solution did not depend on the geometry of the system.

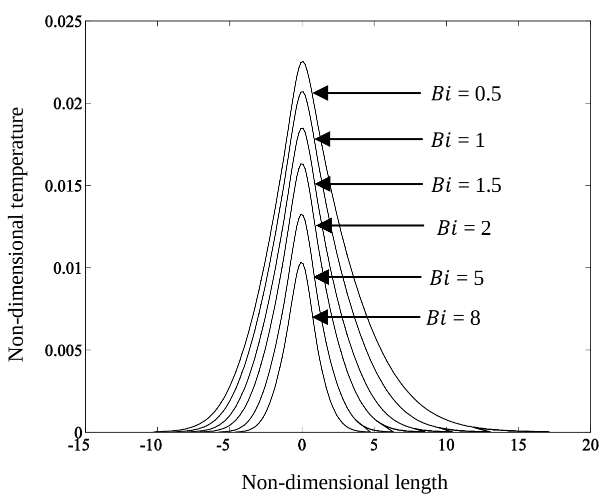


Fig 8. Non-dimensional temperature at $Pe$ = 0.5

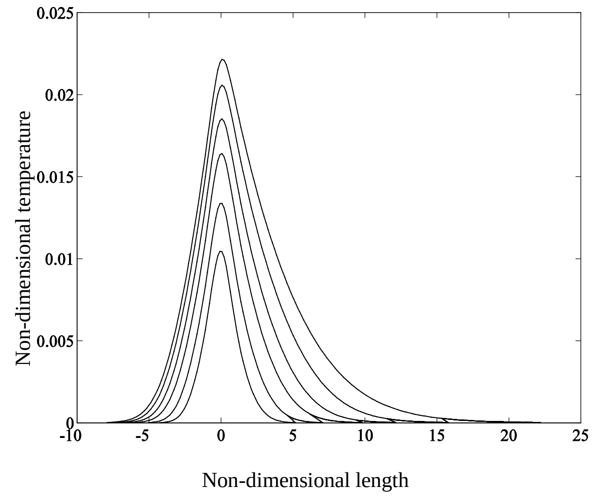


Fig 9. Non-dimensional temperature at $Pe$ = 1

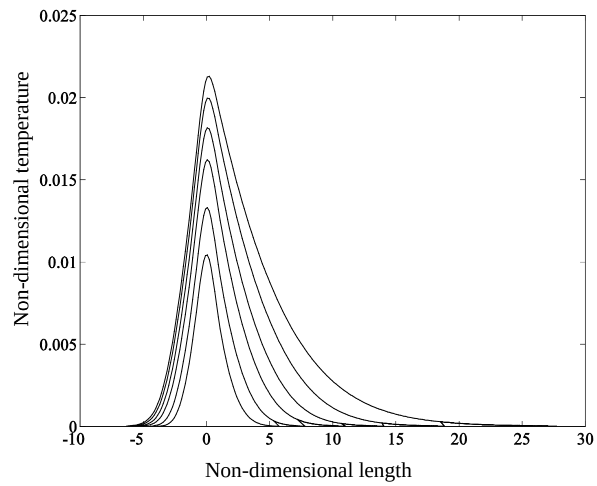


Fig 10. Non-dimensional temperature at $Pe$ = 1.5

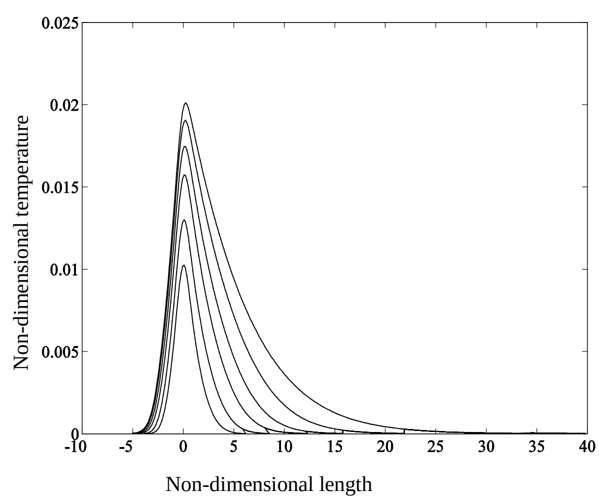


Fig 11. Non-dimensional temperature at $Pe$ = 2

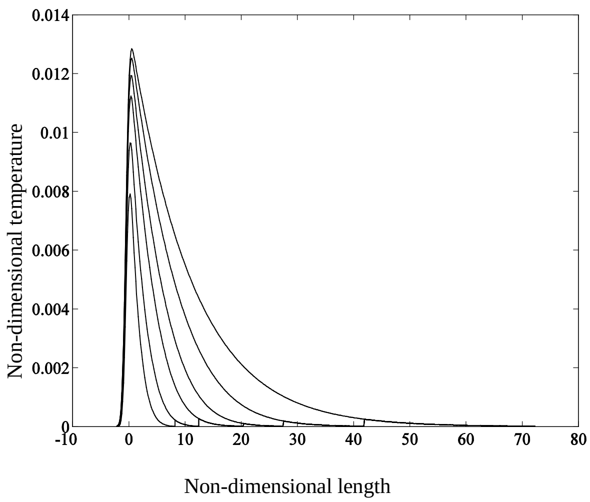


Fig 12. Non-dimensional temperature at $Pe$ = 5

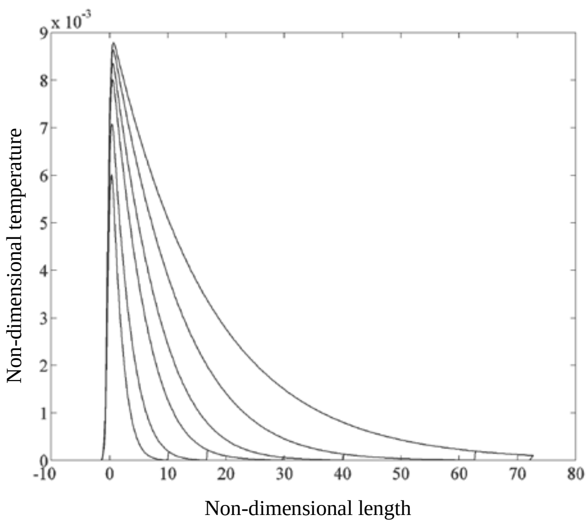


Fig 13. Non-dimensional temperature at $Pe$ = 8

These graphs shows that the temperature profile generated by laser heating of one dimensional moving system is not symmetric around the laser patch. As the $Bi$ number increases the maximum temperature of the laser heating goes down. With the increase of the $Pe$ number the effect of the wire movement start to dominate the temperature distribution. For low values of $Pe$ number the temperature distribution is almost symmetric and the heated region is localized around the laser patch. As the $Pe$ number increases or when the wire speed increases the temperature profile start to slant towards the direction of the wire movement. For the higher Peclet numbers the temperature distribution is totally on the side after laser heating. With this it can be seen that the maximum temperature which the system attains is drastically goes down. For very high $Pe$ numbers it can be seen that at the far ends of the wire the solution is not stable due to the floating point manipulation limit of the computer. This happens because of the computer interpretation of exponentials. The integral contains large positive power of exponential and large negative power of the exponential. A finite answer is reached by the multiplication of these two values.

## Conclusion

It can be seen that the temperature profile of the laser heated wire can be simply modelled as an infinitely long fin problem assuming only one dimensional heat transfer. The effect of temperature dependent thermal conductivity is prominent near the laser irradiated spot. The answer is much easier to evaluate because it is based on simple exponential terms.

## Acknowledgement

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