Supersymmetric Laser Arrays

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Abstract: We present a new strategy for regulating light emission dynamics in high power laser arrays. Our approach is based on engineering the properties of non-Hermitian supersymmetric optical arrays and offers several advantages over previous investigations.

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Laser arrays have been a subject of intense investigations over the last two decades. In particular, potential high power output from coupled waveguide laser system was conceived as an attractive alternative to more bulky geometries. Despite this clear advantage, controlling light emission from these structures remained a hurdle. More specifically, an array made of N coupled elements will exhibit N supermodes. Under normal conditions, all these eigenmodes or at least several of them can reach the lasing threshold simultaneously [1]. This, in turn, was shown to degrade the far field emission pattern of the laser [1] and also lead to a chaotic or quasi-periodic emission dynamics [2]. Several proposals were suggested in order to overcome these problems. However, a practical solution is still largely lacking.

In this work we approach the multi-mode lasing problem from a new angle, and we offer a new alternative for solving this problem. Our strategy is based on engineering the spectral properties of the optical arrays by taking advantage of supersymmetry (SUSY). SUSY was first proposed in the context of quantum field theory as a new paradigm for unifying two different families of elementary particles, namely bosons and fermions. This concept was later translated into quantum mechanics [3]. Recently, it was also realized that optical SUSY structures can offer an unprecedented control over light transport dynamics.

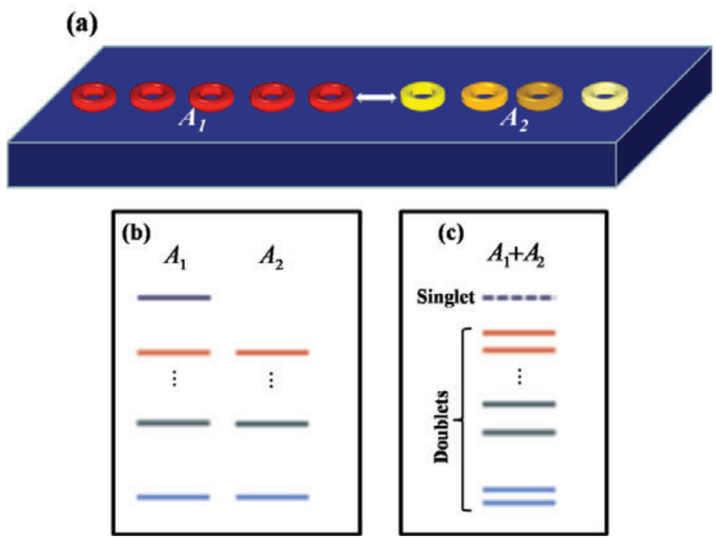

Fig. 1 (a) Supersymmetric laser array made of a main optical lattice (A_1) and its superpartner structure (A_2). (b) and (c) depict the individual spectrum of each array as well as the eigenvalue distribution of the combined structure, respectively. The dashed line in (c) indicates that the singlet state exhibits the lowest lasing threshold.

The supersymmetric laser array proposed in this work is illustrated in Fig.1 (a). It consists of a main optical array of coupled resonators A_1 and its superpartner lattice A_2 [4, 5]. The two structures are coupled together through their inner most elements. Fig.1 (b) depicts the spectra of each array in the absence of any interaction. Evidently, apart from the fundamental state A_1, the eigenvalue spectrum of both arrays is identical. The collective eigenmodes of the full structure is shown in Fig.1 (c). Clearly, the degenerate eigenvalues fuse together into doublets while the isolated fundamental mode of A_1 forms a singlet state. The coupled mode theory between the supermodes (SCMT) of both lattices can be written as:

$$i\frac{dV_{l+1}}{dt}+\omega_l V_{l+1}+\kappa_l U_l=0$$
$$i\frac{dU_l}{dt}+\omega_l U_l+i\gamma U_l+\kappa_l V_{l+1}=0 \quad , \quad (1)$$

where V_{l+1} and U_l are the modal amplitude associated with the supermodes of A_1 and A_2 respectively while ω_l is their resonant frequency. In Eq. (1), γ account for a uniform and stronger optical loss introduced only to the superpartner lattice. From Eq.(1), we find the eigenvalues of the doublets eigenmodes that vary as $\exp\left(i\Omega_l^{\pm}t\right)$ are given by: $\Omega_l^{\pm}=\left(\omega_l\pm\sqrt{\kappa_l^2-\left(\frac{\gamma}{2}\right)^2}\right)+\frac{\gamma}{2}i$. Clearly, within the CMT approximation, the imaginary part associated with the doublets is always larger than that of the singlet state. Thus, it follows that the singlet state exhibits the lowest lasing threshold. This is indicated in Fig.1 (c) by the dashed line.

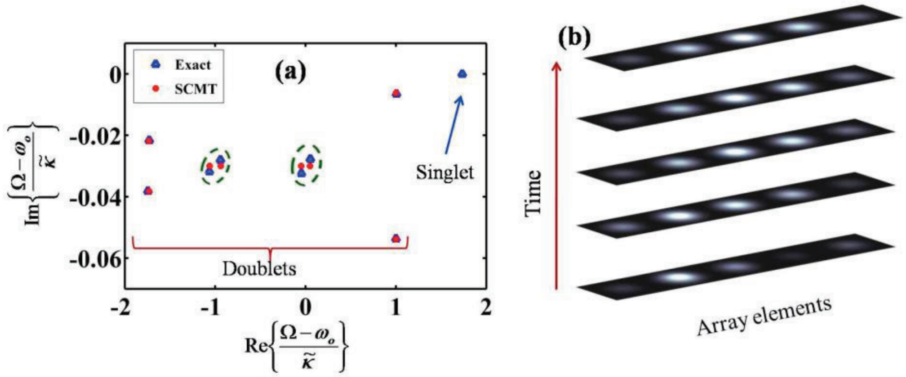

Fig.2. Eigenvalue spectrum (a) and temporal evolution (b) for the SUSY array described in the text

Figure 2 (a) illustrates the eigenvalue spectrum of a supersymmetric array made of a main array that consists of five identical resonators of resonant frequency ω_o and coupling coefficient $\tilde{\kappa}$. The inter-array coupling $\kappa'/\tilde{\kappa}=0.2$ while the extra optical absorption coefficient of the superpartner array is $\gamma/\tilde{\kappa}=0.06$. Comparison between the results of the CMT and the exact diagonalization is also indicated in the figure. Clearly, the singlet state is more immune to losses than the doublets. Figure 2 (b) depict the time dynamics of an arbitrary initial optical intensity distribution in the main array where only the fundamental mode survives after a certain time period. Interestingly, for certain design parameters, one of the doublet eigenmodes starts to lase before the singlet. We use Brillouin-Wigner perturbation theory [6] to explain this unexpected effect and we show that it arises due to the non-resonant interaction between the modes.

References

[1] E. Kapon, J. Katz, and A. Yariv, “Supermode analysis of phase-locked arrays of semiconductor lasers”, Optics Lett., **9**, 125 (1984).

[2] S.S. Wang and H.G. Winful, “Dynamics of phase-locked semiconductor laser arrays”, Appl. Phys. Lett. **52**, 1774 (1988).

[3] F. Cooper, *Supersymmetry in Quantum Mechanics*, World Scientific Pub Co Inc (July 2002).

[4] M.A. Miri, M. Heinrich, R. El-Ganainy and D.N. Christodoulides, “Supersymmetric optical structures”, Phys. Rev. Lett. **110**, 233902 (2013).

[5] M. Heinrich, M.A. Miri, S. Stützer, R. El-Ganainy, S. Nolte, A. Szameit and D.N. Christodoulides, “Supersymmetric mode converters,” Nature Comm. **5**, 3698 (2014)

[6] S. Wilson, I. Hubac, *Brillouin-Wigner Methods for Many-Body Systems*, Springer (2012).