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SUSY-inspired one-dimensional transformation optics

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Transformation optics aims to identify artificial materials and structures with desired electromagnetic properties by means of pertinent coordinate transformations. In general, such schemes are meant to appropriately tailor the constitutive parameters of metamaterials in order to control the trajectory of light in two and three dimensions. Here, we introduce a new class of one-dimensional optical transformations that exploits the mathematical framework of supersymmetry (SUSY). This systematic approach can be utilized to synthesize photonic configurations with identical reflection and transmission characteristics, down to the phase, for all incident angles, thus rendering them perfectly indistinguishable to an external observer. Along these lines, low-contrast dielectric arrangements can be designed to fully mimic the behavior of a given high-contrast structure that would have been otherwise beyond the reach of available materials and existing fabrication techniques. Similar strategies can also be adopted to replace negative-permittivity domains, thus averting unwanted optical losses.

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1. INTRODUCTION

The problem of reconstructing the shape of a potential distribution solely from information carried by its scattering pattern has a long-standing history in a number of diverse disciplines of science and technology [1–5]. Naturally, in such inverse problems, the question of uniqueness is of crucial importance: are the properties of an object fully determined by its corresponding far-field scattering data? In general, the answer is no. In quantum mechanical settings, for example, one can always identify an N-parameter family of different potentials that support the same discrete set of N bound-state eigenvalues and exhibit similar scattering characteristics [4]. Closely related to this subject is the idea of supersymmetry (SUSY) [6–10]. This mathematical framework emerged in quantum field theory as a means to treat fermions and bosons on equal footing [9]. Subsequently, notions of SUSY were utilized to obtain isospectral and phase-equivalent potentials within the context of nonrelativistic quantum mechanics [10].

On the other hand, recent developments in transformation optics and optical conformal mapping have brought about novel methodologies to address inverse problems [11–16]. By virtue of coordinate transformations, such schemes can in principle provide the spatial distribution of electric permittivities and magnetic permeabilities that would perform a desired task such as cloaking [17–22]. As one would expect, the material properties required to implement such configurations might not always be available in practice. Clearly of interest would be to develop alternative strategies that allow one to judiciously control the scattering properties of an object, while at the same time reducing the complexity of the structures involved.

As we will see, SUSY can provide a new avenue for 1D transformation optics that would have been otherwise impossible using other multidimensional approaches (see Fig. 1). Along these lines, we introduce appropriate optical transformations in 1D refractive index landscapes and explore their implications in terms of their far-field response. In addition

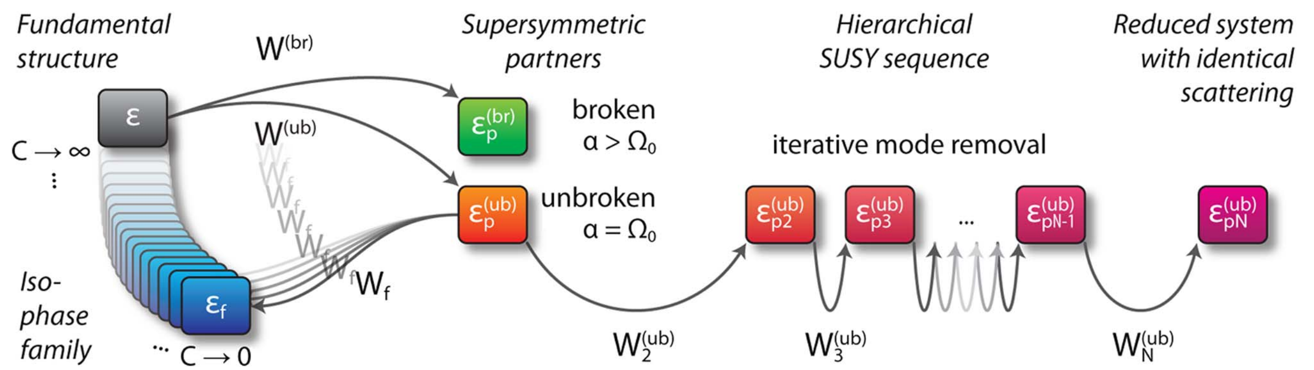

Fig. 1. Schematic overview of the different SUSY optical transformations. Starting from a given fundamental structure ϵ, supersymmetric partners ϵ_p can be constructed. Whereas the broken SUSY system $\epsilon_p^{(br)}$ preserves all bound modes, unbroken SUSY ($\epsilon_p^{(ub)}$) removes the fundamental mode. Regardless, in both cases, the intensity reflection and transmission coefficients of the superpartners are identical to those of the fundamental system. In order to maintain the full complex scattering characteristics, a family ϵ_f of isophase structures can be synthesized. Finally, a hierarchical sequence of higher-order superpartners $\epsilon_{p,2\ldots N}^{(ub)}$ may be utilized to obtain a scattering-equivalent structure, which requires a substantially lower refractive index contrast than that involved in the original system ϵ.

to finding superpartners with similar scattering behavior, systematic SUSY deformations allow us to design systems that exhibit identical complex reflection and transmission coefficients for all incident angles. As a result, two such dielectric objects, however dissimilar, become virtually indistinguishable. Remarkably, the proposed formalism can be employed to synthesize photonic configurations that behave in exactly the same way as high-refractive-index-contrast devices, by only utilizing low-contrast dielectric media. Similar methodologies can be employed to substitute negative-permittivity inclusions with purely dielectric media as a means to obtain the intended functionality without introducing any additional loss.

2. SUPERSYMMETRIC OPTICAL TRANSFORMATIONS

In 1D inhomogeneous settings, the propagation of TE-polarized waves is known to obey the Helmholtz equation [23], $[\partial_{xx} + \partial_{yy} + k_0^2\epsilon(x)]E_z(x,y) = 0$, where k_0 is the vacuum wavenumber and $\epsilon(x)$ is the relative permittivity of a given (fundamental) structure to be emulated via SUSY transformations (see Fig. 1). The analysis of TM waves can be carried out in a similar manner (see Supplement 1). In the TE case, the spatial dependence of the electric field E_z can be described via $E_z(x,y) = \psi(x)e^{i\beta y}$. Here, $\beta = k_0 n_0 \sin\theta$ represents the y component of the wave vector for an incidence angle θ, and $n_0 = \sqrt{\epsilon(X \to \pm\infty)}$ is the background refractive index. By employing the normalized quantities $X = k_0 x$, $Y = k_0 y$, and $\Omega = \beta^2/k_0^2$, the Helmholtz equation then reduces to a 1D Schrödinger-like equation:

$$H\psi(X) = \Omega\psi(X). \quad (1)$$

The resulting Hamiltonian $H = \partial_{XX} + \epsilon(X)$ can be factorized as $H = BA + \alpha$, where the operators A and B are defined as $A = \partial_X + W(X)$, $B = \partial_X - W(X)$, and α is an auxiliary constant of the problem. Here $B = -A^\dagger$, where "†" represents the Hermitian conjugate. The superpotential W can then be obtained as a solution of the Riccati equation [7],

$$\epsilon(X) = +W' - W^2 + \alpha, \quad (2)$$

in terms of the fundamental permittivity profile $\epsilon(X)$. Once W has been determined, one can establish a partner Hamiltonian $H_p = AB + \alpha$, which corresponds to a new distribution in the electric permittivity:

$$\epsilon_p(X) = -W' - W^2 + \alpha. \quad (3)$$

As a direct consequence of this construction, the modes ψ_p of the partner potential ϵ_p are related [24] to the ones of the fundamental through the expressions $\psi_p \propto (\partial_X + W)\psi$ and $\psi \propto (\partial_X - W)\psi_p$. These latter relations hold for guided waves as well as for radiation modes, and each such pair of states is characterized by a common eigenvalue. We note that two options for choosing α exist: (a) Assuming that the structure supports at least one bound state, one may opt to set α equal to the fundamental mode's eigenvalue, i.e., $\alpha = \Omega_0$. (b) The other possibility is to choose $\alpha > \Omega_0$, irrespective of whether the system supports bound states or not. The first case corresponds to an unbroken SUSY: the two potentials share the guided wave eigenvalue spectra except for that of the fundamental mode, which does not have a corresponding state in the partner. In the second case, however, SUSY is broken, and the two arrangements share an identical eigenvalue spectrum, including that of the fundamental mode. As an example, Fig. 2(a) depicts the relative permittivity distribution $\epsilon(X) = 1 + \exp[-(X/5)^8]$, corresponding to a step-index-like waveguide; its unbroken and broken SUSY partners are shown in Figs. 2(b) and 2(c), respectively. W can also be found analytically [7] via

$$W = -\partial_X \ln(\psi_0) \quad (4)$$

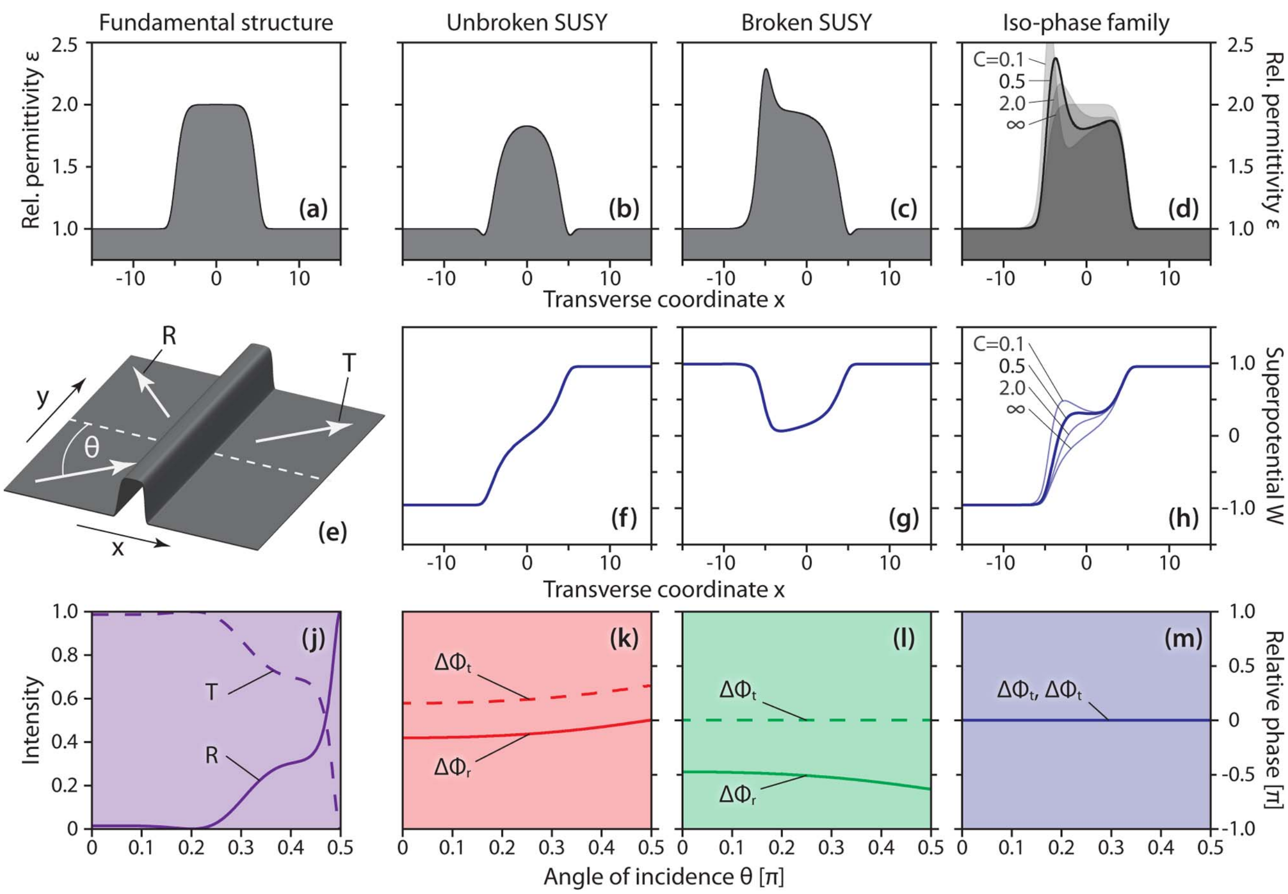

Fig. 2. Relative permittivity distributions of the original and the transformed potentials. (a) The fundamental system has a step-like profile $\epsilon(X) = 1 + \exp[-(X/5)^8]$. (b) Superpartner in the unbroken SUSY regime. (c) Superpartner in the broken SUSY case. (d) Phase-equivalent structures. (e) Scattering geometry. (f–h) Superpotentials W corresponding to panels (b–d). (j) Identical reflectivity R (solid line) and transmittivity T (dashed line) corresponding to Figs. 1(a)–1(d). (k–m) Relative phases of the reflection ($\Delta\Phi_r$, solid line) and transmission ($\Delta\Phi_t$, dashed) coefficients of the structures in (b–d) compared to the fundamental system (a) as a function of the incident angle θ. The scattering characteristics were evaluated by means of the differential transfer matrix method [23].

in the unbroken SUSY case, i.e., for $\alpha = \Omega_0$, when ϵ supports at least one bound state ψ_0. In either regime, Eq. (2) can always be solved numerically to obtain the superpotential W. An alternative approach is to start with an arbitrary superpotential and construct the two superpartner structures ϵ and ϵ_p according to Eqs. (2) and (3). In this scenario, it still remains to be determined whether SUSY is unbroken or broken. This question can be resolved by the so-called Witten index [6]. In general, if $W(X)$ approaches $W_\pm$ at $X \to \pm\infty$, unbroken SUSY requires $W_+ = -W_-$, while a broken SUSY demands that $W_+ = W_-$.

3. ISOPHASE FAMILIES OF OPTICAL POTENTIALS

It is important to note that more than one superpotential can exist for any given distribution $\epsilon(X)$. In fact, as we show here, one can systematically generate an entire parametric family W_f of viable superpotentials that satisfy Eq. (3). To show this, let us start from Eq. (3), which relates the superpartner ϵ_p to the superpotential W. Starting from a particular W, this solution can be generalized by adopting the form $W_f = W + 1/v$, in which case the unknown function v satisfies $(\partial_X - 2W)v = 1$. Direct integration readily leads to $v = e^{+2\int_{-\infty}^{X} W \mathrm{d}X'}\left(C + \int_{-\infty}^{X} e^{-2\int_{-\infty}^{X'} W \mathrm{d}X''}\mathrm{d}X'\right)$, where C is an arbitrary real-valued constant, giving rise to a parametric family W_f of superpotentials $W_f(X;C) = W + \partial_X \ln\left(C + \int_{-\infty}^{X} e^{-2\int_{-\infty}^{X'} W \mathrm{d}X''}\mathrm{d}X'\right)$. If the superpotential W has been specifically obtained from the bound state ψ_0 [from Eq. (4)], then this parametric family can be obtained via

$$W_f(X;C) = W + \partial_X \ln\left(C + \int_{-\infty}^{X} \psi_0^2(X')\mathrm{d}X'\right). \quad \textbf{(5)}$$

Whereas all members of this family lead to the same superpartner ϵ_p, each of them describes a different permittivity distribution ϵ according to Eq. (2). The resulting parametric family [10] of structures $\epsilon_f(X;C)$ is associated with the fundamental distribution ϵ and its ground state ψ_0 as follows:

Table 1. Reflection and Transmission Coefficients for the Different SUSY Transformations[a]

Coefficient	Unbroken SUSY	Broken SUSY	Isophase
Reflection	$r_p = r \cdot \exp\left[-2i \tan^{-1}\left(\frac{n_0 \cos\theta}{W_-}\right)\right]$	$r_p = r \cdot \exp\left[-2i \tan^{-1}\left(\frac{n_0 \cos\theta}{W_-}\right)\right]$	$r_f = r$
Transmission	$t_p = t \cdot \exp\left[-2i \tan^{-1}\left(\frac{n_0 \cos\theta}{W_-}\right)\right]$	$t_p = t$	$t_f = t$

[a] $W_- = W(-\infty)$ designates the asymptotic value of the superpotential on the left side of the structure, and r, t are the coefficients of the original structure.

$$\epsilon_f(X; C) = \epsilon(X) + 2\partial_{XX} \ln\left(C + \int_{-\infty}^{X} \psi_0^2(X')\mathrm{d}X'\right), \quad \textbf{(6)}$$

where C represents a free parameter. Note that here the transformation between the original structure and its superpartner was only used to prove Eq. (6), which itself is completely independent from the superpartner. According to this equation, simply by starting from a given potential and its ground state eigenfunction, a whole family of isospectral potentials can be established. Figure 2(d) depicts such family members for the fundamental structure ϵ shown in Fig. 2(a) when $C = 0.1$, 0.5, or 2.0, respectively. Note that the original permittivity distribution ϵ is in itself a member of this family, since $\epsilon_f \to \epsilon$ for $C \to +\infty$. All the modes ψ_f of any other member are related to its states ψ, according to $\psi_f \propto (\partial_X - W_f)(\partial_X + W)\psi$. As a result, all family members share the same guided wave characteristics, e.g., they have identical sets of eigenvalues as in the case of broken SUSY. This in turn means that an unbroken SUSY partner cannot be a part of the isospectral family associated with its fundamental structure, since the superpartner per definition lacks one guided mode. Nevertheless, SUSY optical transformations can be employed to synthesize dedicated isospectral families for any initial index landscape. Figures 2(f)–2(h) provide an overview of the

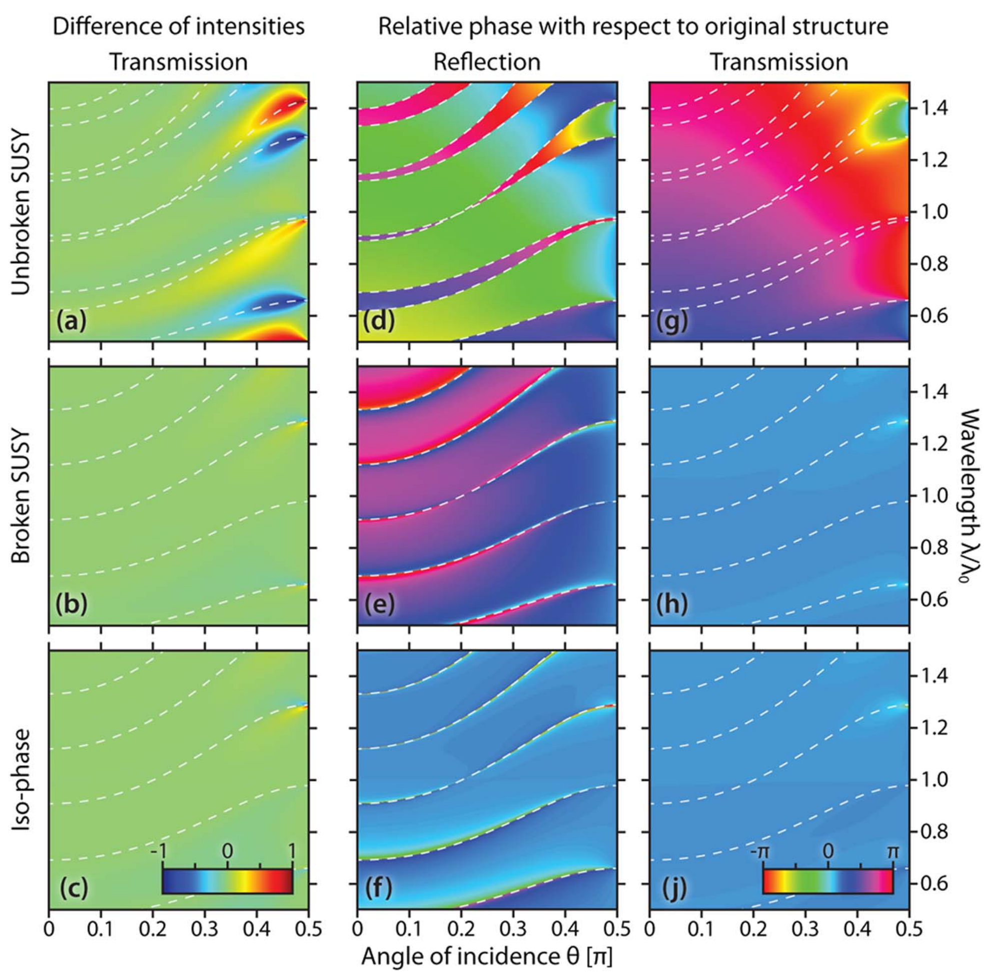

Fig. 3. Reflection/transmission characteristics of structures obtained by SUSY transformations depicted in Fig. 2 as functions of wavelength λ and angle of incidence θ. (a–c) Intensity difference in transmission. (d–f) Relative phases in reflection and (g–j) relative phases in transmission. The dashed lines follow the phase jumps of π, which originate from the interaction with guided modes in the fundamental structure and unbroken-SUSY partner. Top row, unbroken SUSY; middle row, broken SUSY; bottom row, isophase case ($C = 0.5$).

different superpotential functions in the regimes of unbroken and broken SUSY [Figs. 2(b), 2(c)], as well as for the family members shown in Fig. 2(d).

4. SCATTERING CHARACTERISTICS

Let us now turn our attention to the scattering characteristics of structures connected by SUSY transformations. Consider a plane wave $\exp(iXn_0 \cos\theta + iYn_0 \sin\theta)$ incident from the left, i.e., $X \to -\infty$, as shown schematically in Fig. 2(e). For reasons of simplicity, we assume a uniform background medium $n_+ = n_- = n_0$ at $X \to \pm\infty$; the general case of $n_+ \neq n_-$ is discussed in Supplement 1. Here, the reflected and transmitted waves in the far field are given as $r \exp(-iXn_0 \cos\theta + iYn_0 \sin\theta)$ and $t \exp(iXn_0 \cos\theta + iYn_0 \sin\theta)$ in terms of the complex reflection and transmission coefficients r and t [24]. By adopting similar solutions for the partner scatterer ϵ_p, its respective reflection and transmission coefficients r_p and t_p can readily be found. This is done by using the intervening relation $\psi_p \propto (\partial_X + W)\psi$, which connects the scattering states of the original structure to those of the superpartners. A similar approach can be followed for the isospectral configuration by utilizing $\psi_f \propto (\partial_X - W_f)(\partial_X + W)\psi$. Further details concerning this procedure can be found in Supplement 1. Table 1 summarizes the relations between the complex reflection/transmission coefficients of the original and the unbroken and broken superpartners, as well as the isophase families.

Interestingly, the SUSY transformation yields a partner structure with exactly the same absolute values in reflection and transmission, as illustrated in Fig. 2(j). Evidently, all the permittivity distributions from Figs. 2(a)–2(d) display identical reflectivities $R = |r|^2 = |r_p|^2$ and transmittivities $T = 1 - R$ for all angles of incidence. In contrast, the scattering phases depend on whether SUSY is broken or not (see Table 1). In the case of unbroken SUSY, both reflection and transmission coefficients acquire additional phases with respect to the fundamental scattering potential. If on the other hand SUSY is broken, the transmission coefficient is the same in both amplitude and phase. Finally, each member of the parametric family ϵ_f directly inherits all scattering properties of the original structure (in both intensity and phase), i.e., they are phase equivalent to ϵ. Figures 2(k)–2(m) illustrate these relations.

So far, the performance of these systems has been examined at a given operating wavelength λ_0. Of importance would be to investigate to what extent their supersymmetric properties persist when the wavelength λ varies around λ_0. As one would expect, even if two dissimilar profiles exhibit the same phases at a given wavelength, their internal light dynamics may gradually undergo different changes with λ. To elucidate this structural dispersion, we provide the spectral dependence of the difference in transmittivities ΔT (or reflectivities ΔR) between the fundamental structure [Fig. 2(a)] and its superpartners [Figs. 2(b)–2(d)] as a function of the incidence angle θ, as shown in Figs. 3(a)–3(c). As these figures indicate, this difference only becomes notable in the unbroken SUSY regime [Fig. 3(a)], while it is almost absent under broken SUSY and isophase conditions [Figs. 3(b), 3(c)]. The difference in the corresponding reflection phases is similarly presented in Figs. 3(d)–3(f). The dashed lines trace the abrupt phase jumps of π, which mark the interaction with guided modes in the two partners and intersect at the design wavelength λ_0. Evidently, the isophase design displays the greatest resilience with respect to spectral deviations. Note that such phase jumps do not occur in the transmission phases, as can be seen in Figs. 3(g)–3(j). In this latter case, the isophase system again proves to be the least susceptible to spectral deviations. These results demonstrate that SUSY transformations can be robust over a broad spectral range around the design wavelength.

5. REFRACTIVE INDEX ENGINEERING USING SUSY TRANSFORMATIONS

One of the main challenges in designing optical systems is the limited dynamic range of refractive indices associated with available materials. This issue becomes particularly acute when high-contrast arrangements are desirable. For example, the number of grating unit cells required to achieve a certain diffraction efficiency grows with the inverse logarithm of the index contrast n_2/n_1 between the individual layers [23]. As it turns out, SUSY optical transformations can be utilized to reduce the index contrast needed for a given structure. This can be done through a hierarchical ladder of superpartners, i.e., sequentially removing the bound states of the original high-contrast setting [Fig. 4(a)]. In this example, the relative permittivity in the original structure is supposed to vary between 2 and 9, leading to a considerable contrast. Evidently, this range may be difficult to implement in practice in such a wavelength-scale arrangement. On the other hand, each successive step demands less contrast in the corresponding index landscape than the previous one [Fig. 4(b)]. We would like to point out that since reflection and transmission are only relevant for propagating waves, the removal of guided modes is in no way detrimental to scattering-based functionalities. The ultimate result is a low-contrast equivalent structure that fully inherits the intensity reflectivity R and transmittivity T of the original configuration under all incident angles [Figs. 4(c)–4(d)]. For this particular example, we see an approximately four-fold decrease in the required permittivity contrast.

Finally, SUSY transformations can provide a possible avenue in replacing negative-permittivity inclusions (typically accompanied by losses) by purely dielectric materials. In this respect, inverse SUSY transformations, which now add modes with certain propagation constants to a given structure, can instead be used to locally elevate the permittivity (see Supplement 1). Along similar lines, it is possible to find superpotentials that relate a structure with metallic or negative-permittivity regions to an equivalent arrangement with entirely positive ε, as depicted in Fig. 5. Here, we make use of the fact that in a broken-SUSY transformation, the spatial average of ϵ happens to be a conserved quantity. Therefore, changes in the broader vicinity of the original metal–dielectric structure can be used to achieve this goal.

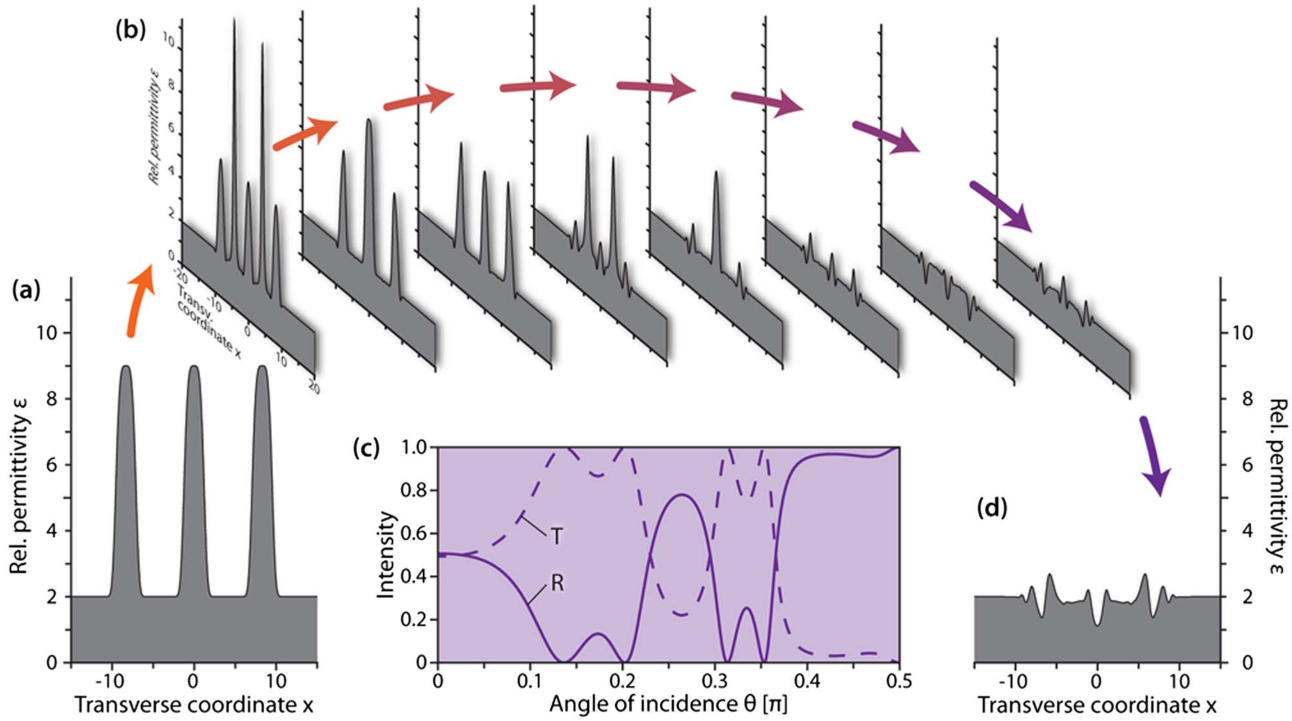

Fig. 4. (a) Hypothetical high-contrast dielectric layer arrangement that supports $N = 9$ guided modes. (b) Hierarchical sequence of partner structures obtained through iterative SUSY transformations. (c) Despite the general trend toward lower-contrast configurations, each intermediate step inherits the reflectivity and transmittivity of the fundamental system (a). (d) The resulting low-contrast structure is free of bound states and faithfully mimics the intensity-scattering characteristics of the original high-contrast configuration for all angles of incidence. Note that the transverse coordinate x scales in units of $\lambda_0/2\pi$.

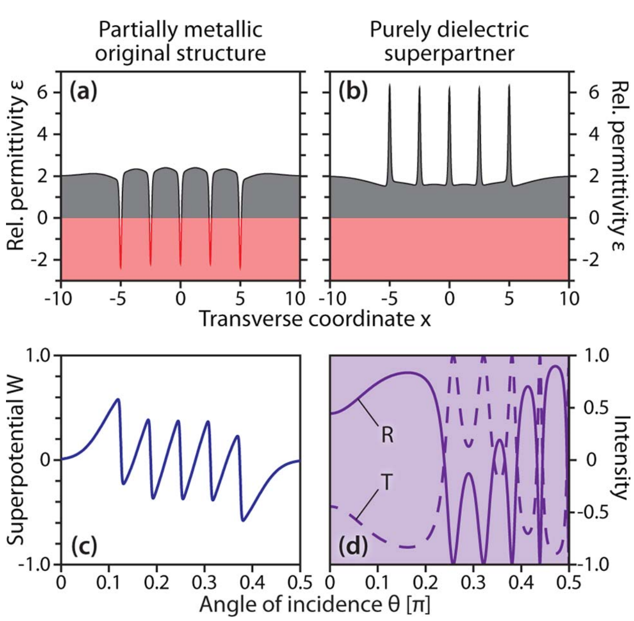

Fig. 5. (a) Metal–dielectric grating arrangement comprising five layers of negative electrical permittivity (red sections). (b) An entirely dielectric superpartner grating constructed in the broken SUSY regime, using the respective superpotential (c). (d) Despite the absence of any metallic regions, the equivalent structure exhibits identical reflectivities/transmittivities.

6. CONCLUSION

We have introduced a new type of supersymmetric optical transformation for arbitrary 1D refractive index landscapes. Compared to conventional transformation optics, our approach poses significantly less stringent requirements on the constituent parameters, and does not involve any modifications to the magnetic response of the materials involved. This method can be utilized to construct photonic arrangements that faithfully mimic the scattering characteristics of high-index-contrast or even metal–dielectric structures. We would like to emphasize that this approach is readily capable of emulating the behavior of arrangements whose permittivity distributions are far beyond the reach of naturally occurring materials. As such, SUSY transformation optics can likewise be employed to narrow the necessary range of effective parameters, and thereby complement the design process of metamaterial devices. SUSY transformation optics may have potential applications in a wide range of scenarios that rely on engineered scattering and transmission properties such as, for example, optical metasurfaces, antireflection coatings, and diffraction gratings. Of interest will be to explore how the aforementioned strategies could be paired up with recently developed transformation schemes for guided-wave photonics based on dielectric materials [25,26]. Finally, the unique characteristics of supersymmetric optical structures may open new opportunities for tailoring the response of non-Hermitian systems beyond PT symmetry [27], and could be employed for a new class of integrated optical mode converters [28].

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See Supplement 1 for supporting content.

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