More on analyzing the reflection of a laser beam by a deformed highly reflective volume Bragg grating using iteration of the beam propagation method

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A further extension of the iteration method for beam propagation calculation is presented that can be applied for volume Bragg gratings (VBGs) with extremely large grating strength. A reformulation of the beam propagation formulation is presented for analyzing the reflection of a laser beam by a deformed VBG. These methods will be shown to be very accurate and efficient. A VBG with generic z-dependent distortion has been analyzed using these methods. © 2009 Optical Society of America

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1. Introduction

Volume Bragg gratings, which are a type of thick grating [1], have been very useful in high-power lasers [2–6]. In this type of Bragg grating there are basically two counterpropagating laser beams; one is the incident beam and the other is the reflected beam. The beam propagation method has been applied to analyze gratings [7,8]. The bidirectional beam propagation method may also be suitable for analyzing VBGs [9–13]. The transfer-matrix method can also calculate bidirectional wave propagation [14].

In [15] an iteration of the beam propagation method is presented and the VBG is divided into two parts. The division of the VBG into two parts solves the difficulty in the convergence of the iteration when the grating strength of the considered VBG is large. In this paper, this iteration method is further extended so that fast convergence can be achieved when the grating strength is so large that even the iteration method presented in [15] does not converge. In addition, a reformulation of the deformed VBG is presented for the calculation using the beam propagation method. In this reformulation distortions, such as background index change, grating period distortion, and laser wavelength shift, are expressed explicitly in the governing equations for both the incident and the reflected beams. For validation, an ideal VBG with very large grating strength has been analyzed and the numerical calculations were compared to analytical formula [16]. A VBG with a z-dependent background index change has also been analyzed with the z axis perpendicular to the grating planes and the results were compared to calculations using a matrix approach described in [17], in which the distorted VBG is represented by a large number of ideal gratings. These validation calculations demonstrate that the methods presented here are very accurate.

As a typical example, generic z-dependent distortion was assumed in a VBG, and it was analyzed using the reformulation and the iteration of the beam propagation method. This calculation may provide some insight on the effects of distortions in VBGs sustaining high-power lasers.

2. Iteration of the Beam Propagation Method

Considering an ideal VBG without distortion, the index distribution can be written as

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$$n = n_0 + \Delta n \cdot \cos(\vec{q}_0 \cdot \vec{r} + \varphi), \tag{1}$$

where n_0 is the background refractive index, Δn is the amplitude of the refractive index modulation, which is generally much smaller than n_0, $\vec{q}_0$ is the grating vector, which is along the z axis and $q_0 = |\vec{q}_0| = 2\pi/\Lambda_0$, where Λ_0 is the grating period of the ideal VBG, and φ is a phase factor of the cosine function. The thickness of the ideal VBG is L_0. The normal incidence of a linearly polarized laser beam that satisfies the Bragg condition is considered and, then, both the input and the reflected laser beams propagate in the z direction perpendicular to the grating planes. The paraxial wave equations for the two counterpropagating beams inside the VBG are [15]

$$\begin{aligned} 2ik_0 n_0 \frac{\partial A}{\partial z} &= k_0^2 n_0 \Delta n \cdot B \cdot e^{i\varphi} + \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}, \\ -2ik_0 n_0 \frac{\partial B}{\partial z} &= k_0^2 n_0 \Delta n \cdot A \cdot e^{-i\varphi} + \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2}, \end{aligned} \tag{2}$$

where A is the slow amplitude of the incident beam, B is the slow amplitude of the reflected beam, and $k_0 = \frac{\omega}{c}$ is the free space wavenumber. For a typical VBG used in high-power laser systems, the diffraction terms are negligible, which are the last two terms (the second-order differentiations with respect to x and y) on the right-hand side in Eqs. (2). The negligibility of the diffraction terms was also ever used in analyzing laser propagation in nonlinear optical media [18]. As a result, Eqs. (2) become

$$\begin{aligned} 2ik_0 n_0 \frac{\partial A}{\partial z} &= k_0^2 n_0 \Delta n \cdot B \cdot e^{i\varphi}, \\ -2ik_0 n_0 \frac{\partial B}{\partial z} &= k_0^2 n_0 \Delta n \cdot A \cdot e^{-i\varphi}. \end{aligned} \tag{3}$$

These two equations can be numerically solved by the fourth-order Runge–Kutta method. Here a measure of the grating strength is defined as $S = (k_0 \Delta n/2) L_0$, which appeared in the coupled-wave theory of Kogelnik for Bragg diffraction [16] and was ever used in [1,15,17]. For grating strength S smaller than $\pi/2$, the two Eqs. (3) are solved separately in sequence, and are iterated until the solutions of A and B converge [15]. This simple iteration method was initially for solving the two beam propagation equations for the two counterpropagating laser beams in a laser gain medium [19,20]. However, when applied for analyzing a VBG with S larger than $\pi/2$, this simple iteration method does not converge. To overcome this convergence difficulty when S is larger than $\pi/2$, it is proposed in [15] to divide the VBG into two parts with equal thickness and the iteration is performed in both of these two parts. When dividing the VBG into two equal parts, the converged solution of the two Eqs. (3) can be obtained quickly for S as large as about 3 [15].

However, when S is larger than π, this two-part division method does not converge anymore. To overcome this difficulty, a four-part division method is proposed that is essentially a nest of the two-part division presented in [15]. Shown in Fig. 1 is the schematic of this four-part division. The VBG is divided into four parts with the same thickness along the z axis. The slow amplitude of the laser beams are labeled A_1 and B_1 in part 1, A_2 and B_2 in part 2, A_3 and B_3 in part 3, and A_4 and B_4 in part 4. The iteration procedure is as follows:

1. Perform the iteration for part 1 and part 2 in the same way as the iteration for the two-part division as described in [15]; stop the calculation when the solutions in part 1 and part 2 converge to the desired accuracy. Save this version of A_1, B_1, A_2, and B_2.
2. Perform the iteration for part 3 and part 4 in the same way as the iteration for the two-part division, as described in [15]; stop the calculation when the solutions in part 3 and part 4 converge to the desired accuracy. Save this version of A_3, B_3, A_4, and B_4.
3. Repeat steps 1 and 2 in sequence until the solutions of A_1, B_1, A_2, B_2, A_3, B_3, A_4, and B_4 all converge.

The following boundary conditions have to be used when performing these iterations:

$$\begin{aligned} &A_1(z=0) = A_{\text{input}}, \qquad B_2\left(z=\frac{L_0}{2}\right) = B_3\left(z=\frac{L_0}{2}\right), \\ &A_3\left(z=\frac{L_0}{2}\right) = A_2\left(z=\frac{L_0}{2}\right), \quad B_4(z=L_0)=0, \end{aligned} \tag{4}$$

where $z = L_0/2$ corresponds to the interface between part 2 and part 3.

Using this four-part division and the iteration method described above, the converged solution of Eqs. (3) can be obtained quickly for S as large as about 4, and the numerically calculated result agrees very well with the analytical solution for an ideal VBG without distortion [16].

In addition, the VBG can be further divided into eight equal parts if S gets even larger and the iteration can be further extended to obtain the converged solution.

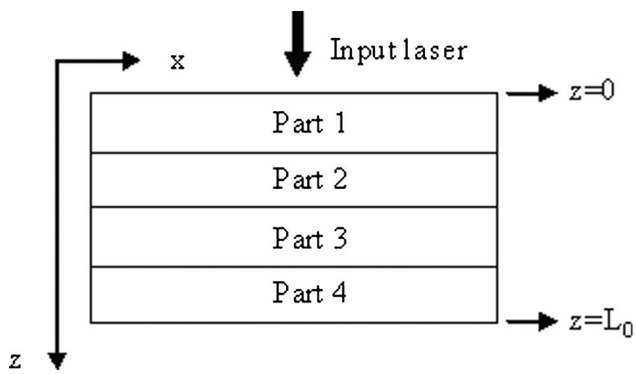

Fig. 1. Schematic of the four-part division of a VBG.

3. Reformulation for Distorted VBG

When applied in a high-power laser system, the VBG may not be an ideal one; instead, there may be some spatially varying distortions in the background index and the grating period. The background index variation could also have significant effects in fiber phase gratings [21]. With these distortions, the index distribution in the VBG can be written as

$$n = n_0 + \Delta n \cdot \cos[(\vec{q}_0 + \Delta\vec{q}) \cdot \vec{r} + \varphi] + \Delta n_T, \quad (5)$$

where $\Delta\vec{q}$ represents the grating period distortion and $|\Delta\vec{q}| \ll |\vec{q}_0|$, Δn_T is the background index change, and $\Delta n_T \ll n_0$.

In [15], a formulation is presented to include the grating period distortion and the background index distortion that might be induced by the nonuniform temperature distribution. Here, the formulation in [15] is reformulated so that the grating period change, background index change, and wavelength shift are each expressed in the governing paraxial wave equations separately and explicitly.

A normally incident linearly polarized laser beam is considered, which means that it is propagating along the z axis. Start with the following scalar wave equation:

$$\nabla^2 E + (k_0 + \Delta k)^2 \varepsilon_r E = 0, \quad (6)$$

where Δk is due to the deviation of the wavelength of the input laser beam and

$$E = A \cdot e^{-ik_0 n_0 \vec{e}_A \cdot \vec{r}} + B \cdot e^{-ik_0 n_0 \vec{e}_B \cdot \vec{r}}. \quad (7)$$

Here, n_0, the background index without distortion, is used as the reference index in the beam propagation method formulation and the slowly varying envelope approximation is valid since all the typical distortions are essentially small perturbations. A is the slow amplitude of the input laser beam, B is the slow amplitude of the reflected laser beam, $\vec{e}_A$ is the unit vector in the direction of propagation of the input beam, and $\vec{e}_B$ is the unit vector in the direction of propagation of the reflected beam. Since normal incidence is considered, $\vec{e}_A$ is in the positive z direction and $\vec{e}_B$ is in the negative z direction.

The input laser beam satisfies the Bragg condition for $\Delta k = 0$, $\Delta n_T = 0$, and $\Delta\vec{q} = 0$ when the VBG is not distorted and the laser wavelength is not shifted. Therefore,

$$k_0 n_0 \vec{e}_A + \vec{q}_0 = k_0 n_0 \vec{e}_B, \quad (8)$$

where $\vec{q}_0$ is the grating vector when the grating is not distorted. Inserting Eqs. (5) and (7) into Eq. (6) results in

$$\begin{aligned}
&\nabla^2[A \cdot e^{-ik_0 n_0 \vec{e}_A \cdot \vec{r}} + B \cdot e^{-ik_0 n_0 \vec{e}_B \cdot \vec{r}}] \\
&\quad + (k_0 + \Delta k)^2 (n_0 + \Delta n_T)^2 \\
&\quad \cdot [A \cdot e^{-ik_0 n_0 \vec{e}_A \cdot \vec{r}} + B \cdot e^{-ik_0 n_0 \vec{e}_B \cdot \vec{r}}] \\
&\quad + (k_0 + \Delta k)^2 (n_0 + \Delta n_T) \Delta n \\
&\quad \cdot [A \cdot e^{-i(k_0 n_0 \vec{e}_A + \vec{q}_0 + \Delta\vec{q}) \cdot \vec{r} - i\varphi} + A \cdot e^{-i(k_0 n_0 \vec{e}_A - \vec{q}_0 - \Delta\vec{q}) \cdot \vec{r} + i\varphi} \\
&\quad + B \cdot e^{-i(k_0 n_0 \vec{e}_B + \vec{q}_0 + \Delta\vec{q}) \cdot \vec{r} - i\varphi} + B \cdot e^{-i(k_0 n_0 \vec{e}_B - \vec{q}_0 - \Delta\vec{q}) \cdot \vec{r} + i\varphi}] \\
&\quad = 0. \qquad (9)
\end{aligned}$$

Since $|\Delta\vec{q}| \ll |\vec{q}_0|$, the terms far from satisfying the Bragg condition can be neglected. Therefore, Eq. (9) becomes

$$\begin{aligned}
&\nabla^2[A \cdot e^{-ik_0 n_0 \vec{e}_A \cdot \vec{r}} + B \cdot e^{-ik_0 n_0 \vec{e}_B \cdot \vec{r}}] \\
&\quad + (k_0 + \Delta k)^2 (n_0 + \Delta n_T)^2 \\
&\quad \cdot [A \cdot e^{-ik_0 n_0 \vec{e}_A \cdot \vec{r}} + B \cdot e^{-ik_0 n_0 \vec{e}_B \cdot \vec{r}}] \\
&\quad + (k_0 + \Delta k)^2 (n_0 + \Delta n_T) \Delta n \\
&\quad \cdot [A \cdot e^{-i(k_0 n_0 \vec{e}_A + \vec{q}_0 + \Delta\vec{q}) \cdot \vec{r} - i\varphi} + B \cdot e^{-i(k_0 n_0 \vec{e}_B - \vec{q}_0 - \Delta\vec{q}) \cdot \vec{r} + i\varphi}] \\
&\quad = 0. \qquad (10)
\end{aligned}$$

Applying Eq. (8) in Eq. (10) results in

$$\begin{aligned}
&\nabla^2[A \cdot e^{-ik_0 n_0 \vec{e}_A \cdot \vec{r}} + B \cdot e^{-ik_0 n_0 \vec{e}_B \cdot \vec{r}}] \\
&\quad + (k_0 + \Delta k)^2 (n_0 + \Delta n_T)^2 \\
&\quad \cdot [A \cdot e^{-ik_0 n_0 \vec{e}_A \cdot \vec{r}} + B \cdot e^{-ik_0 n_0 \vec{e}_B \cdot \vec{r}}] \\
&\quad + (k_0 + \Delta k)^2 (n_0 + \Delta n_T) \Delta n \\
&\quad \cdot [A \cdot e^{-i(k_0 n_0 \vec{e}_B + \Delta\vec{q}) \cdot \vec{r} - i\varphi} + B \cdot e^{-i(k_0 n_0 \vec{e}_A - \Delta\vec{q}) \cdot \vec{r} + i\varphi}] \\
&\quad = 0. \qquad (11)
\end{aligned}$$

Considering normal incidence and applying the slowly varying envelope approximation to Eq. (11), the following paraxial wave equations were obtained:

$$\begin{aligned}
2ik_0 n_0 \frac{\partial A}{\partial z} &= (k_0 + \Delta k)^2 (n_0 + \Delta n_T) \Delta n \cdot B \cdot e^{-i\Delta q z + i\varphi} \\
&\quad + [(k_0 + \Delta k)^2 (n_0 + \Delta n_T)^2 - k_0^2 n_0^2] A + \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}, \\
-2ik_0 n_0 \frac{\partial B}{\partial z} &= (k_0 + \Delta k)^2 (n_0 + \Delta n_T) \Delta n \cdot A \cdot e^{i\Delta q z - i\varphi} \\
&\quad + [(k_0 + \Delta k)^2 (n_0 + \Delta n_T)^2 - k_0^2 n_0^2] B + \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2}. \qquad (12)
\end{aligned}$$

Here, $\Delta q = |\Delta\vec{q}|$. Again, for a typical VBG used in high-power laser systems, the diffraction terms are negligible, which are the last two terms on the right-hand side in the two Eqs. (12). As a result, Eqs. (12) become

$$
\begin{aligned}
2ik_0n_0\frac{\partial A}{\partial z} &= (k_0+\Delta k)^2(n_0+\Delta n_T)\Delta n\cdot B\cdot e^{-i\Delta qz+i\varphi} \\
&\quad + [(k_0+\Delta k)^2(n_0+\Delta n_T)^2 - k_0^2n_0^2]A, \\
-2ik_0n_0\frac{\partial B}{\partial z} &= (k_0+\Delta k)^2(n_0+\Delta n_T)\Delta n\cdot A\cdot e^{i\Delta qz-i\varphi} \\
&\quad + [(k_0+\Delta k)^2(n_0+\Delta n_T)^2 - k_0^2n_0^2]B.
\end{aligned} \tag{13}
$$

The diffraction terms may not be negligible in fiber phase gratings, and then, Eqs. (12) instead of Eqs. (13) have to be solved if scalar formulation is good enough. Since we are dealing with VBG here, Eqs. (13) will be used as the governing equations, which will be solved by the fourth-order Runge–Kutta method and the iteration method discussed in Section 2.

In Eqs. (13), the grating period distortion Δq, background index distortion Δn_T, and Δk induced by the laser wavelength shift are expressed separately and explicitly. In addition, Δq and Δn_T could vary with x, y, and z.

4. Validations

To validate the iteration of the beam propagation method and the reformulation for the distorted VBGs, several example calculations were conducted.

First, the normal reflection of a plane wave by an ideal VBG was analyzed. The wavelength satisfying the Bragg condition is $\lambda = 1.064\,\mu$m, $n_0 = 1.5$, $\Delta n \cong 4.52\times10^{-4}$, $L_0 \cong 2.623$ mm, and $\varphi = 0$. For this VBG the grating strength $S = (k_0\Delta n/2)L_0 \cong 3.5$. Shown in Fig. 2 is the calculated intensity reflection versus deviation from the Bragg wavelength, together with the analytic calculation using coupled-wave theory [16]. From Fig. 2, it can be seen that the results obtained by the iteration of the beam propagation method described here agree very well with the coupled-wave theory.

In the second example calculation, the normal reflection of a plane wave was analyzed by the same VBG, except that a z-dependent background index change was added, which can be written as $\Delta n_T = (5\times10^{-4})\cdot(2z/L_0-1)^2$. Shown in Fig. 3 is the calculated intensity reflection versus deviation from $\lambda = 1.064\,\mu$m, together with the calculation using the matrix method described in [17]. It can be seen that the calculation using the iteration of beam propagation method agrees very well with the calculation using the method described in [17].

5. Modeling a VBG with Generic z-Dependent Distortion

To get some sense of how the distortion in a VBG could change its performance, a VBG with a z-dependent grating period distortion is considered. As pointed out in [2], in a VBG sustaining a high-power laser, the temperature could vary in the z direction in addition to varying in the x and y directions. Therefore, the distortion of the VBG could

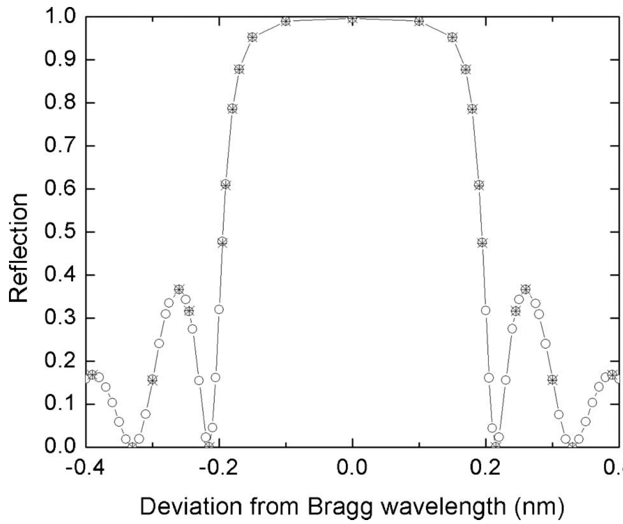

Fig. 2. Calculated intensity reflection (stars) versus deviation from the Bragg wavelength, together with the analytic calculation (open circles) using coupled-wave theory [16].

also vary in the z direction in addition to the x and y directions.

Note that the example calculation presented in this section is only for getting some sense of how the distortion in a VBG could change its performance. Even though the distortions are only assumptions, we hope that the calculation may provide some insight on the effects of distortions in VBGs sustaining high-power lasers.

The normal reflection of a TEM_{00} Gaussian beam by the distorted VBG is considered. The Gaussian beam radius at the entrance surface $w_0 = 3.5$ mm and its intensity pattern is expressed as $I = I_0\cdot\exp(-2r^2/w_0^2)$, its laser wavelength is $\lambda = 1.064\,\mu$m, the grating strength is $S \cong 1.85$ when the VBG is not distorted, background refractive index $n_0 = 1.5$, $\Delta n \cong 2.389\times10^{-4}$, the thickness of

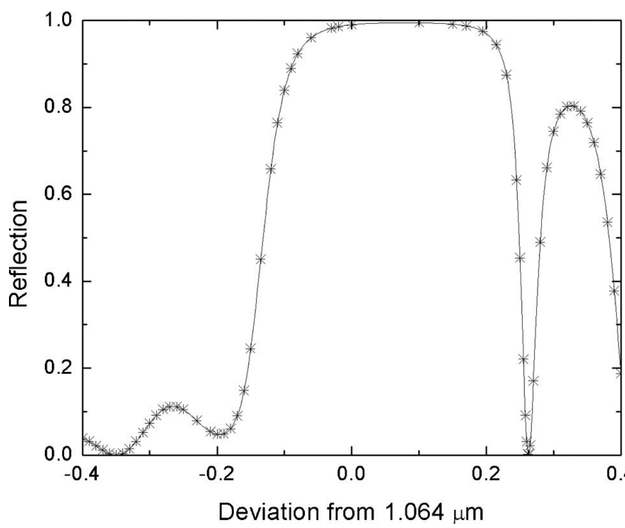

Fig. 3. Calculated intensity reflection (stars) versus deviation from $\lambda = 1.064\,\mu$m, together with the calculation using the matrix method described in [17] (solid curve).

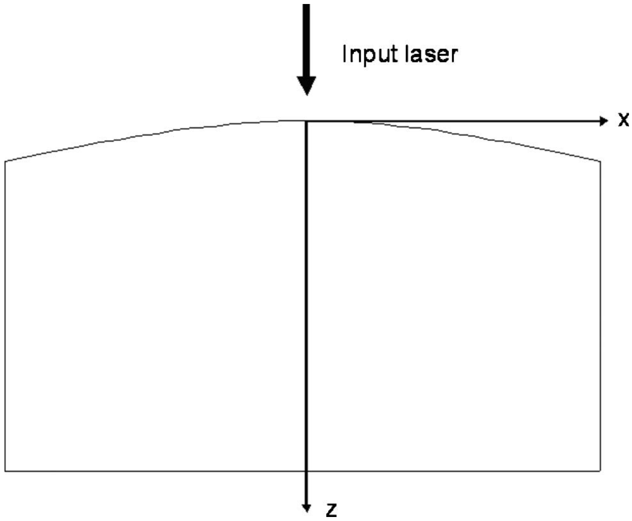

Fig. 4. Schematic of the considered VBG with asymmetric distortion.

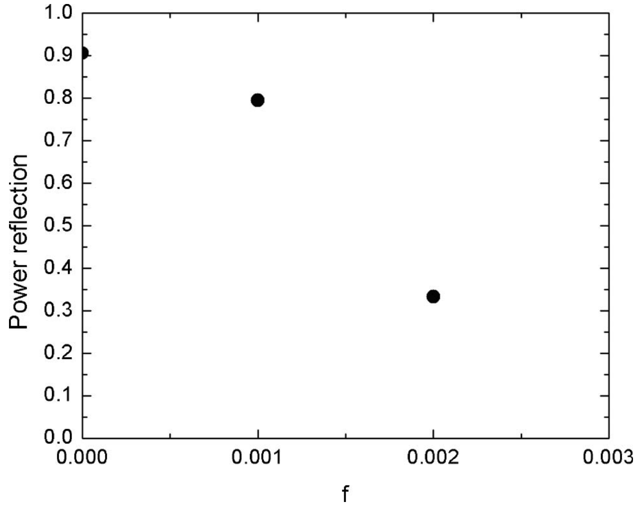

Fig. 5. Calculated power reflection as a function of f.

the grating $L_0 \cong 2.623\,\text{mm}$ when not distorted, and the normally incident laser beam is assumed to satisfy the Bragg condition when the grating is not distorted.

It is assumed that the distortion is asymmetric in the z direction, as shown in the schematic in Fig. 4. The distortion close to the input surface is significant, while there is almost no distortion close to the other surface. A generic shape of the grating thickness distortion is assumed, which is expressed as

$$L(x, y) = L_0 + \Delta L \cdot e^{-\frac{r^2}{w_0^2}}, \qquad (14)$$

where $\Delta L = 0.6\,\mu\text{m}$ is the thickness change in the center where $r = 0$ and w_0 is the same as the Gaussian beam radius. The grating period distortion is assumed to be

$$\Delta q(x, y, z) = -f \cdot q_0 \cdot e^{-\frac{r^2}{w_0^2}} \cdot e^{-\frac{4z}{L_0}}, \qquad (15)$$

where $q_0 = 2\pi/\Lambda_0$, with Λ_0 as the grating period of the ideal VBG when the VBG is not distorted, and f is a dimensionless small positive number. The surface deformation is included in the calculation in a similar way as in [15].

Shown in Fig. 5 is the plot of the calculated power reflection as a function of f. It can be seen that the power reflection is significantly reduced for $f \sim 0.002$.

6. Conclusion

An extension of the iteration of the beam propagation method is proposed, which makes it possible to analyze VBGs with extremely large grating strength. A reformulation of the paraxial wave equations is presented for the laser propagation in a distorted VBG. In this reformulation, distortions such as grating period distortion, background index change, and laser wavelength shift, are expressed explicitly and separately. These methods have been validated to be very accurate and efficient. A VBG with generic z-dependent distortion has been analyzed, which may provide some reasonable insight on the performance change induced by distortions in VBGs sustaining high power laser beams.

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